

## Financial Mathematics Exam—June 2017 (August 2016 revisions)

**IMPORTANT NOTICE** – This version of the syllabus is presented for planning purposes. The syllabus for this exam administration is not considered official until it is posted on the Exam FM home page for the June 2017 exam administration.

**IMPORTANT NOTICE** – This August 2016 version has four major changes. Learning objectives G1 and G2 have been updated to include the topics of: amortizing swaps, accreting swaps, and market value. Learning objective H has been updated to align with the study note on that topic. One section from each of the recommended texts has been deleted (that topic is now covered in one of the new study notes). These changes are highlighted in yellow. The three study notes are now available, with links in these documents. It is possible they will be updated before the syllabus becomes official.

The Financial Mathematics exam is a three-hour exam that consists of 35 multiple-choice questions and is administered as a computer-based test. For additional details, please refer to [Exam Rules](#)

The goal of the syllabus for this examination is to provide an understanding of the fundamental concepts of financial mathematics, and how those concepts are applied in calculating present and accumulated values for various streams of cash flows as a basis for future use in: reserving, valuation, pricing, asset/liability management, investment income, capital budgeting, and valuing contingent cash flows.

The Financial Mathematics Exam assumes a basic knowledge of calculus and an introductory knowledge of probability.

Please check the [Updates](#) section on this exam's home page for any changes to the exam or syllabus.

Each multiple-choice problem includes five answer choices identified by the letters A, B, C, D, and E, only one of which is correct. Candidates must indicate responses to each question on the computer. Candidates will be given three hours to complete the exam.

As part of the computer-based testing process, a few pilot questions will be randomly placed in the exam (paper and pencil and computer-based forms). These pilot questions are included to judge their effectiveness for future exams, but they will NOT be used in the scoring of this exam. All other questions will be considered in the scoring. All unanswered questions are scored incorrect. Therefore, candidates should answer every question on the exam. There is no set requirement for the distribution of correct answers for the multiple-choice preliminary examinations. It is possible that a particular answer choice could appear many times on an examination or not at all. Candidates are advised to answer each question to the best of their ability, independently from how they have answered other questions on the examination.

Since the CBT exam will be offered over a period of a few days, each candidate will receive a test form composed of questions selected from a pool of questions. Statistical scaling methods are used to ensure within reasonable and practical limits that, during the same testing period of a few days, all forms of the test are comparable in content and passing criteria. The methodology that has been adopted is used by many credentialing programs that give multiple forms of an exam.

The ranges of weights shown in the Learning Objectives below are intended to apply to the large majority of exams administered. On occasion, the weights of topics on an individual exam may fall outside the published range. Candidates should also recognize that some questions may cover multiple learning objectives.

## LEARNING OBJECTIVES

- A. Time Value of Money (10-15%)
1. The candidate will be able to define and recognize the definitions of the following terms: interest rate (rate of interest), simple interest, compound interest, accumulation function, future value, current value, present value, net present value, discount factor, discount rate (rate of discount), convertible m-thly, nominal rate, effective rate, inflation and real rate of interest, force of interest, equation of value.
  2. The candidate will be able to:
    - a. Given any three of interest rate, period of time, present value, current value, and future value, calculate the remaining item using simple or compound interest. Solve time value of money equations involving variable force of interest.
    - b. Given any one of the effective interest rate, the nominal interest rate convertible m-thly, the effective discount rate, the nominal discount rate convertible m-thly, or the force of interest, calculate any of the other items.
    - c. Write the equation of value given a set of cash flows and an interest rate.
- B. Annuities/cash flows with payments that are not contingent (15-20%)
1. The candidate will be able to define and recognize the definitions of the following terms: annuity-immediate, annuity due, perpetuity, payable m-thly or payable continuously, level payment annuity, arithmetic increasing/decreasing annuity, geometric increasing/decreasing annuity, term of annuity.
  2. For each of the following types of annuity/cash flows, given sufficient information of immediate or due, present value, future value, current value, interest rate, payment amount, and term of annuity, the candidate will be able to calculate any remaining item.
    - a. Level annuity, finite term
    - b. Level perpetuity
    - c. Non-level annuities/cash flows
      - i) Arithmetic progression, finite term
      - ii) Arithmetic progression, perpetuity
      - iii) Geometric progression, finite term
      - iv) Geometric progression, perpetuity
      - v) Other non-level annuities/cash flows
- C. Loans (15-20%)
1. The candidate will be able to define and recognize the definitions of the following terms: principal, interest, term of loan, outstanding balance, final payment (drop payment, balloon payment), amortization, sinking fund.
  2. The candidate will be able to:
    - a. Given any four of term of loan, interest rate, payment amount, payment period, principal, calculate the remaining item.
    - b. Calculate the outstanding balance at any point in time.
    - c. Calculate the amount of interest and principal repayment in a given payment.
    - d. Given the quantities, except one, in a sinking fund arrangement calculate the missing quantity.
    - e. Perform similar calculations to a-d when refinancing is involved.
- D. Bonds (15-20%)
1. The candidate will be able to define and recognize the definitions of the following terms: price, book value, amortization of premium, accumulation of discount, redemption value, par value/face value, yield rate, coupon, coupon rate, term of bond, callable/non-callable.

