

GIRR Model Solutions

Fall 2023

1. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.
2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.
3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

- (1d) Understand the components of ultimate values.
- (2a) Create development triangles of claims and counts from detailed claim transaction data.
- (3d) Analyze development triangles for investigative testing.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11, and 14.

Commentary on Question:

This question tests the candidate's understanding of claim triangles and identifying anomalies in the data.

Solution:

- (a) Update both triangles to include the missing transactions.

Claim ID 100 – Changes to accident year (AY) 2019 row of each triangle:

1. Continued

		Paid Claims		
AY	12	24	36	48
<hr/>				
2019		0	6	6

		Case Estimates		
AY	12	24	36	48
<hr/>				
2019		5	0	0

		Reported Claims		
AY	12	24	36	48
<hr/>				
2019	0	5	6	6

Claim ID 200 – Changes to accident year (AY) 2020 row of each triangle:

		Paid Claims	
AY	12	24	36
<hr/>			
2020	0	6	6

		Case Estimates	
AY	12	24	36
<hr/>			
2020	17	4	4

		Reported Claims	
AY	12	24	36
<hr/>			
2020	17	10	10

Claim ID 300 – Changes to accident year (AY) 2021 row of each triangle:

		Paid Claims	
AY	12	24	
<hr/>			
2021	0	11	

		Case Estimates	
AY	12	24	
<hr/>			
2021	29	29	

		Reported Claims	
AY	12	24	
<hr/>			
2021	29	40	

1. Continued

Revised triangles:

Accident Year	Reported Claims (000)			
	12	24	36	48
2019	1,148	1,788	2,532	3,416
2020	3,444	4,903	6,857	
2021	5,739	12,210		
2022	8,035			

Accident Year	Paid Claims (000)			
	12	24	36	48
2019	138	466	888	1,431
2020	413	1,275	3,154	
2021	689	4,151		
2022	1,286			

- (b) Identify an anomaly in the triangle of ratios of paid claims to reported claims based on the corrected triangles from part (a).

Accident Year	Ratios of Paid Claims to Reported Claims			
	12	24	36	48
2019	0.12	0.26	0.35	0.42
2020	0.12	0.26	0.46	
2021	0.12	0.34		
2022	0.16			

For calendar year 2022 (i.e., the latest diagonal), the ratios have increased significantly.

1. Continued

- (c) Describe two operational changes that could have caused the anomaly you identified in part (b).

Commentary on Question:

Only operational changes were given credit. Noting a decrease in the adequacy of case estimates is not sufficient without the explanation of what operational change could lead to a decrease in the adequacy of case estimates.

- The insurer implemented new processes to speed-up the settlement of claims.
- A change to the approval process that decreased case estimates.

- (d) Calculate incurred claims for calendar year 2021.

Reported claims as of Dec. 31, 2021: $5,739 + 4,903 + 2,532 = 13,174$

(i.e., the 2021 calendar year diagonal in the revised reported claims triangle)

Reported claims as of Dec. 31, 2020: $3,444 + 1,788 = 5,232$

Ultimate claims as of Dec. 31, 2021: $13,174 + 38,476 = 51,650$

Ultimate claims as of Dec. 31, 2020: $5,232 + 17,722 = 22,954$

CY2021 incurred claims:

= Ultimate claims as of Dec. 31, 2021 – Ultimate claims as of Dec. 31, 2020

= $51,650 - 22,954 = 28,696$

2. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

- (3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 22.

Commentary on Question:

This question tests the candidate's understanding of evaluating the selecting ultimate claims based on various methods.

Solution:

- (a) Evaluate the reasonableness of each of the following methods and datasets for estimating ABC Insurance's ultimate claims:
- (i) Expected method based on paid claims for AY 2017
 - (ii) Bornhuetter Ferguson method based on paid claims for AY 2020
 - (iii) Reported development method for AY 2022
- (i) - The expected method is more often used for immature periods, so possibly not appropriate for AY2017
- Paid methods are affected by change in settlement, so not recommended
- No change in case adequacy so paid methods are reasonable
- Ultimate claims estimate is less than paid claims, so not recommended
Conclusion: this value is not appropriate because ultimate claims are less than paid claims
- (ii) - BF method is reasonable to use for AY2020
- Paid methods are affected by the change in settlement, but change occurred in most recent CY so may not affect AY2020 as much for BF
- No change in case adequacy so paid methods are reasonable
Conclusion: this method is likely reasonable.
- (iii) - Reported method is not affected by change in settlement
- No change in case adequacy so reported methods are reasonable
- Leveraged effect for AY2022: $6,654,576 / 944,060 = 7.05$, which is very high and therefore too much uncertainty
Therefore, this method is likely not appropriate due to the high leverage.

