GI ADV Model Solutions
Fall 2023

1.

Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5f) Calculate the price for a property per risk excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the annual experience rating loss cost for each year in 2019-2022.

Commentary on Question:

This part required understanding how to apply trend and development to individual layer losses. The following outlines the steps involved in the calculation that is included in the Excel solutions spreadsheet.

Step 1: Apply trend to the individual ground-up losses to get the individual trended ground-up losses.
Step 2: Apply the reinsurance terms to the individual trended ground-up losses to get the individual trended layer losses.
Step 3: Multiply the individual trended layer losses by the appropriate development factors to get the individual trended ultimate layer losses.
Step 4: Aggregate the individual trended ultimate layer losses by accident year.
Step 5: Divide the trended ultimate layer losses by the subject premium to get the loss cost for each year from 2019 to 2022.

1. Continued

(b) Calculate the revised expected loss cost for each year in 2019-2022.

Commentary on Question:
This part involved understanding that the trended ultimate layer losses need to be calculated for each scenario and then weighted by the scenario probabilities. The following outlines the steps involved in the calculation that is included in the Excel solutions spreadsheet.

For each scenario (0% increase, 10% increase, 20% increase):
Step 1: Multiply the individual trended ground-up losses (from part (a) Step 1) by the factor from each scenario (1, 1.1, 1.2) to get the individual trended ground-up losses by scenario.
Step 2: Apply the reinsurance terms to the individual trended ground-up losses to get the individual trended layer losses by scenario.
Step 3: Multiply the individual trended layer losses by the appropriate development factors to get the individual trended ultimate layer losses by scenario.
Step 4: Weight the individual trended ultimate layer losses by the scenario probabilities to get the revised individual trended ultimate layer losses.
Step 5: Aggregate the revised individual trended ultimate layer losses by accident year.
Step 6: Divide the revised trended ultimate layer losses by the subject premium to get the loss cost for each year from 2019 to 2022.

(c) Explain why using the average of all years may not be appropriate for pricing the 2024 treaty.

The loss cost is increasing by accident year, so using the average would be inappropriate. There is a need to reevaluate the model.

2. Learning Objectives:

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6c) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of the individual risk rating methods of schedule rating and experience rating.

Solution:

(a) Explain how certain features included in prospective experience rating plans promote equity among insureds regarding the determination of premiums.

Commentary on Question:

There are several different features included in prospective experience rating plans that promote equity among insureds regarding the determination of premiums. Full credit was given for providing an explanation that included at least two features. The model solution is an example of a full credit solution with two features.

- The use of credibility in the experience rating formula ensures that the insured's experience is included only to the extent that it is a reliable predictor. Additionally, the use of at least several years of experience in the experience rating formula ensures that one bad year does not have an overly large influence on the premium.

(b) Describe split rating as it pertains to the NCCI experience rating plan.

Split rating separates actual claims into primary and excess amounts. Primary claims represent an insured's frequency and excess claims represent an insured's severity.

(c) Explain why the use of prospective experience rating for an insured does not eliminate the need for schedule rating of that insured.

Commentary on Question:

There are several reasons for this. Full credit was given for providing an explanation that included at least two reasons. The model solution is an example of a full credit solution with two features.

2. Continued

Schedule rating differs in that it modifies the rates based on a subjective assessment of the insured's risk characteristics. Furthermore, risk characteristics from schedule rating adjustments may not have been in place during the experience period used for experience rating.

(d) Identify two examples of risk characteristics used in schedule rating plans.

Commentary on Question:
There are many examples to choose from. Only two were required for full credit. The model solution is an example of a full credit solution.

- Quality of police and fire protection
- Condition and upkeep of the premises and equipment

3. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) State two reasons why this is the case.

The Cape Cod method requires that fewer parameters be estimated.
The Cape Cod method uses the information in the exposure base.

(b) Calculate the maximum likelihood estimates of *ULT* for each of the four accident years.

For each row of data provided, calculate *G*(at the end of the interval). The end of the interval is "To" months minus 6. The MLE for each accident year is the accident year total divided by *G*(at the end of the interval) from the row showing the accident year valued at the end of 2022.

- ULT_{2019} = 5,000 / $G(42)$ = 5,843.894
- ULT_{2020} = 7,000 / $G(30)$ = 8,643.159
- ULT_{2021} = 6,800 / $G(18)$ = 9,433.850
- ULT_{2022} = 5,300 / $G(6)$ = 11,327.367

(c) Calculate $\hat{\sigma}^2$, the estimate of the scale factor.

