LTAM Exam

Fall 2021

Solutions For Form B

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Solutions to Multiple Choice Questions

Fall 2021, Multiple Choice Solutions, Form B

MC1: Answer B

A universal life policy is not a traditional life policy.

MC2: **Answer E**

$$\mu_x = -\frac{d}{dx}\log(S_0(x)) = \frac{d}{dx}0.001\left(e^{0.1x}-1\right) = 0.0001e^{0.1x}$$

$$\Rightarrow \mu_{80} = 0.298$$

MC3: Answer C

$$g(s) = \log\left(-\log(s)\right); \quad g'(s) = -\frac{1}{s\log s}$$

$$g\left(\hat{S}(10)\right) = -4.1922$$

$$V\left[g(\hat{S}(10)\right] \approx \left(g'(\hat{S}(10))\right)^2 V[S(10)] \approx \left(\frac{1}{0.985\log(0.985)}\right)^2 \times 0.01^2 = 0.4512 = 0.6717^2$$

CI for $g(S(10))$ is: $-4.1922 \pm 1.96\times 0.6717 = \left(-5.5087, -2.8757\right)$

CI for $S(10)$ is: $\left(\exp\left(-e^{-2.8757}\right), \exp\left(-e^{-5.5087}\right)\right) = \left(0.9452, 0.9960\right)$

MC4: Answer E

$$H(1.1) = \frac{1}{9}+\frac{1}{8} = 0.2361$$

$$\widehat{S}(1.1) = e^{-0.2361} = 0.7897$$

MC5: Answer B

$$p_x'^{(2)} = \left(p_x^{(\tau)}\right)^{q_x^{(2)}/q_x^{(\tau)}} = \left(0.85\right)^{0.1/0.15} = 0.89732$$

$$\Rightarrow q_x'^{(2)} = 1 - p_x'^{(2)} = 1 - 0.89732 = 0.1027$$

MC6: Answer D

The easiest way is trial and error.

PV of benefits is:

$\leq 100,000v = 92592$ with probability of 1.0

$\leq 100,000v^2 = 85734$ with probability of 0.96

$\leq 100,000v^3 = 79,383$ with probability of 0.88

$\boxed{\leq 100,000v^4 = 73,502 \text{ with probability of } 0.78}$

Or

$$Z = 100,000v^{K_{80}+1} \leq 75,000$$

$$v^{K_{80}+1} \leq 0.75 \Longrightarrow K_{80}+1 \geq \frac{\ln(0.75)}{\ln[(1.08)^{-1}]} = 3.74 \Longrightarrow K_{80} \geq 2.74 \Longrightarrow K_{80} \geq 3$$

$$\Pr[K_{80} \geq 3] = {}_3p_{80} = 0.78$$

MC7: Answer A

$$EPV = 20,000\left(1+0.66v\right)+100,000(0.22v)+5,000\left(0.12v+\left(0.66\times 0.12+0.22\times 0.36\right)v^2\right)$$
$$= 54,814$$

The original question had a typo as the 0.4 in the table of annual transition probabilities was a 0.42. This value has been fixed in the published exam. The incorrect value did not effect the answer as that value was not used in the calculations.

MC8: Answer C

$$a_{40:50:\overline{22}|} = a_{40:50} - {}_{22}E_{40:50}\cdot a_{62:72}$$
$$= [\ddot{a}_{40:50}-1] - \left(\frac{l_{62}}{l_{40}}\right)\left(\frac{l_{72}}{l_{50}}\right)(1.05)^{22}[\ddot{a}_{62:72}-1]$$
$$= \left(16.5558-1\right) - 0.29836\left(10.5396-1\right)$$
$$= 12.7096$$

MC9: Answer B

$$\frac{V[L']}{V[L]}=\frac{0.49}{0.16}=\frac{\left(1+\frac{\pi}{\delta}\right)^2\left({}^2\bar{A}_x-(\bar{A}_x)^2\right)}{\left(1+\frac{0.02}{\delta}\right)^2\left({}^2\bar{A}_x-(\bar{A}_x)^2\right)}\Rightarrow 3.0625=\frac{\left(1+\frac{\pi}{\delta}\right)^2}{\left(1+\frac{0.02}{\delta}\right)^2}$$

$$\Rightarrow 1.75=\frac{\left(1+\frac{\pi}{\delta}\right)}{\left(1+\frac{0.02}{\delta}\right)}=\frac{\delta+\pi}{\delta+0.02}\Rightarrow \pi=0.08$$

MC10: Answer E

$$P\ddot{a}_{55}=200,000v^{\frac{1}{3}}\bar{A}_{55}+1935+0.25P+0.05\,P\ddot{a}_{55}$$

$$\bar{A}_{55}=\frac{i}{\delta}A_{55}=0.24107$$

We multiply this by $v^{\frac{1}{3}}$ since we are paying the benefit 4 months after the death instead of at the moment of death.

$$P=\frac{200,000v^{\frac{1}{3}}\bar{A}_{55}+1935}{0.95\,\ddot{a}_{55}-0.25}=\frac{(200,000)(1.05)^{-1/3}(0.24107)+1935}{(0.95)(16.0599)-0.25}=3289.90$$