2. Continued

- (b) Recommend ultimate claims from a method and dataset for AY 2021. Justify your recommendation.

Recommendation: Any reported method, or combination of any reported methods, is acceptable.

Justification:

- Reported methods is not affected by change in settlement
- Paid methods are affected by change in settlement, so are not recommended
- Leveraged effect for AY2021: $6,159,764 / 1,772,745 = 3.47$. This is likely a reasonable amount of leverage.

- (c) Evaluate the reasonableness of the AY 2021 ultimate claims estimate using the paid development method after adjustment.

- Paid methods are reasonable with the adjustment for the change in settlement
- No change in case adequacy so paid methods are reasonable
- Leveraged effect for AY2021 without adjustment: $4,747,208 / 841,930 = 5.64$.
 - Ultimate claims with the adjustment are likely higher, so the leveraged effect would be even higher, therefore not recommended.
- Conclusion: this value is likely not appropriate because of the leveraged effect.

3. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

- (6g) Calculate loadings for catastrophes and large claims.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26 and 31.

Commentary on Question:

This question tests the candidate's understanding of a loading for large claims that is used in ratemaking.

Solution:

- (a) State two reasons for using a large claim loading approach when estimating ultimate claims at total limits for ratemaking.

Any two of the following are acceptable:

- The loading factor smooths the influence of large claims over time
- The actuary can introduce a greater volume of experience
- The claims at a limited value are more reliable

- (b) Calculate the large claim loadings at 500,000 limit, adjusted to the cost level for each accident year.

Average earned date in rating period: July 1, 2024 (i.e., 9 months after effective date)

July 1, 2022 to July 1, 2024 = 24 months

Accident Year	Trending Period (months)	(1) Severity 5.0%	(2) Trend Factor at: 7.0%	(3) = (2)/(1) Trend Factors for Loading for Large Claims	(4) = 1.28 / (3) Loadings for Large Claims Adjusted to Cost Level of AY
2019	60	1.276	1.403	1.099	1.165
2020	48	1.216	1.311	1.078	1.187
2021	36	1.158	1.225	1.058	1.210
2022	24	1.103	1.145	1.038	1.233

3. Continued

- (c) Calculate ultimate claims at total limits for each accident year using selected ultimate claims at a 500,000 limit and the large claim loadings from part (b).

Accident Year	(4) Selected Ultimate Claims at 500,000 Limit	(5) Loadings for Large Claims Adjusted to Cost Level of AY	(6) = (4)(5) Indicated Ultimate Claims at Total Limits based on Projections at 500,000 Limits
2019	9,850,000	1.165	11,472,916
2020	10,365,000	1.187	12,302,726
2021	11,275,000	1.210	13,637,761
2022	12,385,000	1.233	15,265,711

- (d) Describe how the calculations in part (b) are affected when the experience is less than fully credible.

Commentary on Question:

This question asks to describe how the calculations in part (b) are affected when the experience is less than fully credible. Providing an explanation of what credibility is does not answer the question.

The calculations are affected in two ways:

- Need to develop credibility-weighted trend rates
- Need to develop credibility-weighted loadings

4. Learning Objectives:

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

- (4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

Commentary on Question:

This question tests the Wendy Johnson count-based method to calculate unallocated loss adjustment expenses.

Solution:

- (a) Verify that the forecasted incremental reported count for AY 2021 at 36 months is 95.

Commentary on Question:

The correct formula for determining the AY 2021 cumulative counts to 36 months is $(820 - 536)(0.770 - 0.654) / (1 - 0.654) + 536 = 631.2$. Using the formula $0.770 \times 820 = 631.4$ is not correct, even though the answer is close to 631.2.

AY 2021 cumulative counts to 24 months:	536
AY 2021 cumulative counts to 36 months:	
$(820 - 536)(0.770 - 0.654) / (1 - 0.654) + 536 =$	631
AY 2021 incremental reported counts at 36 months = 631 - 536 =	95

- (b) Estimate unpaid ULAE as of December 31, 2022 using a simple three-year average of historical experience.