2. Given sufficient partial information about the items listed below, the candidate will be able to calculate the any of the remaining items.
    - a. Price, book value, amortization of premium, accumulation of discount
    - b. Redemption value, face value
    - c. Yield rate
    - d. Coupon, Coupon rate
    - e. Term of bond, point in time that a bond has a given book value, amortization of premium, or accumulation of discount
- E. General Cash Flows and Portfolios (10-15%)
1. The candidate will be able to define and recognize the definitions of the following terms: yield rate/rate of return, dollar-weighted rate of return, time-weighted rate of return, current value, duration (Macaulay and modified), convexity (Macaulay and modified), portfolio, spot rate, forward rate, yield curve, stock price, stock dividend.
  2. The candidate will be able to:
    - a. Calculate the dollar-weighted and time-weighted rate of return.
    - b. Calculate the duration and convexity of a set of cash flows.
    - c. Calculate either Macaulay or modified duration given the other.
    - d. Use duration to approximate the change in present value due to a change in interest rate.
      - i. Using 1<sup>st</sup>-order linear approximation based on modified duration.
      - ii. Using 1<sup>st</sup>-order approximation based on Macaulay duration.
    - e. Calculate the price of a stock using the dividend discount model.
- F. Immunization (10-15%)
1. The candidate will be able to define and recognize the definitions of the following terms: cash flow matching, immunization (including full immunization), Redington immunization.
  2. The candidate will be able to:
    - a. Construct an investment portfolio to fully immunize a set of liability cash flows.
    - b. Construct an investment portfolio to match present value and duration of a set of liability cash flows.
    - c. Construct an investment portfolio to exactly match a set of liability cash flows.
- G. Interest Rate Swaps (0-10%)
1. The candidate will be able to define and recognize the definitions of the following terms: swap rate, swap term or swap tenor, notional amount, market value of a swap, settlement dates, settlement period, counterparties, deferred swap, amortizing swap, accreting swap, interest rate swap net payments.
  2. The candidate will be able to:
    - a. Calculate the swap rate in an interest rate swap, deferred or otherwise, and with either constant or varying notional amount.
    - b. Calculate the market value of an interest rate swap, deferred or otherwise, and with either constant or varying notional amount.
- H. Determinants of Interest Rates (0-10%)
1. The candidate will be able to define and recognize the components of interest rates including: real risk-free rate, inflation rate, default risk premium, liquidity premium, and maturity risk premium.
  2. The candidate will be able to explain how the components of interest rates apply in various contexts, such as commercial loans, mortgages, credit cards, bonds, government securities.
  3. The candidate will be able to explain the roles of the Federal Reserve and the FOMC in carrying out fiscal policy and monetary policy and the tools used by the Federal Reserve and

- the FOMC including targeting the Federal Funds rate, setting reserve requirements, and setting the discount rate.
4. The candidate will be able to explain the theories of why interest rates differ by term, including liquidity preference (opportunity cost), expectations, preferred habitat, and market segmentation.
  5. The candidate will be able to explain how interest rates differ from one country to another (e.g., U.S. vs. Canada).