MC11: Answer B

$$170\times 0.92\times\left(\frac{l_{[41]}+l_{[41]+1}v+l_{[41]+2}v^2}{l_{[41]}}\right)=35+\left(S+1080\right)\left(\frac{d_{[41]}v+d_{[41]+1}v^2+d_{[41]+2}v^3}{l_{[41]}}\right)$$

$$\Rightarrow 156.4\times 2.8527=35+(S+1080)\times 0.008208$$

$$\Rightarrow S=49,015$$

MC12: Answer A

$$P\left(\frac{l_{57}+l_{58}v+l_{59}v^2}{l_{57}}\right)=100,000\left(\frac{d_{57}v^{\frac{1}{2}}+d_{58}v^{1\frac{1}{2}}+d_{59}v^{2\frac{1}{2}}}{l_{57}}\right)$$

$$P\left(\frac{99,960.2+97,642.2+95,351.5}{99,960.2}\right)=100,000\left(\frac{(243.2)(1.05)^{-0.5}+(264.4)(1.05)^{-1.5}+(287.6)(1.05)^{-2.5}}{99,960.2}\right)$$

$$\Rightarrow P=264.0$$

MC13: Answer C

$$50,000+12F\left(\ddot{a}_{65}^{(12)00}+\ddot{a}_{65}^{(12)01}+\ddot{a}_{65}^{(12)02}\right)=12\left(3,500\,\ddot{a}_{65}^{(12)00}+8,000\,\ddot{a}_{65}^{(12)01}+20,000\,\ddot{a}_{65}^{(12)02}\right)+50,000\bar{A}_{65}^{03}$$

$$50,000+12F\left(12.495+0.789+0.252\right)$$
$$=12\left(3,500(12.495)+8,000(0.789)+20,000(0.252)\right)+50,000(0.127)$$

$$\Rightarrow F=3800.8$$

MC14: Answer D

$$P^*=1000\frac{A_{36}}{\ddot{a}_{36:\overline{19}|}};\quad \ddot{a}_{36:\overline{19}|}=\frac{a_{35:\overline{20}|}}{v\,p_{35}}=\frac{13.0240-1}{(1.05)^{-1}(1-0.000391)}=12.6301$$

$$P^*=\frac{(1000)(0.10101)}{12.6301}=8.00$$

$$_{10}V=1000A_{45}-8.0\,\ddot{a}_{45:\overline{10}|}=(1000)(0.15161)-(8.00)(8.0751)=87.0$$

MC15: Answer C

$$\frac{d}{dt}\,{}_tV = \delta\,{}_tV + P - E - \mu_{x+t}\left(100{,}000 - {}_tV\right)$$

$$\mu_{50+20} = Bc^{70} = (0.0003)(1.075)^{70} = 0.047393$$

at $t = 20$: $\frac{d}{dt}\,{}_tV = (0.05)(52{,}225) + (0.95)(3400) - 100 - 0.047393(100{,}000 - 52{,}225) = 3477$

MC16: Answer D

$${}_5V^{(1)} = 1000\,\bar{a}_{65}^{11} + 10{,}000\,\bar{A}_{65}^{12} - 850\,\bar{a}_{65}^{10}$$

$$= (1000)(8.8123) + (10{,}000)(0.56810) - (850)(0.0395) = 14{,}460$$

MC17: Answer C

${}_9V$ = PV of Death Benefit for next year + PV of Annuity Benefit $-$ PV of Premium

$$= 100{,}000\left(1 - p_{59:59}\right)v + p_{59:59}v\left(10{,}000\right)\left(2\ddot{a}_{60} - \ddot{a}_{60:60}\right) - 12{,}570$$

$$= (100{,}000)[1-(1-0.003048)^2](1.05)^{-1}$$
$$+ (1-0.003048)^2(1.05)^{-1}(10{,}000)(2\times 14.9041 - 13.2497) - 12{,}750$$

$$= 144{,}750$$

MC18: Answer D

$$\Pr_5^{(0)} = \left({}_4V^{(0)} + P^{(0)} - Expense\right)(1+i) - p_{x+4}^{01}(DisBen + {}_5V^{(1)}) - p_{x+4}^{02}(DeathBen) - p_{x+4}^{00} \times {}_5V^{(0)}$$

$$\left(218 + 100 - 5\right)(1.05) - 0.02(500 + 370) - 0.01(2000) - (1 - 0.02 - 0.01)(228)$$

$$= 70.1$$

MC19: Answer A

$$NC = 0.015 \times FAS_{65} \times {}_{10}E_{55} \times \ddot{a}_{65}^{(12)} = 0.015 \times \underbrace{90{,}000 \times 1.025^{9}}_{FAS_{65}} \times 0.59342 \times 13.087$$

$$= 13{,}093$$

MC20: Answer B

$$\ddot{a}_{67,0}^{B} = 31.75 = 1 + \frac{1.04 \times 1.06}{1.07} \times p_{67} \times \ddot{a}_{68,1}^{B} \Rightarrow \ddot{a}_{68,1}^{B} = 30.07$$

$$\ddot{a}_{68,1}^{B} = 30.07 = 1 + \frac{1.04 \times 1.06}{1.07} \times p_{68} \times \ddot{a}_{69,2}^{B} \Rightarrow \ddot{a}_{69,2}^{B} = 28.45$$

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Solutions to Written Answer Questions

Question 1 Model Solution

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a)