Calendar Year	Weighted Counts	Avg ULAE Per Wtd. Count	Trended Period (Yrs.)	Trend to 2023 @2%	Avg ULAE Trended to 2023
2020	789.10	811.05	3	1.06121	860.69
2021	806.35	837.11	2	1.04040	870.92
2022	813.90	850.23	1	1.02000	867.23
Selected average ULAE per weighted count at 2023 level					866.28

4. Continued

Projection of Unpaid ULAE Using Count-Based ULAE Method:

Calendar Year	Counts			Weighted Total	Trending Period in Years	Trend from 2023 at 3.0%	Trended Average ULAE	Estimated Unpaid ULAE
	Newly Reported	Open	Closed					
2023	416	558	674	534.10	0	1.0000	866.28	462,682
2024	282	336	504	339.30	1	1.0300	892.27	302,748
2025	190	173	353	195.25	2	1.0609	919.04	179,443
2026	124	72	225	100.30	3	1.0927	946.61	94,945
2027	75	22	125	45.55	4	1.1255	975.01	44,412
2028	25	0	47	10.95	5	1.1593	1,004.26	10,997
Total								1,095,226

5. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 19.

Commentary on Question:

This question tests the candidate's understanding of estimating IBNR using the Cape Cod and Generalized Cape Cod methods.

Solution:

- (a) Calculate the IBNR for all accident years using the Cape Cod method.

Accident Year	On-Level Earned Premiums	Reported CDFs	Expected % Reported	Used-Up On-Level Earned Premiums	Reported Claims
2019	15,700	1.100	90.91%	14,273	8,200
2020	15,200	1.500	66.67%	10,133	6,200
2021	15,800	2.200	45.45%	7,182	3,500
2022	16,300	4.000	25.00%	4,075	1,500
Total				35,663	19,400

Accident Year	Claim Trend Factors	Tort Reform Factors	Adjusted Claims
2019	1.0927	1.100	9,856
2020	1.0609	1.100	7,235
2021	1.0300	1.000	3,605
2022	1.0000	1.000	1,500
Total			22,197

Adjusted Expected Claim Ratio = $22,197 / 35,663 = 62.24\%$

5. Continued

Accident Year	Expected Claims	Expected % Unreported	Expected Unreported Claims	Ultimate Claims	IBNR
2019	8,130	9.09%	739	8,939	739
2020	8,107	33.33%	2,702	8,902	2,702
2021	9,548	54.55%	5,208	8,708	5,208
2022	10,145	75.00%	7,609	9,109	7,609
Total	35,929			35,658	16,258

- (b) Calculate the accident year 2021 IBNR using the Generalized Cape Cod method and a decay factor of 0%.

Generalized Cape Cod with 0 decay factor = Development Method

2021 IBNR:	3,500	Reported claims
	2.20	CDF
	<u>7,700</u>	Ultimate claims
	4,200	IBNR (Ultimate – Reported)

6. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

Commentary on Question:

This question tests the candidate's understanding of premium trend.

Solution:

- (a) Describe why you would adjust actual historical premiums to current rate levels before analyzing premium trend.

Using unadjusted (actual historical) premiums could result in estimates of trend that were actually due to rate changes.

- (b) Describe an advantage of using written premiums instead of earned premiums for a premium trend analysis.

Written premiums reflect shifts in the mix of exposures more quickly than earned premiums.

- (c) Describe why an adjustment for inflation is required if premiums are based on inflation-sensitive exposures.

Without such adjustment, the premium trend could double-count what is in fact change due to inflation.

- (d) Describe why an increasing proportion of insureds replacing their old vehicles with new vehicles might affect premium trend factors.

Newer vehicles would have higher rate group factors, leading to increased premiums and therefore increasing premium trend.

6. Continued

- (e) Describe how a premium trend analysis for an insurer's book of business is different from a premium trend analysis for a self-insurer.

Commentary on Question:

Some candidates explained that self-insurers would typically use pure premiums instead of historical earned premiums. This does not explain how a premium trend analysis for an insurer's book of business is different from a premium trend analysis for a self-insurer.

The difference is that a self-insurer is essentially a single policy, not a series of policies written over the period. Therefore, the average written dates would reflect the actual date the policy is written.

7. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

- (3h) Explain the effect of changing conditions on the projection methods cited in (3e).
- (3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

Commentary on Question:

This question tests the candidate's understanding of the appropriateness of various methods of estimating ultimate claims under changing conditions.

Solution:

- (a) Explain whether the Bornhuetter Ferguson method or Cape Cod method is more responsive to a deterioration in claims experience.

In the Bornhuetter Ferguson method, the expected claims are based on an a priori estimate and do not change unless the actuary deliberately makes a change. In the Cape Cod method, the expected claims are a function of the reported claims to date and will reflect the deterioration somewhat. Thus, the Cape Cod method is more responsive to a change in claims experience.

- (b) Describe how this change affects the reported claims development triangle evaluated as of December 31, 2022, assuming the following:
 - (i) The court decision affects only new claims.
 - (ii) The court decision affects new and open claims.
 - (i) The change affecting all new claims would occur on a row (accident year) basis and would be immediate with the effective date as claim adjusters estimate new claims that occurred after the effective date.
 - (ii) The change affecting all open claims would occur on a diagonal (or calendar year) basis and would have more of a phased-in effect as all claim estimates get re-evaluated by the claim department over time.

