**Smoothing and Measuring Discrete Risks**

Vytaras Brazauskas, Ponmalar Ratnam

**Abstract**

Many risk measures can be defined through the quantile function of the underlying loss variable (e.g., a class of distortion risk measures). When the loss variable is discrete or mixed, however, the definition of risk measures has to be broadened, which makes statistical inference trickier. To facilitate a straightforward transition from the risk measurement literature of continuous loss variables to that of discrete, in this paper we study smoothing of quantiles for discrete variables. Smoothed quantiles are defined using the theory of fractional or imaginary order statistics, which was originated by Stigler (1977). To prove consistency and asymptotic normality of sample estimators of smoothed quantiles, we utilize the results of Wang and Hutson (2011) and generalize them to vectors of smoothed quantiles.

Further, we thoroughly investigate extensions of this methodology to discrete populations with infinite support (e.g., Poisson and zero-inflated Poisson distributions).

Finally, applications of smoothed quantiles to risk measurement (e.g., estimation of distortion risk measures such as value-at-risk, conditional tail expectation, and proportional hazards transform) are discussed and illustrated using automobile accident data. Comparisons between the classical (and linearly interpolated) quantiles and smoothed quantiles are performed as well.