Commentary on Question:

AY = Accident Year

1. For each row of data provided, calculate *G*(at the start of the interval) and *G*(at the end of the interval).
2. For each row of data provided, calculate the expected increment as the MLE of ULT_{AY} times [*G*(at the end of the interval) minus *G*(at the start of the interval)].

3. Continued

3. For each row of data provided, calculate the square of [the increment minus the expected increment] divided by the expected increment.
4. Then, the scale factor equals the sum of the amounts in step 3 divided by [the number of data points minus the number of parameters estimated]. This equals 42.198.

(d) Estimate the process standard deviation of the loss reserve for all accident years combined.

1. The total ultimate loss estimate equals $ULT_{2019} + ULT_{2020} + ULT_{2021} + ULT_{2022}$.
2. The total loss reserve estimate equals the total ultimate estimate minus the total paid.
3. The estimate of the process standard deviation of the loss reserve for all AYs combined is the square root of [the total loss reserve estimate times the estimate of the scale factor]. This equals 685.883.

(e) A likelihood ratio test indicates that $\omega = 1$ is a plausible value. Using this value and re-estimating the other parameters leads to a significant reduction in the estimated scale factor.

Explain why this reduction is to be expected.

The slight change in this parameter will lead to small changes in the MLEs of the other parameters. As a result, the numerator of the scale factor will be similar. However, the denominator will change from 4 to 5, leading to a significant reduction.

4. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1c) Identify alternative models that should be considered depending on the results of the tests.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Construct the fitted incremental triangle and compute the sum of squared errors (SSE) for each of the four alternative models. (Omit the first column from the fitted triangles and the SSE calculations.)

Commentary on Question:

AY = accident year, DY = development year, CL = chain ladder, ult = ultimate, q(w, d) = incremental loss for AY w from DY d −1 to DY d

Step 1: Create the cumulative loss triangle from the incremental loss triangle.
Step 2: Calculate the age-to-age factors triangle from the cumulative loss triangle.
Step 3: Calculate the volume-weighted average age-to-age factors for each development interval from the age-to-age factors triangle. (Mack)
Step 4: Calculate the unweighted average age-to-age factors for each development interval from the age-to-age factors triangle. (Unweighted)
Step 5: Calculate the age-to-ultimate factors at each development age for both the Mack and Unweighted age-to-age factors.
Step 6: For each of (i) to (iv), the fitted incremental triangle is calculated for DYs 2 to 8 and AYs 1 to (9 – DY).

(i) Mack CL

- The Mack CL fitted incremental triangle is calculated as the cumulative loss for AY, (DY – 1) times (Mack age-to-age factor at (DY – 1) minus 1).

4. **Continued**

(ii) Unweighted CL

- The unweighted CL fitted incremental triangle is calculated as the cumulative loss for AY, (DY – 1) times (unweighted age-to-age factor at (DY – 1) minus 1).

(iii) Additive CL

- The additive CL fitted incremental triangle is calculated as the average incremental loss for a DY.

(iv) BF

- The BF fitted incremental triangle is calculated using estimates of the lag factor f and accident year parameter h. These are obtained through an iterative procedure.
- Iterations to obtain f and h estimates are as follows:
  1. Begin with an initial estimate of f using Mack CL age-to-ultimate factors:
     $f(1)$ = 1 / 1-to-ult factor, and for j = 2 to 8
     $f(j)$ = ((j – 1)-to- j factor – 1) / (j – 1)-to-ult factor.
  2. Estimate h as follows:
     For $i = 1$ to 8, $h(i) = \sum f(j)q(i, j) / \sum f(j)^2$, $j = 1$ to $(9 - i)$.
  3. Estimate f as follows:
     For $j = 2$ to 8, $f(j) = \sum h(i)q(i, j) / \sum h(i)^2$, for $i = 1$ to $(9 - j)$ and
     $f(1) = 1 - \sum f(j)$, for $j = 2$ to 8.
  4. Repeat steps 2 and 3 until convergence occurs.
- The BF fitted incremental triangle is calculated as f(DY)h(AY) for DY > 1 and AY from 1 to (9 – DY).

For each of (i) to (iv), the SSE is the sum of the squares of the (incremental loss triangle minus the fitted incremental triangle) for DY > 1.

(b) Compute one test statistic, based on the SSE, for each model.