7. Continued

- (c) Describe why the Cape Cod method could be appropriate when estimating claims under scenario (b)(i) above.

The Cape Cod method allows for tort reform adjustments, so the benefit level change can be treated as tort reform. This would adjust prior accident years to the current benefit level.

- (d) Describe why a Berquist-Sherman data adjustment could be appropriate when estimating claims under scenario (b)(ii) above.

The benefit change on a diagonal is similar to the effect of a case adequacy change. The Berquist-Sherman adjustment uses the latest diagonal to restate prior calendar year data (diagonals) consistent with current benefit level.

8. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.
5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

- (3g) Estimate ultimate values using the methods cited in (3e).
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 26.

Commentary on Question:

This question tests the development-based frequency-severity method for estimating ultimate claims.

Solution:

- (a) Explain why a linear trend model may not be appropriate when trend is decreasing.

If the trend is decreasing, as frequency trends often are, then eventually the application of a linear trend will result a negative value, which cannot occur for GI frequency, severity, or pure premium.

- (b) Recommend an annual claim frequency trend to use for this line of business. Justify your recommendation.

Recommended trend: -1.11%

Justification: the increase for 2022 might be an anomaly, so exclude that year from the average.

8. Continued

- (c) Calculate projected ultimate claims using the development-based frequency-severity method and your recommended annual claim frequency trend.

Accident Year	Indicated Frequency	Frequency Trend	Trended Frequency	Calculated Ultimate Counts
2016	9.170%	0.935221	8.576%	1,472.57
2017	9.000%	0.945718	8.511%	1,482.03
2018	8.960%	0.956334	8.569%	1,476.93
2019	8.900%	0.967068	8.607%	1,465.38
2020	8.720%	0.977923	8.527%	1,468.39
2021	8.650%	0.988900	8.554%	1,486.51
2022	8.760%	1.000000	8.760%	1,462.54
Average, excluding 2022				
All years			8.557%	
Latest 3 years			8.563%	
Selected frequency at 2022 cost level			8.684%	

Accident Year	Ultimate Severity	Severity Trend	Trended Ultimate Severity	Calculated Ultimate Severity	Ultimate Claims
2016	3,750.00	1.418519	5,319.45	3,764.58	5,543,602
2017	3,993.00	1.338226	5,343.53	3,990.45	5,913,955
2018	4,230.00	1.262477	5,340.28	4,229.88	6,247,220
2019	4,489.00	1.191016	5,346.47	4,483.67	6,570,290
2020	4,679.00	1.123600	5,257.32	4,752.69	6,978,783
2021	5,048.00	1.060000	5,350.88	5,037.85	7,488,816
2022	5,409.00	1.000000	5,409.00	5,340.12	7,810,150
Average, excluding 2022					46,552,817
All years			5,326.32		
Latest 3 years			5,318.23		
Selected frequency at 2022 cost level			5,340.12		

9. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

Learning Outcomes:

- (2d) Adjust historical earned premiums to current rate levels.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 13.

Commentary on Question:

This question tests the candidate’s ability to adjust premium to current rate levels for ratemaking purposes.

Solution:

- (a) Provide one reason why the company would want to write more 6-month policies in this situation.

Rates have been increasing since the change, so the higher premiums will be earned faster to help keep up with the needed rate changes.

- (b) Calculate the premium on-level factors for calendar years 2019 through 2022 to use in estimating expected claim ratios for the ratemaking analysis.

On-level factors for all 12-month policies:

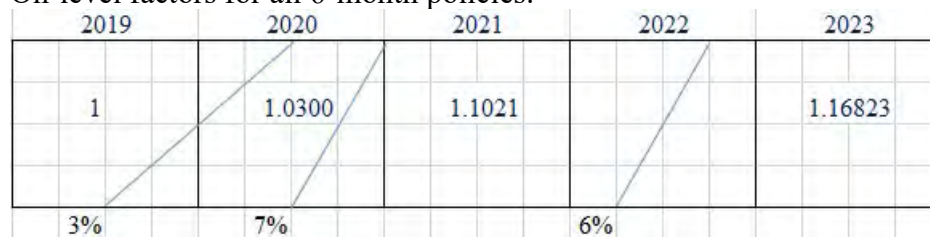
2019	2020	2021	2022	2023
1	1.0300	1.1021		1.16823
3%	7%		6%	