(i)

$$
\begin{aligned}
(IA)_x &= \sum_{k=0}^{\infty}(k+1)\,{}_{k|}q_x v^{k+1} \\
&= \sum_{k=0}^{\infty} k\ {}_{k|}q_x\ v^{k+1} + \sum_{k=0}^{\infty}\ {}_{k|}q_x\ v^{k+1} \\
&= \sum_{k=1}^{\infty} k\ {}_{k|}q_x\ v^{k+1} + \sum_{k=0}^{\infty}\ {}_{k|}q_x\ v^{k+1} \\
&= \sum_{j=0}^{\infty} (j+1)\ {}_{j+1|}q_x\ v^{j+2} \quad + A_x \qquad\qquad \text{where k=j+1} \\
&= v\,p_x\ \sum_{j=0}^{\infty} (j+1)\ {}_{j|}q_{x+1}\ v^{j+1} \quad + A_x \\
&= \ v\,p_x\,(IA)_{x+1} \quad + A_x
\end{aligned}
$$

or

$$
\begin{aligned}
(IA)_x &= v\,q_x + 2v^2\,p_x\,q_{x+1} + 3v^3\ {}_2p_x\,q_{x+2} + 4v^4\ {}_3p_x\,q_{x+3} + \ldots \\
&= v\,q_x + v^2\,p_x\,q_{x+1} + v^3\ {}_2p_x\,q_{x+2} \ + v^4\ {}_3p_x\,q_{x+3} + \ldots \\
&\qquad + v^2\,p_x\,q_{x+1} + 2v^3\ {}_2p_x\,q_{x+2} + 3v^4\ {}_3p_x\,q_{x+3} + \ldots \\
&= A_x \\
&\qquad + v\,p_x\,\{v q_{x+1} + 2v^2\ p_{x+1}\,q_{x+2} + 3v^3\ {}_2p_{x+1}\,q_{x+3} + \ldots\} \\
&= A_x + \ v\,p_x\,(IA)_{x+1}
\end{aligned}
$$

(ii)

$$(IA)_{50} = A_{50} + v\,p_{50}\,(IA)_{51} = 5.8255$$

$$(IA)_{51} = \frac{5.8255 - A_{50}}{v\,p_{50}} = \frac{5.8255 - 0.18931}{0.998791/1.05} = 5.92516$$

Comments: Candidates did very well on this part. Where points were deducted, it was primarily from part i not being sufficiently robust where candidates did not really "show" the equation to be true.

b)

$$
\begin{aligned}
(200{,}000 + 1000)A_{50} + 5000((IA)_{50} - A_{50}) &= P((0.95)\ \ddot{a}_{50:\overline{20|}} - 0.1) \\
(195{,}000 + 1000)A_{50} + 5000(IA)_{50} &= P((0.95)\ \ddot{a}_{50:\overline{20|}} - 0.1)
\end{aligned}
$$

$$37{,}104.76 + 29{,}127.5 = P((0.95)(12.8428) - (0.1))$$

$$P = \frac{66{,}232.26}{12.10066} = 5473.44$$

Comments: Candidates did very well on this part with most getting full credit.

c)

$$ {}_2V = (205{,}000 + 1000)A_{52} + 5000(IA)_{52} - (0.95)P\, \ddot{a}_{52:\overline{18}|} $$
where
$$ A_{52} = 0.20664 $$
$$ (IA)_{52} = \frac{(IA)_{51} - A_{51}}{v\, p_{51}} = \frac{5.925163 - 0.1978}{0.998669/1.05} = 6.02175 $$

$$ \ddot{a}_{52:\overline{18}|} = \ddot{a}_{52} - v^{18}\, {}_{18}p_{52}\, \ddot{a}_{70} $$
$$ = 16.6606 - 1.05^{-18}\left(\frac{91{,}082.4}{98{,}326.2}\right)(12.0083) = 16.6606 - 4.6221 = 12.0385 $$

$$ {}_2V = 206{,}000(0.20664) + 5000(6.02175) - (0.95)(5473.44)(12.0385) $$
$$ = 42{,}567.84 + 30{,}108.75 - 62{,}597.41 = 10{,}079.18 $$

Or, recursively,

$$ {}_1V = \frac{(5473.44)(0.85)(1.05) - (0.001209)(201{,}000)}{0.998791} = 4647.6 $$

$$ {}_2V = \frac{[4647.6 + (5473.44)(0.95)](1.05) - (0.001331)(206{,}000)}{0.998669} = 10{,}078.9 $$

Comments: Candidates also did well on this part but the results were not as good as the first two parts.

d)

(i) Total gain: using the actual experience (*) during year 11,

$$ 1000({}_{10}V + P - e_{10}^{*})(1 + i_{10}^{*}) - 1000\, q_{60}^{*}(250{,}000 + E_{10}^{*}) - 1000\, p_{60}^{*}\, {}_{11}V $$
=1000(63,208 + 0.94(5473.44)) (1.06) – 6 (250,000+ 1100) – 994(71,217)
= 1000[72,454.2156 -1506.6 -70,789.698]
= 157,917.62 i.e. a gain per policy in force at 10 of 157.92

(ii) Gain by source:

__Gain from interest__:

Actual interest earned – expected interest earned,

$$ 1000(63{,}208 + 0.95(5473.44))\,(1.06) - [1000(63{,}208) $$
$$ + 0.95(5473.44))\,(1.05)] $$
= 1000(68,407.768)(1.06−1.05)
= 684,077.68 i.e. a gain from interest of 684,077.68

Alternatively, using actual experience for interest only, total gain would have been