## TEXT REFERENCES

Knowledge and understanding of the financial mathematics concepts are significantly enhanced through working out problems based on those concepts. Thus, in preparing for the Financial Mathematics exam, whichever of the source textbooks candidates choose to use, candidates are encouraged to work out the textbook exercises related to the listed readings.

### Suggested Textbooks for Learning Objectives in Sections A-F

There is not a single textbook required for the learning objectives in Sections A-F. The texts listed below are representative of the textbooks available to cover the material on which the candidate may be tested. Not all topics may be covered at the same level in each text. Listed sections may include introductory material, summary material, and problems that are not part of the learning objectives. The candidate may wish to use one or more texts in his/her preparation for the examination.

**IMPORTANT NOTICE – The curriculum committee reviews textbooks from time to time. It is possible that the final syllabus for this exam will have additions or deletions to the list below.**

Broverman, S.A., Mathematics of Investment and Credit (Sixth Edition), 2015, ACTEX Publications, ISBN 978-1-62542-485-3:

Chapter 1 (excluding 1.2.1, 1.7, and 1.8)

Chapter 2 (excluding 2.4.2 and 2.4.3)

Chapter 3 (excluding 3.2.1, 3.2.2 and 3.4)

Chapter 4 (excluding 4.3.2)

Chapter 5 (excluding 5.1.4, the investment year method portion of 5.3.1, 5.3.2 and 5.3.3)

Chapter 6 (excluding 6.2 and 6.4)

Chapter 7 (excluding 7.1.6 and 7.3)

In addition, candidates are not responsible for formula (7.6) and its uses. Thus, excluded is the part of 7.1.2 from the paragraph starting “Although” on page 369 to the end of that section, the part of 7.1.3 from the paragraph starting “It” on page 372 to the end of that section, and the last sentence of 7.1.5.

Chapter 9 (9.1 only)

At various places in the sections of this text that are listed above there are statements indicating that more information is available in sections that are not listed above. Candidates are not responsible for this additional information.

Daniel, J.W., and Vaaler, L.J.F., *Mathematical Interest Theory* (Second Edition), 2009, The Mathematical Association of America, ISBN: 978-0883857540 : **[Candidates may also use the First Edition of *Mathematical Interest Theory* (Publisher: Prentice Hall, ISBN: 0-13-147285-2). The same chapter references apply.]**

Chapter 1 (excluding 1.13 and 1.14)

Chapter 2

Chapter 3 (excluding 3.10, 3.12, and the investment year method portion of 3.13)

Chapter 4

Chapter 5

Chapter 6 (excluding 6.10)

Chapter 7 (excluding 7.2, 7.3, and 7.4)

Chapter 8 (8.3 only)

Chapter 9 (excluding 9.5)

Kellison, S.G., *The Theory of Interest* (Third Edition), 2009, Irwin/McGraw-Hill, ISBN: 125921544X or 978-1259215445:

Chapter 1

Chapter 2

Chapter 3 (excluding 3.9)

Chapter 4

Chapter 5 (excluding 5.7 and 5.8)

Chapter 6 (excluding 6.8, 6.9, and 6.11)

Chapter 7 (excluding the investment year method portion of 7.7, 7.8, 7.9, and 7.10)

**(Section 9.4 has been removed)**

Chapter 10 (excluding 10.6 and 10.7)

Chapter 11 (excluding 11.4 and 11.9)

Ruckman, C.; and Francis, J., *Financial Mathematics: A Practical Guide for Actuaries and other Business Professionals* (Second Edition), 2005, BPP Professional Education, ISBN: 0-9753136-4-9:

Chapter 1

Chapter 2

Chapter 3

Chapter 4

Chapter 5 (excluding 5.2 and the investment year method portion of 5.4)

Chapter 6 (excluding 6.1.6, 6.1.7, 6.4, and 6.5)

Chapter 7 (excluding 7.4)

Chapter 8 (excluding 8.4, 8.5, and 8.6)

Chan, Wai-Sum, and Tse, Yiu-Kuen, *Financial Mathematics for Actuaries*, Updated Edition, 2013, McGraw-Hill Education (Asia), ISBN: 978-1259011481 [Candidates may also use the 2011 Edition of *Financial Mathematics for Actuaries* with [Supplementary Notes for Financial Mathematics for Actuaries](#) for Chapter 8. The same chapter references apply.]