Commentary on Question:
The syllabus includes three test statistics based on the SSE. Candidates could compute any one of these test statistics to earn full credit. The model solution is an example of a full credit solution computing the AIC test statistic.

For each model, AIC = SSE × exp(2p/n), where n is the number of observations and p is the number of parameters in the model.

4. **Continued**

(c) Identify the best model based on the value of this test statistic for each model.

Commentary on Question:
The model solution is an example of a full credit solution based on the calculation of the AIC test statistic in part (b).

The best model is the one with the lowest test statistic, AIC.
This would be the BF method for this data set.

5. Learning Objectives:

7. The candidate will understand the application of game theory to the allocation of risk loads.

Learning Outcomes:

(7a) Allocate a risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

EL = Expected Losses, Var = Variance, SD = standard deviation,
CoV = coefficient of variation, Corr = Correlation, RL = Risk Load

Solution:

(a) Calculate the renewal risk loads for each of the three accounts (X, Y and Z) using the Marginal Surplus method.

The RL multiplier for the Marginal Surplus method is the return on marginal surplus times the z-score divided by (1 plus the return on marginal surplus).

$$SD(XYZ) = [Var(X) + Var(Y) + Var(Z) + 2Corr(X,Y)SD(X)SD(Y) + 2Corr(X,Z)SD(X)SD(Z) + 2Corr(Y,Z)SD(Y)SD(Z)]^{0.5}$$

$SD(X) = CoV(X)\times EL(X)$, $SD(Y) = CoV(Y)\times EL(Y)$, $SD(Z) = CoV(Z)\times EL(Z)$
Marginal $SD(X) = SD(XYZ) - [Var(Y) + Var(Z) + 2Corr(Y,Z)SD(Y)SD(Z)]^{0.5}$
Marginal $SD(Y) = SD(XYZ) - [Var(X) + Var(Z) + 2Corr(X,Z)SD(X)SD(Z)]^{0.5}$
Marginal $SD(Z) = SD(XYZ) - [Var(X) + Var(Y) + 2Corr(X,Y)SD(X)SD(Y)]^{0.5}$

For each account, RL = Marginal SD × RL multiplier for Marginal Surplus method.

(b) Calculate the renewal risk loads for each of the three accounts using the Marginal Variance method.

The RL multiplier for the Marginal Variance method is the RL multiplier for Marginal Surplus method divided by SD(XYZ).

Marginal $Var(X) = Var(XYZ) - [Var(Y) + Var(Z) + 2Corr(Y,Z)SD(Y)SD(Z)]$
Marginal $Var(Y) = Var(XYZ) - [Var(X) + Var(Z) + 2Corr(X,Z)SD(X)SD(Z)]$
Marginal $Var(Z) = Var(XYZ) - [Var(X) + Var(Y) + 2Corr(X,Y)SD(X)SD(Y)]$

5. Continued

For each account, RL = Marginal Var × RL multiplier for Marginal Variance method.

(c) Demonstrate that the renewal risk loads for accounts X, Y and Z, as calculated in both parts (a) and (b), are not renewal additive.

For the combined total of all accounts, the RL is 454 under both the Marginal Surplus and Marginal Variance methods.

The sum of the RLs equals 408 under the Marginal Surplus method and 637 under the Marginal Variance method. Neither equals the RL for the combined total of all accounts. As such, they are both not renewal additive.

6. Learning Objectives:

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4d) Explain retrospective rating in graphical terms.

(4e) Explain Table M and Table L construction in graphical terms.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Commentary on Question:

This question tested a candidate's knowledge of Lee's graphical approach to retrospective rating.

Solution:

Identify which of the quantities above is equal to each of the following:

(i) 1

(ii) r_1, the entry ratio associated with the minimum premium

(iii) r_2, the entry ratio associated with the maximum premium

(iv) k, the loss elimination ratio

(v) $\psi(r_1)$, the Table M savings

(vi) $\psi^*(r_1)$, the Table L savings

(vii) $\phi(r_2)$, the Table M charge

(viii) $\phi^*(r_2)$, the Table L charge

(i) area_2
(ii) area_8
(iii) area_5
(iv) area_4
(v) area_6
(vi) area_7
(vii) area_1
(viii) area_3

7. Learning Objectives:

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4g) Estimate the premium asset for retrospectively rated polices for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Discussion of Estimating the Premium Asset on Retrospectively Rated Policies, Feldblum

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

EPLE = Cumulative Expected Percentage of Loss Emerged, IEPLE = Incremental EPLE, ILCR = Incremental Loss Capping Ratio

Solution:

(a) Calculate the implied Cumulative Premium Development to Loss Development (CPDLD) ratios for the first to fourth retrospective rating adjustments using the formula approach.