$1000({}_{10}V^g + P - e_{10})(1.06) - 1000\, q_{60}(250{,}000 + E_{10}) - 1000\, p_{60\ 11}V^g$
$= 1000(63{,}208 + .95(5473.44))(1.06)\ - 3.398\ (250{,}000 + 1000)$
$\qquad - 996.602\ (71{,}217)$
$= \ 684{,}331.446$

Since the expected gain is

$$1000(63{,}208 + .95(5473.44))(1.05)\ - 3.398\ (250{,}000 + 1000) - 996.602\ (71{,}217)$$

$= 253.8$
the gain from interest is $684{,}331.446 - 253.8 = 684{,}077.68$

Gain from mortality:
Expected mortality cost – actual mortality cost, using the actual interest
$[1000 q_{60}(250{,}000 + E_{10}) + 1000 p_{60\ 11}V^g]$
$\qquad -[1000\ q^*_{60}(250{,}000 + E_{10}) - 1000\ p^*_{60\ 11}V^g\ ]$
= 852,898 + 70,975,004.6 – (1,506,000 + 70,789,698)
= - 467,795.37   i.e. a loss from mortality of 467,795.37

Alternatively, using actual mortality and interest rate, the gain would have been
$1000({}_{10}V^g + P - e_{10})(1.052) - 1000\, q_{60}(S + 16P + E_{10}) - 1000\, p_{60\ 11}V^g$
= 38,138,976.56 – 792,610 – 37,258,812.78
= 216,536.08
So the gain from mortality is 216,536.08 – 684,331.446 = –467,795.37 (a loss)

Gain from expenses:
Expected expenses – Actual expenses, valued at year end, using actual interest & mortality

$(1000({}_{10}V^g\ + P - e_{10})(1.06)\ + 1000\ q^*_{60}(250{,}000 + E_{10}))$
$\qquad -[1000({}_{10}V^g + P - e^*_{10})(1.06)\ + 1000\ q^*_{60}(250{,}000 + E^*_{10})]$
= 1000(0.05P-0.06P)(1.06) + 1000(0.006)(251,000 – 251,100)
= –58,618.46   i.e. a loss from expenses of 58,618.46

Alternatively, the total gain calculated in (i) is 157,917.62
So, the gain from expenses is 157,917.62 – 216,536.08 = –58,618.46
i.e. a loss from expenses of 58,618.46

Comments: Candidates struggled with this portion. A common error was not doing the calculations in the order given or not incorporating some of the actual experience.

Question 2 Model Solution

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a)

- To set premiums
- To set reserves
- To measure profitability
- To stress test profitability
- To determine distributable surplus (for participating contracts)

Comments: Most candidates that tried this part received full credit for this part. If candidates lost credit it was usually because all of their answers were related to measuring profitability without mentioning other things.

b)

$$
\begin{aligned}
{}_2V^n &= EPV\ of\ Future\ Benefits - EPV\ of\ Future\ Premiums \\
&= S\ v\ q_{x+2} - P = 1{,}000{,}000(1.05^{-1})(0.3) - 177{,}313.20 \\
&= \ 285{,}714.29 - 177{,}313{,}20 = 108{,}401.09
\end{aligned}
$$

Or

$$({}_1V^n + P)(1.05) - S\ q_{x+1} = p_{x+1}\ {}_2V^n$$

$${}_2V^n = \frac{(95{,}754 + 177{,}313.20)(1.05) - 1{,}000{,}000\ (0.2)}{0.8} = \frac{86{,}720.56}{0.8} = 108{,}400.70$$

Comments: Most candidates received full credit for this part. Common errors involved an incorrect discount rate, including expenses, or including withdrawals.

c)

(i) $$
\begin{aligned}
Pr_0 &= -4000 - (0.15)G \\
&= -4000 - (0.15)(210{,}000) = -35{,}500
\end{aligned}
$$

(ii) Lapse rate: w=0.1

$$
\begin{aligned}
Pr_1 &= (0.95G - E_0)(1 + i^*) - 1{,}000{,}000\ q_x\ - p_x\ \ (1 - w)\ {}_1V^n \\
&= (0.95(210{,}000) - 100)(1.07) - 1{,}000{,}000(0.1)\ - (0.9)(0.9)(95{,}754) \\
&= 213{,}358 - 100{,}000 - 77{,}560.74 = 35{,}797.26
\end{aligned}
$$

Comments: Part i was done well by most candidates. In part ii, many candidates failed to include withdrawals in part ii.

d)

(i) $\Pi_0 = Pr_0 = -35{,}500$

$\Pi_1 = Pr_1 = 35{,}797.26$

$\Pi_2 = p_x^{(\tau)}\, Pr_2 = (0.9)(0.9)(37{,}766) = 30{,}590.46$

$\Pi_3 = {}_2p_x^{(\tau)}\, Pr_3 = p_x^{(\tau)} p_{x+1}^{(\tau)}\, Pr_3 = (0.9)(0.9)(0.8)(0.9)(29{,}347) = 17{,}115.17$

(ii) $NPV = \sum_t \Pi_t\, v_{0.12}^t$

$$= -35{,}500 + \frac{35{,}797.26}{1.12} + \frac{30{,}590.46}{1.12^2} + \frac{17{,}115.17}{1.12^3} = 33{,}030.61$$

(iii) $PM = \frac{NPV}{G\left(1+v_{0.12}^1\, p_x^{(\tau)} + v_{0.12}^2\, {}_2p_x^{(\tau)}\right)}$

$$= \frac{33{,}030.61}{210{,}000(1 + 0.7232143 + 0.4649235)} = \frac{33{,}030.61}{495{,}508.93} = 0.07188 \quad or\ 7.19\%$$