Chapter 1

Chapter 2 (excluding 2.4)

Chapter 3

Chapter 4 (excluding 4.5, the investment year method portion of 4.6, 4.7, and 4.8)

Chapter 5

Chapter 6

Chapter 7

Chapter 8 (excluding 8.6, 8.7, and 8.8)

## ADDITIONAL REFERENCES

**IMPORTANT NOTICE** – There are three new study notes. Two of them are currently available. It is possible that edits will be made prior to them becoming official. It is expected that the third note will be available in August 2016. An announcement of availability will be posted to the Exam FM home page. These notes will be available as free downloads.

There are three study notes that are required reading for this examination. They are:

- For Learning Objective E.2.d.ii – A [study note](#) on using Macaulay duration to approximate the effect of interest rate changes on present values. [It is Using Duration and Convexity. Sections 1-4 are required reading for this examination.](#) (11-3-16)
- For Learning Objective G – A [study note](#) on interest rate swaps. [It is Interest Rate Swaps. The entire note is required reading.](#)
- For Learning Objective H – A [study note](#) on the determinants of interest rates. [It is Determinants of Interest Rates. The entire note is required reading.](#) (11-3-16)

## OTHER RESOURCES:

**IMPORTANT NOTICE** – It is likely the notation and terminology note will be updated to reflect the new readings. The links below relate to the current syllabus.

[Notation and terminology used for Exam FM](#)

[All released exam papers](#), since 2000, can be found here.

Interest Theory Sample [Questions](#) and [Solutions](#)

[Review of Calculator Functions for the Texas Instruments BA-35](#)

[Review of Calculator Functions for the Texas Instruments BA II Plus](#)

Although several different [calculators](#) are allowed for this exam, the BAII Plus is recommended due to its ability to solve for interest rates.

[Online Sample Exam FM](#)

## Models for Financial Economics Exam—July 2017

**IMPORTANT NOTICE – This version of the syllabus is presented for planning purposes. The syllabus for this exam administration is not considered official until it is posted on the Exam MFE home page.**

The Models for Financial Economics is a three-hour exam that consists of 30 multiple-choice questions. Also, a [normal distribution calculator](#) will be available during the test by clicking a link on the item screen. Details are available on the [Prometric Web Site](#).

The purpose of the syllabus is to develop the candidate's knowledge of the theoretical basis of certain actuarial models and the application of those models to insurance and other financial risks. A thorough knowledge of calculus, probability, and interest theory is assumed.

Formulas are provided for the density and distribution functions for the standard normal and lognormal random variables. For paper and pencil examinations, tables of the standard normal distribution function are provided. Since the tables will be provided to the candidate at the examination, candidates will not be allowed to bring copies of the tables into the examination room. For CBT candidates, a normal distribution calculator is provided. See the link below for more information.

Note: It is anticipated that candidates will have done the relevant exercises in the textbooks.

Check the [Updates](#) section of the web site for any changes to the exam or syllabus.

The ranges of weights shown are intended to apply to the large majority of exams administered. On occasion, the weights of topics on an individual exam may fall outside the published range. Candidates should also recognize that some questions may cover multiple learning outcomes.

Each multiple-choice problem includes five answer choices identified by the letters A, B, C, D, and E, only one of which is correct. Candidates must indicate responses to each question on the computer.

As part of the computer-based testing process, a few pilot questions will be randomly placed in the exam (paper and pencil and computer-based forms). These pilot questions are included to judge their effectiveness for future exams, but they will NOT be used in the scoring of this exam. All other questions will be considered in the scoring. All unanswered questions are scored incorrect. Therefore, candidates should answer every question on the exam. There is no set requirement for the distribution of correct answers for the SOA/CIA multiple-choice preliminary examinations. It is possible that a particular answer choice could appear many times on an examination or not at all. Candidates are advised to answer each question to the best of their ability, independently from how they have answered other questions on the examination.