Rate Change History

Effective Date of Rate Change	Rate Change %	Rate Level Index	Percent Premium Earned in Each CY at Rate Level			
			2019	2020	2021	2022
	0.0%	1.00000	87.50%	12.50%	-	-
1-Jul-19	3%	1.03000	12.50%	75.00%	12.50%	-
1-Jul-20	7%	1.10210	-	12.50%	87.50%	71.88%
1-Apr-22	6%	1.16823	-	-	-	28.13%
Total			100.00%	100.00%	100.00%	100.00%
Average Rate Level in each CY:			1.00375	1.03526	1.09309	1.12070
On-Level Factors:			1.1639	1.1284	1.0687	1.0424

9. Continued

On-level factors for all 6-month policies:



Rate Change History

Effective Date of Rate Change	Rate Change %	Rate Level Index	<u>Percent Premium Earned in Each CY at Rate Level</u>			
			2019	2020	2021	2022
	0.0%	1.00000	87.50%	12.50%	-	-
1-Jul-19	3%	1.03000	12.50%	62.50%	-	-
1-Jul-20	7%	1.10210	-	25.00%	100.00%	50.00%
1-Apr-22	6%	1.16823	-	-	-	50.00%
Total			100.00%	100.00%	100.00%	100.00%
Average Rate Level in each CY:			1.00375	1.04428	1.10210	1.13516
On-Level Factors:			1.1639	1.1187	1.0600	1.0291
Combined On-Level factors: (50% weights)			1.1639	1.1236	1.0644	1.0358

(c) Explain why the on-level factors needed for reserving would be lower than the on-level factors calculated in part (b).

- On-level factors for reserving are adjusted to the 2022 average rate level.
- This level is a lower value than the current rate level for ratemaking, leading to lower on-level factors.

(d) Provide one situation where actuaries would need to determine an estimate of ultimate premiums.

Any one of the following is acceptable:

- adjustments to ultimate are required when analyzing policy year data that is not yet completed
- when conducting actuarial work for lines of business where the premiums are subject to audit of exposures following the completion of the policy year
- for lines of business that are subject to retrospective experience rating adjustments

10. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 17 and 18.

Commentary on Question:

This question tests the estimation of ultimate claims using the expected method and the Bornhuetter Ferguson method.

Solution:

- (a) Describe two situations where the expected method is most often used when estimating ultimate claims.

Any two of the following four situations:

- For immature experience periods, particularly in the case of long-tail lines of business
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future

- (b) Describe the primary assumption of the expected method.

The primary assumption of the expected method is that actuaries can better project ultimate values based on an a priori estimate than from the experience observed to date.

10. Continued

- (c) Calculate the expected claim ratios for each year at the 2022 cost level using reported claims.

Accident Year	Earned Premium	Ultimate Claims Based on Reported	Claim Trend Factors at 3%	Premium On Level Factors	On-Level Earned Premium	Trended Ultimate Claims Based on Reported	Trended On-Level Claim Ratio
2018	14,750	11,753	1.1255	1.103	16,269	13,228	81.31%
2019	15,895	13,006	1.0927	1.098	17,453	14,212	81.43%
2020	17,400	14,507	1.0609	1.060	18,444	15,390	83.44%
2021	18,705	15,836	1.0300	1.034	19,341	16,311	84.33%
2022	20,010	16,544	1.0000	1.000	20,010	16,544	82.68%

- (d) Calculate the pure premiums for each year at the 2022 cost level using reported claims.

Accident Year	Trended Pure Premium
2018	67.84
2019	69.33
2020	68.40
2021	69.41
2022	70.10

- (e) Calculate the accident year 2021 ultimate claims using the Bornhuetter Ferguson method and:

- (i) A selected expected claim ratio of 82% at the 2022 cost level
- (ii) A selected pure premium of 69 at the 2022 cost level

10. Continued

Implicit development factor (Ultimate/Reported)	1.412
Expected % undeveloped $(1 - 1/1.412) =$	29.2%

(i) Using expected claim ratio

2021 Earned Premium	18,705
Claim ratio at 2021 cost level $= 82\% \times 1/1.03 =$	0.823
2021 Expected Claims $= 0.823 \times 18,705 =$	15,398
BF estimate of ultimate claims $= 11,213 + 15,398 \times 0.292 =$	15,708

(ii) Using expected pure premium

2021 Expected Claims $= 69 \times 235 / 1.03 =$	15,743
BF estimate of ultimate claims $= 11,213 + 15,743 \times 0.292 =$	15,809

11. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 30.

Commentary on Question:

This question tests the candidate's understanding of expenses used for ratemaking, including trending of fixed expenses.

Solution:

- (a) Identify why a separate trending procedure for fixed expenses may not be required when analyzed on a per-exposure basis.