Comments: Many candidates failed to include withdrawals and some candidates did not use the appropriate discount rates in parts ii and iii.

e)

The NPV in terms of G is ($v=v_{0.12}$)

$$\begin{aligned} NPV = & -4000 - (0.15) \\ & +\{(0 + 0.95G - 100)(1.07) - (1{,}000{,}000)(0.1) - (0.9)(0.9)(95{,}754)\}v \\ & +\{(95{,}754 + 0.95G - 100)(1.07) - (1{,}000{,}000)(0.2) \\ & \quad - (0.8)(0.9)(108{,}401.09)\}\,(0.9)^2\, v^2 \\ & +\{(108{,}401.09 + 0.95G - 100)(1.07) - 300{,}000\}\,(0.9)^2\,(0.9)(0.8)\, v^3 \end{aligned}$$

$$\begin{aligned} NPV = & \; G\left(-0.15 + 0.95(1.07)(v + (0.9)^2 v^2 + (0.9)^3(0.8)v^3)\right) \\ & -4000 - 177{,}667.74\, v - 142{,}316.19\, v^2 - 107{,}377.52\, v^3 \end{aligned}$$

$$NPV = 1.8359304\, G - 352{,}514.70$$

$$PM = \frac{1.8359304\, G - 352{,}514.70}{2.18814\, G} = 0.1$$

$\Rightarrow\ G = \frac{352{,}514.70}{1.8359304 - 0.218814} = \mathbf{217{,}989.69}$

Comments: Very few candidates attempted this part of the problem. For those who did, most simply wrote down a basic formula for PM without trying to write NPV in terms of the new premium. Candidates who attempted to write NPV in terms of the new premium were rewarded significantly, even if the attempt was incomplete/incorrect.

Question 3 Model Solution

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a)

μ_{x+t}^{01} is the force of mortality under Makeham's Law with parameters A, B and c*

Where A=a=0.004, B=b=0.015 and c* = e^c = $e^{0.005}$ = 1.0050125

$$
\begin{aligned}
{}_5p_{60}^{00} &= exp\left\{-5A - \frac{B}{ln\ c^*}\ (c^*)^{60}((c^*)^5 - 1)\right\} \\
&= exp\left\{-0.02 - \frac{0.015}{0.005} e^{60(0.005)}\left(e^{5(0.005)} - 1\right)\right\} \\
&= \ exp\{-0.1225155\} = 0.8846922
\end{aligned}
$$

The probability of a diagnosis is 1 – 0.8846922 = 0.11531 or 11.531%

Or

$$
{}_5p_{60}^{00} = \exp\left\{-\int_0^5 \mu_{60+t}^{01}\ dt\right\} = \exp\left\{-\int_0^5\left(0.004 + 0.015e^{0.005(60+t)}\right) dt\right\}
$$

$$
= \exp\left\{-\left(0.004t + \frac{0.015}{0.005}e^{0.005(60+t)}\right)\Big|_{t=0}^{t=5}\right\}
$$

$$
= \exp\left\{3e^{0.005(60)} - 0.004(5) - 3e^{0.005(65)}\right\} = 0.8846922
$$

The probability of a diagnosis is 1 – 0.8846922 = 0.11531 or 11.531%

Comments: Candidates did okay on this question. The most successful attempts recognized the Makeham model and generally solved the problem correctly, others had varying levels of success setting up the correct equations. Most common mistakes were not recognizing the Makeham formulation, confusing c and e^c*, and not taking the probability complement in the last step.*

b)

$$
\begin{aligned}
{}_tp_x^{12} &= \int_0^t {}_rp_x^{11}\ \mu_{x+r}^{12}\ \ {}_{t-r}p_{x+r}^{22}\ dr \\
&= \int_0^t e^{-0.2r}(0.2)e^{-0.4(t-r)}dr \\
&= e^{-0.4t}\ (0.2)\int_0^t\ e^{0.2r}\ dr = e^{-0.4t}\ (0.2)\ \frac{e^{0.2t} - 1}{0.2} = e^{-0.2t} - e^{-0.4t}
\end{aligned}
$$

Comments: Candidates did fairly well on this question.

c)

$$_5p_x^{13} = 1 - {}_5p_x^{11} - {}_5p_x^{12}$$
$$_5p_x^{11} = e^{-5(0.2)} = 0.367879$$
$$_5p_x^{12} = e^{-5(0.2)} - e^{-5(0.4)} = 0.232544 \qquad \text{from part (b)}$$

$$_5p_x^{13} = 0.399577$$

Or

$$_5p_x^{13} = \int_0^5 {}_tp_x^{12}\, \mu_{x+t}^{23} dt = 0.4 \int_0^5 (e^{-0.2t} - e^{-0.4t}) dt$$
$$= 0.4 \left(\frac{e^{-0.2t}}{-0.2} + \frac{e^{-0.4t}}{0.4} \right) \Big|_{t=0}^{t=5}$$
$$= 0.4(0.998941) = 0.399577$$

Comments: Candidates did okay on this question. Those that identified one of the approaches generally finished the problem successfully.