$PDLD_1 = (BPF \times TM / (ELR \times EPLE_1)) + (TM \times LCF \times ILCR_1)$
For i = 2 to 4, $PDLD_i = TM \times LCF \times ILCR_i$

$IEPLE_i = EPLE_i - EPLE_{i-1}$

$$CPDLD_j = \sum_{i=j}^{4} PDLD_i \times IEPLE_i \div \sum_{i=j}^{4} IEPLE_i$$

(b) Provide two situations in which one would favor the formula approach to estimating PDLD ratios over the empirical approach assuming there is sufficient data to use the empirical approach.

1. Retro rating parameters are changing significantly over time.
2. Historical EPLEs are not indicative of current EPLEs.

7. Continued

(c) Calculate the premium asset on retrospectively rated policies as of December 31, 2022.

For each policy year:

Expected Future Loss Emergence
= Ultimate Losses – Losses Reported at Prior Retro Adjustment

Estimated Future Premium
= Expected Future Loss Emergence × CPDLD

Estimated Total Premium
= Estimated Future Premium + Premium Booked from Prior Adjustment

Premium Asset
= Estimated Total Premium − Premium Booked as of Dec. 31, 2022

8. Learning Objectives:

3. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(3a) Describe a risk margin analysis framework.

(3b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(3c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Identify two reasons that quantitative methods should not be used to assess these correlation effects.

Commentary on Question:

There are more than two reasons. The model solution is an example of a full credit solution.

- It's difficult to separate the past correlation effects between independent risk and systemic risk.
- Correlations associated with external systemic risk sources may differ materially from correlations associated with past episodes of systemic risk.

(b) Complete the following internal systemic risk balanced scorecard:

Complete in the Excel spreadsheet.

Risk Source	Risk Indicator	Score (1 or 5)	Reason for receiving the score	Weight
Parameter selection error	Ability to identify and use best predictors	5		30%
	Best predictors are stable over time	1		
Specification error				40%
Data error				30%

8. Continued

Commentary on Question:
There are many ways to complete this table and have it represent a proper solution that earns full credit, The model solution is an example of a full credit solution.

Risk Source	Risk Indicator	Score (1 or 5)	Reason for receiving the score	Weight
Parameter selection error	Ability to identify and use best predictors	5		30%
	Best predictors are stable over time	1		
Specification error	Range of results produced by models	1	Large variance between the two model results suggests great uncertainty in our ability to model. More modeling approaches may need to be considered.	40%
	Ability to model using more granular data	5	Claim level data is available and can help better understand key predictors.	
Data error	Timeliness, consistency and reliability of information from business	5	Regular communication between actuaries and the portfolio managers can ensure timeliness and reliability of information.	30%
	Data subject to appropriate reconciliation	5	The data is reconciled against another source with differences well understood.	

(c) Select the internal systemic risk CoV using the completed internal systemic risk balanced scorecard from part (b).

First calculate the weighted average score using part (b):

$$5\times(0.3/2) + 1\times(0.3/2) + 1\times(0.4/2) + 5\times(0.4/2) + 5\times(0.3/2) + 5\times(0.3/2) = 3.6$$

Looking up the weighted average score of 3.6 in the CoV scale table gives an Internal Systemic Risk CoV of 6.5%.

9. **Learning Objectives:**

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5k) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain why the "10% - 10% rule" is often not considered appropriate for determining the existence of sufficient risk transfer in a reinsurance agreement.

It does not capture risk transfer in many valid types of reinsurance, such as those with a high loss amount offsetting a low probability of loss.

(b) Define the expected reinsurer deficit (ERD) metric as used for determining the existence of sufficient risk transfer in a reinsurance agreement.

The ERD measure is derived from the probability distribution of net economic outcomes. ERD = pT/P where p = probability of net income loss, T = average severity of net economic loss when it occurs, and P = expected premium.

(c) Determine the reinsurance premium. [Using Excel's Goal Seek function is an acceptable method for determining this amount.]

Commentary on Question:

The model solution was set up to be solved using Excel's Goal Seek function. However, it was also acceptable to solve this problem using trial and error by changing P such that it provides an ERD of 5%.