Since the CBT exam will be offered over a period of a few days, each candidate will receive a test form composed of questions selected from a pool of questions. Statistical scaling methods are used to ensure within reasonable and practical limits that, during the same testing period of a few days, all forms of the test are comparable in content and passing criteria. The methodology that has been adopted is used by many credentialing programs that give multiple forms of an exam.

## LEARNING OBJECTIVES – MODELS FOR FINANCIAL ECONOMICS

- I. Introductory Derivatives (10%-15%)
  - A. Stock as an underlying asset

The candidate will be able to define and recognize the definitions of the following terms:  
Nondividend-paying stocks, stocks paying discrete dividends, stocks paying dividends continuously at a rate proportional to the price, short selling, long and short positions, bid and ask prices, bid-ask spread, net profit of long and short positions
  - B. Forward contracts and prepaid forward contracts on stocks

The candidate will be able to:

    1. Define and recognize the definitions of the following terms:  
Forward contract, prepaid forward contracts, outright purchase, fully leveraged purchase, payoff of long and short forward, net profit of long and short forward
    2. Determine forward and prepaid forward prices by the principle of no-arbitrage.
    3. Construct a synthetic forward from the underlying stock and a risk-free asset.
    4. Identify arbitrage opportunities when the no-arbitrage forward price is different from the market forward price.
    5. Recognize that forward price is less than the expected future stock price.
  - C. Futures contracts

The candidate will be able to:

    1. Define and recognize the definitions of the following terms:  
Marking to market, margin balance, maintenance margin, margin call
    2. Evaluate an investor's margin balance based on changes in asset values.
- II. General Properties of Options (25%-30%)
  - A. Option Contracts

The candidate will be able to:

    1. Define and recognize the definitions of the following terms:  
Call and put options, expiration date, strike price / exercise price, moneyness, European option, American option, Bermudan option, payoff and net profit of long and short option positions
    2. Explain the cash flow characteristics of the following exotic options:  
Asian (both arithmetic and geometric), barrier, compound, lookback
  - B. Option strategies and risk management

The candidate will be able to:

    1. Recognize that a long put can be used as an insurance strategy for a long stock position and a long call can be used as an insurance strategy for a short stock position.
    2. Identify and explain how the following option strategies can be used as tools to manage financial risk or speculate on price or volatility:  
option spreads (bull, bear, box, ratio), collar, zero-cost collar, straddle, strangle, butterfly spread
    3. Evaluate the payoff and profit of the strategies above.
  - C. General Properties of Options

The candidate will be able to

    1. Apply put-call parity to European options on the following underlying assets:  
Stock (no dividends, discrete and continuous dividends), currency, futures contract
    2. Recognize generalized parity for European exchange options, put-call duality for European currency options, and put-call parity for barrier and compound options.
    3. Compare options with respect to maturity and strike.
    4. Identify factors affecting the early exercising of American options and the situations where the values of European and American options are the same.



### III. The Binomial Option Pricing Model (10%)

The candidate will be able to:

- A. Price options under a one-period binomial model on a nondividend-paying stock by:
  - 1. applying the principle of no-arbitrage, and identify arbitrage opportunities if any.
  - 2. applying the risk-neutral pricing formula.
- B. Extend the one-period binomial model on stocks in the following directions:
  - 1. to other underlying assets, including stock paying dividends continuously at a rate proportional to its price, currency, and futures contract.
  - 2. to a multi-period setting for pricing European and American options.
- C. Construct a binomial model from market stock price data using historical volatility and the following methods:
  - Forward binomial tree, Cox-Ross-Rubinstein tree, lognormal tree
- D. Understand option pricing using real probabilities and calculate the appropriate risk-adjusted interest rate for discounting.