d)

i) $$\bar{a}_{x:\overline{5}|}^{11} = \int_0^5 e^{-\delta t}\ {}_tp_x^{11}\ dt = \int_0^5 e^{-(0.05)t}\ e^{-(0.2)t}\ dt$$
$$= \frac{1 - e^{-5(0.25)}}{0.25} = 2.853981$$

ii) $$\bar{a}_{x:\overline{5}|}^{12} = \int_0^5 e^{-\delta t}\ {}_tp_x^{12}\ dt = \int_0^5 e^{-(0.05)t} \left(e^{-(0.2)t} - e^{-(0.4)t} \right) dt$$
$$= 2.853981 - \frac{1 - e^{-5(0.45)}}{0.45} = 2.853981 - 1.988002 = 0.865979$$

iii) $$\bar{a}_{x:\overline{5}|}^{11} + \bar{a}_{x:\overline{5}|}^{12} + \bar{a}_{x:\overline{5}|}^{13} = \int_0^5 e^{-\delta t}\ ({}_tp_x^{11} + {}_tp_x^{12} + {}_tp_x^{13})\ dt$$
$$= \int_0^5 e^{-\delta t}\ dt = \bar{a}_{\overline{5}|}$$

Comments: Candidates who attempted this part did well on this question in general, with most who attempted it receiving full marks. Part iii) was the biggest stumbling block with many candidates not providing a rigorous enough demonstration.

e)

Under Option A the QL index is

$0.5\,\bar{a}^{11}_{x:\overline{5}|} + 0.2\,\bar{a}^{12}_{x:\overline{5}|} + 1.0\,\bar{a}^{13}_{x:\overline{5}|}$

Where the annuities are now calculated with $\mu^{23}_x = 1.0$

$\bar{a}^{11}_{x:\overline{5}|} = 2.853981$ from part d)i)

$$\bar{a}^{12}_{x:\overline{5}|} = \int_0^5 e^{-(0.05)t}\ {}_tp^{12}_x \quad dt = \int_0^5 e^{-(0.05)t} \int_0^t e^{-0.2r}(0.2)e^{-1.0(t-r)}dr \quad dt$$

$$= \int_0^5 e^{-(0.05)t} \left(e^{-\,t}\,(0.2)\,\frac{e^{0.8t}-1}{0.8}\right)\ dt$$

$$= 0.25\int_0^5 e^{-(0.05)t}\,(e^{-0.2\,t} - \ e^{-\,t})\ \ dt$$

$$= 0.25\left(2.853981 - \frac{1-e^{-5(1.05)}}{1.05}\right) = 0.25(2.853981 - 0.947383)$$

$$= 0.476649$$

$$\bar{a}^{13}_{x:\overline{5}|} = \bar{a}_{\overline{5}|} - \bar{a}^{11}_{x:\overline{5}|} - \bar{a}^{12}_{x:\overline{5}|}$$

$$= \frac{1-v^5}{\delta} - 2.853981 - 0.476649 = 4.423984 - 2.853981 - 0.476649$$

$$= 1.093354$$

$$\mathrm{QL}^A \ = \ 0.5(2.853981) + 0.2(0.476649) + 1.0(1.093354) = 2.61567$$

Under Option B the QL index is

$0.55\,\bar{a}^{11}_{x:\overline{5}|} + 0.3\,\bar{a}^{12}_{x:\overline{5}|} + 1.0\,\bar{a}^{13}_{x:\overline{5}|}$ where the annuities are calculated as in part (d)

$$\mathrm{QL}^B = 0.55(2.853981) + 0.3(0.865979) + 1.0(4.423984 - 2.853981 - 0.865979)$$

$$= 1.569690\ + 0.259794 + 0.704024 = 2.53351$$

Based on the improved QL, the health authority should select Option A.

Comments: This problem was not attempted by a majority of the candidates; those that did generally received most of the available credit (most common reason for missing points was not attempting a part of the question)

Question 4 Model Solution

---

a)

(i) $_{10}p_{50}^{A} = \frac{96{,}634.1}{98{,}576.4} = 0.9802965$

$_{10}p_{50}^{B} = ({}_{10}p_{50}^{A})^{0.8} = 0.9842059$

$_{10}p_{50}^{C} = {}_{10}p_{50}^{A}\, e^{-(0.01)(10)} = 0.887009$

(ii) Probability of a refund for a contract chosen randomly

$$P(\text{Refund}) = \frac{250(1-0.9802965)+150(1-0.9842059)+100(1-0.887009)}{500}$$

$$= \frac{4.925875 + 2.369115 + 11.2991}{500} = \frac{18.59409}{500} = 0.0371882$$

$$\text{P(contract B| refund)} = \frac{P(\text{contract B and refund})}{P(\text{refund})}$$
$$= \frac{\frac{150}{500}(1-0.9842059)}{0.037188} = 0.12741$$

Or

$$\text{P(contract B| refund)}$$
$$= \frac{150(1-0.9842059)}{250(1-0.9802965)+150(1-0.9842059)+100(1-0.887009)}$$
$$= \frac{2.369115}{4.925875 + 2.369115 + 11.2991}$$
$$= \frac{2.369115}{18.59409} = 0.12741$$

Comments: As a whole, most candidates did well on this part especially for part i where the vast majority of candidates received full credit.

b)

$$\begin{aligned} V[N] &= 250\,{}_{10}p_{50}^{A}(1-{}_{10}p_{50}^{A}) + 150\,{}_{10}p_{50}^{B}(1-{}_{10}p_{50}^{B}) + 100\,{}_{10}p_{50}^{C}(1-{}_{10}p_{50}^{C}) \\ &= 4.828818 + 2.331697 + 10.022403 = 17.182918 \end{aligned}$$