Amounts in millions.
Min = minimum of, Max = maximum of, RD = reinsurer deficit

9. Continued

For each row in the table, we need to calculate the following:

- Layer Loss = Min[800 and Max[0 and (UVW Direct Loss minus 200)]]
- Reinsured Amount = Layer Loss times 0.85
- PV of RD = Max[0 and − (P − Reinsured Amount/(1.035^3))]
- Probability of RD = Probability if PV of RD > 0, else 0%

Set up cells with P, p, T, ERD and (ERD – 5%).

- P is the value that is changed by goal seek so that the target cell of (ERD – 5%) is equal to 0. Any reasonable starting value should work. The model solution used a starting value of 250.
- p is the sum of the column for Probability of RD.
- T is the sum of the product of PV of RD times Probability of RD divided by p.
- ERD is pT/P.
- The target cell is ERD – 5% with a goal of 0 by changing P.

(d) Explain why UVW would likely not need to test for risk transfer with respect to this reinsurance agreement.

Testing for risk transfer is not required when risk transfer is self-evident. Risk transfer in most excess of loss reinsurance contracts is self-evident. This aggregate excess of loss reinsurance agreement is priced such that it is exposed to significant risk.

10. **Learning Objectives:**

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

(2b) Understand the differences in development patterns and trends for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate total IBNR for the layer as of December 31, 2022 using Siewert's formula.

Step 1: Calculate estimated CDFs for 250,000 and 750,000 limits.
At each age of development, the estimated CDF for a limit is the total limits CDF times the severity relativity for a limit at 84 months of development divided by the severity relativity for a limit at the age of development.

Step 2: Project ultimate claims for 250,000 and 750,000 limits.
For each accident year, the projected ultimate claims for a limit is the reported claims for the limit times the estimated CDF for that limit at its age of development.

Step 3: Estimate the ultimate claims for the layer 500,000 excess of 250,000.
For each accident year, this is the projected ultimate claims for 750,000 limits minus the projected ultimate claims for 250,000 limits.

Step 4: Estimate the IBNR for the layer 500,000 excess of 250,000.
For each accident year, this is the projected ultimate claims for the layer minus the reported claims for the layer. The reported claims for the layer is the reported claims for 750,000 limits minus the reported claims for 250,000 limits. The sum of these amounts by accident year gives the total IBNR for the layer.

(b) Describe a peculiarity with the CDFs derived from Siewert's formula in part (a).

Commentary on Question:

The model solution is an example of a full credit solution.

10. Continued

The estimated CDFs for 250,000 limits are higher than the CDFs for 750,000 limits for accident year 2021 and accident year 2022. It is unusual for lower limits to have a higher CDF.

(c) Calculate the layer IBNR for AY 2022 as of December 31, 2022 using the ILF method.

Commentary on Question:
Claim amounts are shown in thousands.

The estimated ILF is at the Jan. 1, 2020 cost level. Therefore, 2.5 years of trend is required to take the factor to an AY 2022 level (i.e., assumed to be the average date of the year, July 1, 2022).

Residual trend factor for the 750,000 limit is (1.022)/(1.01) = 1.0119.
ILF trended is $1.19 \times 1.0119^{2.5} = 1.2257$.
AY 2022 Ultimate claims at 750,000 limits is 5,019 × 1.2257 = 6,152.
AY 2022 Ultimate claims for the layer is 6,152 – 5,019 = 1,133.
AY 2022 Layer IBNR is AY 2022 ultimate claims for the layer minus AY 2022 reported claims for the layer = 1,133 – (3,978 – 3,721) = 876.

11. Learning Objectives:

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6b) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Identify two advantages of claims-made coverage.

- Less uncertainty in pricing as ultimate amounts do not require pure IBNR.
- Less effect on annual claim amounts due to sudden changes in either the trend or the reporting pattern.

(b) Identify two advantages of occurrence coverage.

- Greater opportunity for investment income.
- Less risk of coverage gap.

(c) Compare the size of expected ultimate claims for report year 2024 to expected ultimate claims for accident year 2024.

Commentary on Question:

One may use percentages or assume a base value for claims. The model solution uses a base value assuming that the total claims for report year 2021 equals 100.