Commentary on Question:

Some candidates misunderstood this question and answered assuming no trend would be needed for fixed expenses as opposed to asking why a separate trending procedure for fixed expenses may not be required.

When the forces affecting changes in expenses (i.e., the expense trend) are similar to those driving changes in premiums, a separate trend adjustment for fixed expenses may not be necessary.

11. Continued

- (b) Recommend an annual fixed expense trend. Justify your recommendation.

Calendar Year	Fixed Expense to Earned Premiums at Current Rates Ratio	Year-to-Year Change
2016	6.84%	
2017	7.03%	2.77%
2018	7.13%	1.52%
2019	7.29%	2.24%
2020	7.51%	2.96%
2021	7.68%	2.26%
2022	7.92%	3.13%
Average - All years		2.48%
Average - excl hi-lo		2.56%
Recommended fixed expense trend:		2.56%

Justification: 2018 appears to be an anomaly. Exclude highest and lowest to smooth out the variation.

- (c) Recommend a fixed expense ratio to be used in ratemaking. Justify your recommendation.

Average earned premium date in 2022 1-Jul-22
 Average earned premium dates in future rating period: # months:
 for 12-month policies 1-Nov-24 28

Calendar Year	Trending Period (months)	Trending Period (years)	Expense Trend at 2.56%	Trended Fixed Expenses	Trended Fixed Expense Ratio
2016	100	8.33	1.2343	569,624	8.44%
2017	88	7.33	1.2035	594,138	8.46%
2018	76	6.33	1.1735	622,353	8.37%
2019	64	5.33	1.1442	653,791	8.34%
2020	52	4.33	1.1157	694,861	8.38%
2021	40	3.33	1.0878	723,949	8.35%
2022	28	2.33	1.0607	769,701	8.40%

Average - all years 8.39%

Recommended trended fixed expense ratio: 8.39%

Justification: No significant outliers and no significant trend, so all years average is reasonable.

12. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

- (5b) Identify the time periods associated with trending procedures.
- (5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
- (5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
- (5e) Calculate trend factors for claims and exposures.
- (6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
- (6k) Demonstrate the use of credibility in ratemaking.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 26 and 32.

Actuarial Standards of Practice, Actuarial Standards Board of the American Academy of Actuaries, No. 25, Credibility Procedures, 2013.

Commentary on Question:

This question tests basic ratemaking using a pure premium approach, including the application of credibility.

Solution:

- (a) Calculate the trended pure premiums for each accident year.

Past pure premium trend: $(1.06)(1 - 0.01) - 1 = 4.94\%$

Future pure premium trend: $(1.06)(1.01) - 1 = 7.06\%$

Past trend period: For AY2022, from average accident date in AY2022 (July 1, 2022) to December 31, 2022 = 0.5 years

Future pure premium trend period: From average accident date in 2022 to average accident date in future rating period: 1/1/2023 to 3/1/2025 = 26 months, or 2.167 years

12. Continued

Accident Year	Earned Exposures	Ultimate Claims	Pure Premium (PP)	Past Trending Period (yrs.)	Future Trending Period (yrs.)
2018	10,146	13,085,953	1,289.76	4.5	2.167
2019	10,127	14,011,147	1,383.54	3.5	2.167
2020	10,298	14,968,858	1,453.57	2.5	2.167
2021	10,291	15,499,745	1,506.15	1.5	2.167
2022	10,573	18,068,228	1,708.90	0.5	2.167

Accident Year	Future Trending Period (yrs.)	Past PP Trend	Future PP Trend	Total PP Trend	Trended Ultimate Pure Premium
2018	2.167	1.2423	1.1593	1.4402	1,857.54
2019	2.167	1.1838	1.1593	1.3724	1,898.80
2020	2.167	1.1281	1.1593	1.3078	1,900.99
2021	2.167	1.0750	1.1593	1.2462	1,877.03
2022	2.167	1.0244	1.1593	1.1876	2,029.46

- (b) Recommend a trended pure premium. Justify your recommendation.

Commentary on Question:

Other weights are possible.

AY	Trended Ultimate Pure Premium	Weights
2018	1,857.54	22.50%
2019	1,898.80	22.50%
2020	1,900.99	22.50%
2021	1,877.03	22.50%
2022	2,029.46	10.00%

Averages

- all years straight	1,912.76
- weighted	1,898.18
Recommended:	1,898.18

Justification: AY2022 is possibly an anomaly so less weight for that year. Include all years due to credibility (i.e., all years is 4,341 ultimate counts, so still not fully credible even using all years).

12. Continued

- (c) Calculate the pure premium to use for the complement of credibility.