SD[N] = 4.1452

Comments: Most candidates did not know how to complete this calculation. Many candidates attempted to calculate $V[N]=E[N^2]-\{E[N]\}^2$ which can be done but is a much more difficult calculation.

c)

(i) $$\begin{aligned} {}_{10}p^{A}(50,0) &= {}_{8}p^{A}(50,0)\left(1-q^{A}(58,8)\right)\left(1-q^{A}(59,9)\right) \\ &= (0.987068)(1-q^{A}(58,0)(0.98)^{8})(1-q^{A}(59,0)(0.98)^{9}) \\ &= (0.987068)(1-(0.002736)(0.98)^{8})(1-0.003048)(0.98)^{9}) \\ &= (0.987068)(0.997672)(0.997459) = 0.982268 \end{aligned}$$

(ii) $${}_{10}p^{B}(50,0) = ({}_{10}p^{A}(50,0))^{0.8} = 0.985789$$
$${}_{10}p^{C}(50,0) = {}_{10}p^{A}(50,0)\,e^{-(0.01)(10)} = 0.888792$$

$$\begin{aligned} &\mathrm{P}(\text{contract B}|\text{refund}) \\ &= \frac{150(1-0.985789)}{250(1-0.982268)+150(1-0.985789)+100(1-0.888792)} \\ &= \frac{2.13165}{4.433+2.13165+11.1208} = \frac{2.13165}{17.68545} = 0.12053 \end{aligned}$$

Comments: Candidates did better on this part than on Part b but still did not do very well. In particular, very few candidates were able to do Part iii.

d)

Single-factor age-based mortality scales assume that improvement factor for a given age will persist for ever. This is not true especially for older ages.

Single-factor age-based mortality scales assume that the improvement factor for a given age is constant over time. Experience (heatmaps) indicate that the improvement factors vary over time. In practice, improvements depend on age, calendar year and year of birth (cohort effects).

Comments: Most candidates did not answer this part of the question.

Question 5 Model Solution

---

a)

$$E[L^g] = 100{,}000A_{50} + 900 + 100\ddot{a}_{50} - G[(0.95)\ddot{a}_{50} - 0.07\ddot{a}_{50:\overline{10|}} - 0.68]$$
$$= -0.5\,G$$

$$G = \frac{100{,}000\,A_{50} + 900 + 100\,\ddot{a}_{50}}{(0.95)\ddot{a}_{50} - 0.07\ddot{a}_{50:\overline{10|}} - 0.68 - 0.5}$$

$$G = \frac{100{,}000(0.18931) + 900 + 100(17.0245)}{0.95(17.0245) - 0.07(8.055) - 0.68 - 0.5} = \frac{21{,}533.45}{14.429425} = 1492.33$$

Comments: Almost all the candidates received full credit on this part. The candidates who did not generally used the equivalence principle instead of setting the $E[L^g] = -0.5G$

b)

$${}_{10}V^g = 100{,}000\,A_{60} - [(0.95)\,G - 100]\,\ddot{a}_{60}$$
$$= 100{,}000\,(0.29028) - [(0.95)(1492.33) - 100]\,(14.9041)$$

= 9388.67

Comments: Candidates did even better on the part. Those that did not receive full credit either did not try this part or made a minor calculation error.

c)

(i) $${}_{10}V^{Mod} = 100{,}000\,A_{60} - \pi_2\,\ddot{a}_{60} = 100{,}000(0.29028) - \pi_2\,(14.9041)$$
$$= {}_{10}V^g = 9388.67$$

$$\Rightarrow \pi_2 = \frac{29{,}028 - 9388.67}{14.9041} = 1317.71$$

(ii) $$\pi_1\,\ddot{a}_{50:\overline{10|}} + \pi_2\,{}_{10}E_{50}\,\ddot{a}_{60} = 100{,}000\,A_{50}$$
$$\pi_2\,\ddot{a}_{50} + (\pi_1 - \pi_2)\ddot{a}_{50:\overline{10|}} = 100{,}000\,A_{50}$$

$$\Rightarrow \pi_1 = \frac{100{,}000(0.18931) - (1317.71)(17.0245 - 8.055)}{8.055} = 882.905$$

Comments: Candidates did reasonably well on this problem. Most were able to calculate the premium after 10 years but significantly fewer candidates were able to calculate the premiums for the first 10 years. A common mistake was to include expenses.

d)

We know that the EPV of the insurance benefits are the same under both reserving methods (same policy) and the reserves at time 10 are equal.
Then the reserves at time 5 plus the EPV of premiums between 5 and 10 must be equal. Since the premium used under the modified method is $\pi_1 = 882.905$ is less than the gross premium net of expenses (.88G – 100=1213.24), we must have ${}_5V^{Mod} > {}_5V^{g}$.

Or

Under the modified method there is less premium received and used to accumulate future reserves than under the gross premium method. For the reserves under both methods to be equal at time 10, we must start with a larger reserve at time 5 under the modified method.

Comments: Most candidates skipped this part or generally could not explain the reserve relationships.

Question 6 Model Solutions

(a)

F: account balance

$$F = \int_0^{25} c\, S_{35+t}\, e^{\delta(25-t)}\, dt$$

$$F = \int_0^{25} (0.12)\,(50{,}000)\, e^{0.02t}\, e^{0.06(25-t)}\, dt$$

$$= \ 6000\, e^{0.06(25)} \left(\frac{1 - e^{-0.04(25)}}{0.04}\right) = 424{,}945.17$$

Comments: This part was done okay overall. However, many candidates treated the salary as an annual lump sum. Also, some candidates confused the projection of the salary between enrollment and payment, and its accumulation from payment until retirement.