### IV. The Black-Scholes Option Pricing Model (25-30%)

The candidate will be able to:

- A. Recognize the underlying assumptions behind the Black-Scholes model.
  - B. Explain the properties of a lognormal distribution and calculate the following for future stock prices under the Black-Scholes model:
    - 1. probabilities and percentiles
    - 2. means and variances
    - 3. conditional expectations  $E(S_t | S_t > K)$  and  $E(S_t | S_t < K)$
  - C. Deduce the analytic pricing formulas for the following European options using risk-neutral pricing formulas:
    - 1. cash-or-nothing calls and puts
    - 2. asset-or-nothing calls and puts
    - 3. ordinary calls and puts (the Black-Scholes formula)
    - 4. gap calls and puts
  - D. Explain the concepts underlying the risk-neutral approach to valuing derivative securities.
  - E. Implement the risk-neutral pricing formula using Monte-Carlo simulation:
    - 1. Simulate standard normal random variates by inverse transformation.
    - 2. Estimate prices of path-independent and path-dependent options, and compute the standard deviation of the estimate.
    - 3. Use the following variance reduction techniques to accelerate convergence:
      - Antithetic variate, stratified sampling, control variate
  - F. Generalize the Black-Scholes formula to price exchange options.
  - G. Estimate a stock's expected rate of appreciation and historical volatility from stock price data.
  - H. Understand the concept of implied volatility.
- ### V. Option Greeks and Risk Management (10%-15%)
- The candidate will be able to:
- A. Interpret and compute the following under the Black-Scholes model:
    - 1. Option Greeks (Delta, Gamma, Theta, Vega, Rho, and Psi)
    - 2. Option elasticity, Sharpe ratio and instantaneous risk premium for both an option and a portfolio of options and the underlying stock.
  - B. Approximate option prices using delta, gamma and theta.
  - C. Recognize the relationship among delta, gamma and theta (the Black-Scholes equation)
  - D. Explain and demonstrate how to control stock price risk using the methods of delta-hedging and gamma-hedging.

## VI. Interest Rate Derivatives (10%)

The candidate will be able to:

- A. Price interest rate derivatives under a binomial tree for interest rates.
- B. Recognize the features of a Black-Derman-Toy tree.
- C. Price interest rate caplets, floorlets and bond calls and puts by applying the Black formula.
- D. Apply put-call parity to European options on zero-coupon bonds.

**Note:** Concepts, principles and techniques needed for Exam MFE are covered in the reference listed below. Candidates and professional educators may use other references, but candidates should be very familiar with the notation and terminology used in the listed references.

**IMPORTANT NOTICE – The curriculum committee reviews textbooks and readings from those textbooks from time to time. It is possible that the final syllabus for this exam will have additions or deletions to the list below.**

### Text – Models for Financial Economics

*Derivatives Markets* (Third Edition), 2013, by McDonald, R.L., Pearson Education,  
ISBN: 978-0-32154-308-0

Chapter 1,

Chapter 2, Sections 2.1-2.4,

Chapter 3,

Chapter 5, Sections 5.1-5.2, Section 5.3 (through the middle of p.136),

Section 5.4 (through the top of p.143),

Chapter 9,

Chapter 10, Sections 10.1-10.5, Section 10.6 (through the middle of p.315),

Chapter 11, Sections 11.1-11.3, Appendix 11.A,

Chapter 12, Sections 12.1-12.5, Appendix 12.A, Appendix 12.B,

Chapter 13,

Chapter 14, Sections 14.1-14.3, Section 14.4 (through the bottom of p.419),

Sections 14.5-14.6,

Chapter 18, Sections 18.1-18.5, Appendix 18.A,

Chapter 19, Sections 19.1-19.5

Chapter 23, Section 23.1 (but with only those definitions in Tables 23.1 and 23.2 that are relevant to Section 23.1), the top half of p.714 (Re: Lookback calls and puts),

Chapter 25, Section 25.1 (through the bottom of p.754), Section 25.4 (through the middle of p.773), Section 25.5 (through the middle of p.781),

Appendices B.1, and C

Unless otherwise stated, chapter appendices are not included in the required readings from this text.

### Other Resources – Models for Financial Economics

**IMPORTANT NOTICE – It is possible that some of the resources listed below will be updated to reflect the new syllabus. The links below relate to the current syllabus.**

[Exam MFE Formulas and Tables for paper/pencil](#)

Formulas and Tables for CBT:

- A [normal distribution calculator](#) will be available during the test by clicking buttons on the item screen.
- [Formula document](#)

[Some Remarks on \*Derivatives Markets\*](#)

[All released exam papers](#), since 2000 can be found here.

[Exam MFE Sample Questions and Solutions \(1-76\)](#)