Pure premium trend (future, since future trend started Jan. 1, 2023)	7.06%
Average accident date of prior filing	Jul. 1, 2023
Average accident date of forecast period	Mar. 1, 2025
Trending period in months	20
Pure premium used for complement of credibility: [1,700(1.076) ^(20/12)] =	1,904.70

- (d) Calculate the credibility-weighted indicated rate.

Commentary on Question:

The number of claims for credibility need to match the years that were included in the part (b) selection. For example, if only the most recent three years were included in the selection in part (b), then the number of claims to use for credibility in this part should be: 2,610 = 875 + 852 + 883.

Weighted average pure premium (from part (b)):	1,898.18
Number of claims to use for credibility:	4,341
Credibility: $(4,341 / 4,654)^{0.5}$	96.58%
Credibility-weighted pure premium: $1,898.18 \times 96.58\% + 1,904.70(1 - 96.58\%)$	1,898.40
Indicated rate: $(1,898.40(1.04) + 125) / (1 - 0.18 - 0.05)$	2,726.41

- (e) Identify one adjustment that is necessary when relying on a complement of credibility that is a pure premium based on industry experience.

Either of the following is acceptable:

- adjusted to reflect the insurer's mix of business
- adjusted to the cost level of the forecast period

13. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

- (3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
- (3f) Demonstrate knowledge of good practice related to projecting ultimate values.
- (3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 15.

Commentary on Question:

This question tests the candidate's understanding of estimating ultimate claims using the development method.

Solution:

- (a) Identify a potential problem with your colleague's recommendation.

Due to the large claims in 2019 and 2021, the 12-24 and 24-36 age-to-age factors are too high, therefore ultimate claims would be overstated.

- (b) Describe an alternative approach to your colleague's recommendation.

Recommend adjusting for the large claims (i.e., removing them from the development factor analysis).

{alternatively, could use average of the factors that exclude the AY2021 12-24 and AY2019 24-36 factors.}

13. Continued

- (c) Estimate total ultimate claims based on the development method and your alternative from part (b).

Construction of right triangle that excludes large claims:

Accident Year	Reported Claims (000)						
	12	24	36	48	60	72	84
2016	1,826	2,742	2,948	3,174	3,239	3,248	3,248
2017	2,296	3,656	3,928	4,230	4,458	4,506	
2018	3,064	4,932	5,465	6,104	6,373		
2019	2,327	3,675	4,022	4,624			
2020	2,691	4,495	4,924				
2021	2,497	4,025					
2022	3,740						

e.g., AY2021 @ 24 months: 4,025 = 5,025 – 1,000

Development factors:

	12-24	24-36	36-48	48-60	60-72	72-84
	1.5016	1.0751	1.0767	1.0205	1.0028	1.0000
	1.5923	1.0744	1.0769	1.0539	1.0108	
	1.6097	1.1081	1.1169	1.0441		
	1.5793	1.0944	1.1497			
	1.6704	1.0954				
	1.6119					
Simple average	1.5942	1.0895	1.1050	1.0395	1.0068	1.0000
Volume-weighted average	1.6002	1.0916	1.1081	1.0416	1.0074	1.0000
Selected:						
- Age-to-age:	1.5942	1.0895	1.1050	1.0395	1.0068	1.0000
- Age-to-ultimate	2.0086	1.2599	1.1564	1.0465	1.0068	1.0000

13. Continued

AY	Reported Claims	Age-to-Ultimate Factors	Ultimate Claims
2016	3,248	1.0000	3,248
2017	4,506	1.0000	4,506
2018	6,373	1.0068	6,416
2019	4,624	1.0465	4,839
2020	4,924	1.1564	5,694
2021	4,025	1.2599	5,071
2022	3,740	2.0086	7,512
Ultimate claims, excluding large claims			37,287
Ultimate claims, including large claims:			38,787

- (d) Describe how you would adjust for the large claims when estimating ultimate claims based on the paid development method for this line of business.

The ultimate values would need to include the case estimates for large claims.

14. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

- (3c) Identify the types of development triangles that can be used for investigative testing.
(3d) Analyze development triangles for investigative testing.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 14 and 20.

Commentary on Question:

This question tests investigative analysis of various development triangles.

Solution:

- (a) Identify two other examples of actions that could result in shifts in a reported claim patterns.

Any two of the following are acceptable:

- new procedures for the payment of claims such as direct deposit to a claimant's bank account instead of issuance of checks
- changes in the distribution of policy limits purchased by insureds; (or offered by the company)
- changes in the distribution of deductibles purchased by insureds; (or offered by the company)
- changes in the use of partial settlements or ex gratia payments
- shifts in the attitude toward defense of questionable claim files
- change in the definition of reported claims

- (b) Verify your colleague's assumption.