(b)

(i) X: monthly income

$$
\begin{aligned}
424{,}945.17 &= 12X\left(\ddot{a}_{\overline{10}|}^{(12)} + {}_{10}E_{60}\,\ddot{a}_{70}^{(12)}\right)\\
&= 12X\left[7.92949 + (0.57864)\left(12.0083 - \tfrac{11}{24}\right)\right]\\
&= 12X\,[7.92949 + (0.57864)(11.54997)]\\
&= 12X\,(14.61276)
\end{aligned}
$$

$$\ddot{a}_{\overline{10}|}^{(12)} = \frac{1-v^{10}}{d^{(12)}} = \frac{1-1.05^{-10}}{0.04869} = 7.92949$$

⇨ $X = 424{,}945.17/[(12)(14.61276)] = 2423.37$

(ii) Final 1-year salary is

$$\int_{24}^{25}(50{,}000)e^{0.02t}dt = 81{,}617.17$$

$$R = \frac{Pension\ income\ in\ year\ post\ retirement}{Salary\ in\ year\ prior\ to\ retirement} = \frac{29{,}080.42}{81{,}617.17} = 0.3563$$

R=35.63%

Comments: Common mistakes were using UDD instead of Woolhouse to calculate the monthly annuity or using an annual annuity factor. Additionally, the replacement ratio compares the first year in retirement to the last year in employment. Many candidates calculate a rate of salary instead of the salary during the last year.

(c)

$$
\begin{aligned}
750{,}000 &= 12X\left(\ddot{a}^{(12)}_{60:60} + \frac{2}{3}\left(\ddot{a}^{(12)}_{\overline{60:60}} - \ddot{a}^{(12)}_{60:60}\right)\right) \\
&= 12X\left(\ddot{a}^{(12)}_{60:60} + \frac{2}{3}\left(\left(2\ddot{a}^{(12)}_{60} - \ddot{a}^{(12)}_{60:60}\right) - \ddot{a}^{(12)}_{60:60}\right)\right) \\
&= 12X\left(\frac{4}{3}\ddot{a}^{(12)}_{60} - \frac{1}{3}\ddot{a}^{(12)}_{60:60}\right) \\
&= 12X\left(\frac{4}{3}\left(\ddot{a}_{60} - \frac{11}{24}\right) - \frac{1}{3}\left(\ddot{a}_{60:60} - \frac{11}{24}\right)\right) \\
&= 12X\left(\frac{4}{3}\left(14.9041 - \frac{11}{24}\right) - \frac{1}{3}\left(13.2497 - \frac{11}{24}\right)\right) \\
&= 12X\,(14.9972)
\end{aligned}
$$

X=4167.44

Comments: There were many ways to calculate the annuity values in this part, but it was not well done. Candidates do not seem to have a good handle on annuities whose payment depends on a two-life status.

(d)

(i) (1) (4167.44) + 1000 = 5167.44
(ii) (2/3)(4167.44) + 1000 = 3778.29
(iii) (2/3)(4167.44) + 500 = 3278.29

Comments: Many candidates did not try this part, but those who did it mostly did it well.

(e)

$$750{,}000 = 12X\left(\frac{4}{3}\ddot{a}_{60}^{(12)} - \frac{1}{3}\ddot{a}_{60:60}^{(12)}\right)$$

$$- 12\,(1000)\left(\frac{4}{3}\ddot{a}_{65}^{(12)} - \frac{1}{3}\ddot{a}_{65:65}^{(12)}\right){}_5E_{60:60} \qquad \text{(i.e. both alive at 65)}$$

$$- 12\,(1000)\,2\left(\frac{2}{3}\ddot{a}_{65}^{(12)}\right){}_5E_{60}\,({}_5q_{60}) \quad \text{(i.e. only one alive)}$$

Where $\frac{4}{3}\ddot{a}_{60}^{(12)} - \frac{1}{3}\ddot{a}_{60:60}^{(12)} = 14.9972$ from part c).

$$750{,}000 = 12X\,(14.9972)$$

$$- 12\,(1000)\left(\frac{4}{3}\left(13.5498 - \frac{11}{24}\right) - \frac{1}{3}\left(11.6831 - \frac{11}{24}\right)\right)(0.75057)$$

$$- 12\,(1000)\,2\left(\frac{2}{3}\left(13.5498 - \frac{11}{24}\right)\right)(0.76687)(0.02126)$$

$$= 12X\,(14.9972) - 123{,}516.35 - 3414.95$$

$\Rightarrow X =$ 4872.74

$${}_5E_{60:60} = ({}_5E_{60})^2(1+i)^5 = (0.76687)^2(1.05)^5 = 0.75057$$

$${}_5E_{60:60} = v^5({}_5p_{60})^2 = (1.05)^{-5}\left(\frac{94{,}579.7}{96{,}634.1}\right)^2 = (1.05)^{-5}(0.97874)^2 = 0.75057$$

$${}_5q_{60} = 1 - {}_5p_{60} = 1 - \frac{94{,}579.7}{96{,}634.1} = 1 - 0.97874 = 0.02126$$

Comments: Few attempted it and very few got it right. One has to remember to think of all the possible cases at age 65 and treat them correctly.