- Create a matrix of report year (RY) claims by accident year (AY) lag.
- The first column is RY 2021 with values by lag equal to 40, 25, 20 and 15.
- The second column is RY 2022 with values by lag equal to RY 2021 values multiplied by the trend factor of 1.085.
- Each additional column is the prior column increased by the trend factor and represents the next RY.
- Since we are interested in AY 2024, we need only show lags 1, 2 and 3 for RY 2025, lags 2 and 3 for RY 2026 and lag 3 for RY 2027.
- RY 2024 claims is the sum over lags 0 to 3 for RY 2024 = 127.73.
- AY 2024 claims are calculated as RY 2024 lag 0 + RY 2025 lag 1+ RY 2026 lag 2 + RY 2027 lag 3 = 140.28.

11. Continued

RY 2024 expected ultimate claims are 91.1% of the AY 2024 expected ultimate claims.

(d) Explain why members with claims-made policies for prior years will have a coverage gap if they decide to get coverage with the association on January 1, 2024.

This is because they will lack coverage for claims incurred prior to January 1, 2024, but not yet reported. This would entail claims incurred in 2023 with accident year lags 1 to 3, claims incurred in 2022 with accident year lags 2 and 3, and claims incurred in 2021 with accident year lag 3.

12. **Learning Objectives:**

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6a) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain how insurance policy deductibles assist in reducing both moral and morale hazard.

Moral hazard involves the character of the insured regarding the potential to gain from insurance. Morale hazard involves an insured's indifference to loss because they have insurance protection. A deductible shifts some of the burden from a loss to the insured. This will assist in reducing acts of both moral and morale hazard as there will not be full compensation for the loss.

(b) Define the following terms:

(i) Franchise deductible

(ii) Disappearing deductible

(i) Franchise deductible – the insurer pays the full amount of the loss if the covered loss exceeds the deductible amount; otherwise the insurer pays nothing.

(ii) Disappearing deductible – a combination of a straight deductible and a franchise deductible.

(c) Determine the total amount paid by the insurance company if the following loss amounts occurred on <u>each</u> of policies A to D:

(i) 3,500

(ii) 350,000

12. **Continued**

Commentary on Question:
Max = maximum of, Min = minimum of, limit = insured limit, L = loss amount

For policies with a coinsurance requirement, the coinsurance penalty is 1 minus the limit divided by (coinsurance requirement times property value).

Insurance company paid after coinsurance, limit and deductible for L
= Max(0 and (Min(limit and L × (1 – coinsurance penalty)) – deductible))

13. Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5h) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Demonstrate that the mean and coefficient of variation of aggregate losses are 2 billion and 1.118, respectively.

Commentary on Question:

CoV = coefficient of variation, x = loss size in billions, p = probability,
λ = Poisson mean for annual number of losses

Mean = $\sum px = 0.4(1) + 0.3(2) + 0.2(3) + 0.1(4) = 2$
$\sum px^2 = 0.4(1^2) + 0.3(2^2) + 0.2(3^2) + 0.1(4^2) = 5$
CoV = $(\lambda \sum px^2)^{0.5}$ / Mean = 2.2361 / 2 = 1.118

(b) Complete the following aggregate loss probability table:

Aggregate Losses (billion)	**Probability**
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

13. **Continued**

Commentary on Question:
L = aggregate loss in billions, Prob(L) = probability of L,
p(x) = probability of loss size x

- For L = 0, Prob(0) = $\lambda^0 \times e^{-\lambda} / L! = 0.3679$.
- For L = 1, Prob(1) = $(\lambda / 1) \times (Prob(0) \times p(1))$
- For L = 2, Prob(2) = $(\lambda / 2) \times (Prob(1) \times p(1) + 2 \times Prob(0) \times p(2))$
- For L = 3, Prob(3) = $(\lambda / 3) \times (Prob(2) \times p(1) + 2 \times Prob(1) \times p(2) + 3 \times Prob(0) \times p(3))$
- For L > 3, Prob(L) = $(\lambda / L) \times (Prob(L-1) \times p(1) + 2 \times Prob(L-2) \times p(2) + 3 \times Prob(L-3) \times p(3) + 4 \times Prob(L-4) \times p(4))$

(c) Calculate the method of moments estimates for μ and σ^2.

Commentary on Question:
ln = natural logarithm

σ^2 = ln(aggregate CoV^2 + 1) = ln(1.118^2 + 1) = 0.8109
μ = ln(aggregate mean) − σ^2 / 2 = ln(2) – 0.8109 / 2 = 0.2877