First need to determine if the case adequacy was strengthened in calendar year (CY) 2021: Analyze change in average case estimates.

Accident Year	Average Case Estimates					
	12	24	36	48	60	72
2017	5,141	6,014	6,700	7,120	8,155	28,343
2018	5,670	6,456	6,931	8,510	8,670	
2019	5,821	6,742	8,372	9,033		
2020	6,158	7,923	8,828			
2021	7,588	8,303				
2022	8,159					

14. Continued

Accident Year	Change in Average Case Estimates					
	12	24	36	48	60	72
2017-2018	10.3%	7.3%	3.4%	19.5%	6.3%	
2018-2019	2.7%	4.4%	20.8%	6.1%		
2019-2020	5.8%	17.5%	5.4%			
2020-2021	23.2%	4.8%				
2021-2022	7.5%					

Analysis: There appears to have been strengthening in CY 2021 to support colleague's recommendation.

(c) Critique your colleague's recommendation.

Any one of the following is acceptable:

- even though the change was in CY 2021, could still use the most recent diagonal to reflect the most recent data point
- common practice to use most recent diagonal, so could use that
- even though it is more common to use most recent diagonal, using CY 2021 still acceptable

(d) Construct a reported claims triangle adjusted for the change in case adequacy, basing the adjustments on the calendar year 2022 diagonal.

Accident Year	Adjusted Average Case Estimates					
	12	24	36	48	60	72
2017	6,011	6,503	7,349	7,994	8,157	28,343
2018	6,390	6,912	7,812	8,497	8,670	
2019	6,792	7,348	8,305	9,033		
2020	7,220	7,811	8,828			
2021	7,675	8,303				
2022	8,159					

Accident Year	Adjusted Reported Claims					
	12	24	36	48	60	72
2017	3,602,331	4,775,496	5,805,686	6,784,638	7,372,495	7,702,277
2018	3,848,406	5,063,164	6,447,061	7,430,019	8,060,259	
2019	4,198,283	5,567,827	6,905,880	8,051,684		
2020	4,622,567	5,976,718	7,664,425			
2021	4,890,709	6,611,842				
2022	5,320,155					

15. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

Learning Outcomes:

- (2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
(2d) Adjust historical earned premiums to current rate levels.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

Commentary on Question:

This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to understand earned premiums adjusted to current rate level.

Solution:

- (a) Calculate the written premiums for 2022.

(1) Policy Number	(2) Policy Premium	(3) Policy Effective Date	(4) Policy Expiration Date	(5) % Written in 2022	(6) = (2)(5) 2022 Written Premiums
501	5,000	July 1, 2020	June 30, 2022	0%	0
502	3,600	April 1, 2021	March 31, 2024	33%	1,200
503	2,400	January 1, 2022	December 31, 2024	33%	800
504	4,800	September 1, 2022	August 31, 2024	50%	2,400
Total					4,400

- (b) Calculate the earned premiums for 2022.

(1) Policy Number	(7) % Earned in 2022	(8) = (2)(7) 2022 Earned Premiums
501	6/24 = 25.0%	1,250
502	12/36 = 33.3%	1,200
503	12/36 = 33.3%	800
504	4/24 = 16.7%	800
Total		4,050

15. Continued

- (c) Calculate the unearned premiums as of December 31, 2022.

(1) Policy Number	(9) Equivalent Annual Premium	(10) Written Date in 2022	(11) # Months Unearned as of Dec 31, 2022	(12) = (9)(11)/12 UEP as of Dec. 31, 2022
501	2,500	Jul. 1, 2022	0	0
502	1,200	Apr. 1, 2022	3	300
503	800	Jan. 1, 2022	0	0
504	2,400	Sep. 1, 2022	8	1,600
Total				1,900

- (d) Recalculate the 2022 earned premium for policy 504.

Annual premium written on Sep. 1, 2022:	2,400
Number of months of earned premium during 1st year: (i.e., Sep. 1, 2022 to Sep. 1, 2023)	6
Monthly earned premium:	400
# of months in 2022 vehicle was operated:	1
2022 earned premium:	400

- (e) Recalculate the unearned premium as of December 31, 2022 for policy 504.

# of months unearned as of Dec. 31, 2022 (excluding months in 2023 vehicle was not operated):	5
Unearned premium as of Dec. 31, 2022:	2,000

- (f) Describe why the parallelogram approximation would not be appropriate when adjusting historical premiums to current rate levels for policies such as policy 504.

It would not be appropriate because premiums are not earned evenly throughout the policy term.