Living to 100: Mortality Modelling

Modelling, Measurement and Management of Longevity Risk

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The 'Modelling, Measurement and Management of Longevity and Morbidity Risk' research programme is being funded by the ARC, the SoA and the CIA.

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ARC research program themes

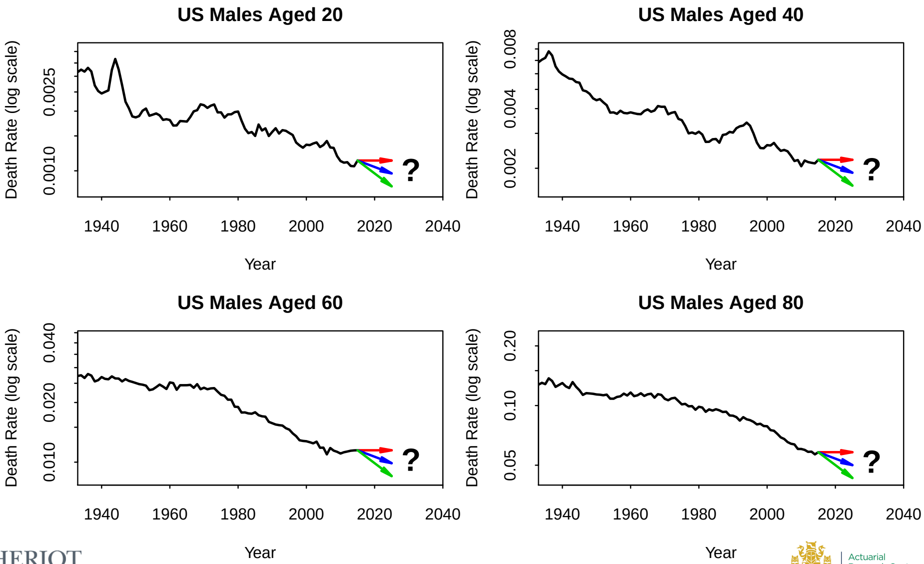
- Improved models for mortality
- Key drivers of mortality
- Management of longevity risk
- Morbidity risk modelling for critical illness insurance

Outline

- Part 1: All cause mortality modelling
  - Introduction to stochastic mortality models
  - Why?
  - Example applications
- Part 2: Key drivers
  - Education level
  - Cause of death
  - Health inequalities

Part 1: All Cause Mortality Modelling

US Historical Death Rates



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Graphical Diagnostics

- Mortality is falling
- Different improvement rates at different ages
- Different improvement rates over different periods
- Improvements are random
  - Short term fluctuations
  - Long term trends
- All *stylised facts*
- Other countries:
  - Some similarities
  - Some different patterns

Why do we need stochastic mortality models?

Data $\Rightarrow$ future mortality is uncertain

- Good risk management
- Setting risk reserves
- Regulatory capital requirements (e.g. Solvency II)
- Life insurance contracts with embedded options
- Pricing and hedging mortality-linked securities

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Aims:

- to develop the best models for forecasting future uncertain mortality;
  - general desirable criteria
  - complexity of model $\leftrightarrow$ complexity of problem;
  - longevity versus brevity risk;
- measurement of risk;
- valuation of future risky cashflows.

Management

Aims:

- active management of mortality and longevity risk;
    - internal (e.g. product design; natural hedging)
    - over-the-counter deals (OTC)
    - securitisation
- part of overall package of good risk management.

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Stochastic Mortality Models

Two basic examples:

- Lee-Carter Model (1992)
- Cairns-Blake-Dowd Model (CBD) (2006)

Stochastic model:

- Central forecast
- Uncertainty around the central forecast

Good ERM $\Rightarrow$ Use a combination of stochastic projections *plus* some deterministic scenarios or stress tests

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Death rate:

$$m(t,x) = \frac{D(t,x)}{E(t,x)} = \frac{\text{deaths}(t,x)}{\text{average population}(t,x)}$$

Year t; Age x.

$$\text{LC:} \quad \log m(t,x) = \alpha(x) + \beta(x)\kappa(t)$$

- $\alpha(x)$ = base table; age effect
- $\beta(x)$ = age effect
- $\kappa(t)$ = period effect

The Lee-Carter Model

$$\log m(t,x) = \alpha(x) + \beta(x)\kappa(t)$$

- Estimate $\alpha(x)$, $\beta(x)$, $\kappa(t)$ from historical data
- "Traditional" model:
  - Fit a random walk model to historical $\kappa(t)$
  - Simulate future scenarios for $\kappa(t)$
  - Calculate future mortality scenarios given $\kappa(t)$
- Alternative models for $\kappa(t)$ can be used

The CBD Model

$$q(t,x) = \text{Probability of death in year } t \text{ given initially exact age } x.$$

$$q(t,x) \approx 1 - \exp[-m(t,x)]$$

$$\text{logit } q(t,x) = \log\left(\frac{q}{1-q}\right) = \kappa_1(t) + \kappa_2(t)(x - \bar{x})$$

- $\kappa_1(t) =$ period effect; affects level
- $\kappa_2(t) =$ period effect; affects slope
- $\bar{x} =$ mean age
- Captures big picture at higher ages

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Comparison

- LC $\Rightarrow$ all mortality rates dependent on a single $\kappa(t)$
  $\Rightarrow$ rates at all ages perfectly correlated
- CBD $\Rightarrow$ simpler age effects (1 and $x - \bar{x}$)
  but two period effects
  $\Rightarrow$ richer correlation structure
- CBD linearity $\Rightarrow$
  not good for younger ages
- Historical data:
  Different improvements at different ages over different time periods
  $\Rightarrow$ need more than one period effect

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Applications: Scenario Generation

Example: the Lee Carter Model

- (Applied to a synthetic dataset)
- $\log m(t,x) = \alpha(x) + \beta(x)\kappa(t)$
- Choose a time series model for $\kappa(t)$
- Calibrate the time series parameters using data up to the current time (time 0)
- Generate $j = 1, \ldots, N$ stochastic scenarios of $\kappa(t)$

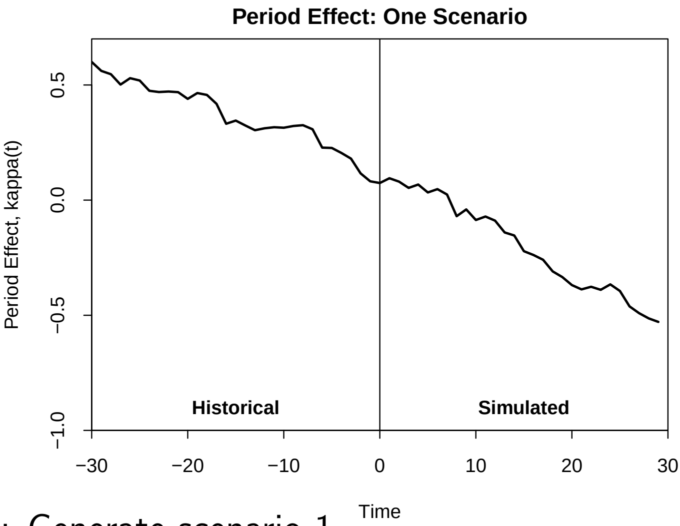
$$\kappa_1(t), \ldots, \kappa_N(t)$$

Applications: Scenario Generation

- Generate N scenarios for the future $m(t,x)$ $m_j(t,x)$ for $j = 1, \ldots, N$, $t = 0, 1, 2, \ldots,$ $x = x_0, \ldots, x_1$
- Generate N scenarios for the survivor index, $S_j(t,x)$
- Calculate financial functions

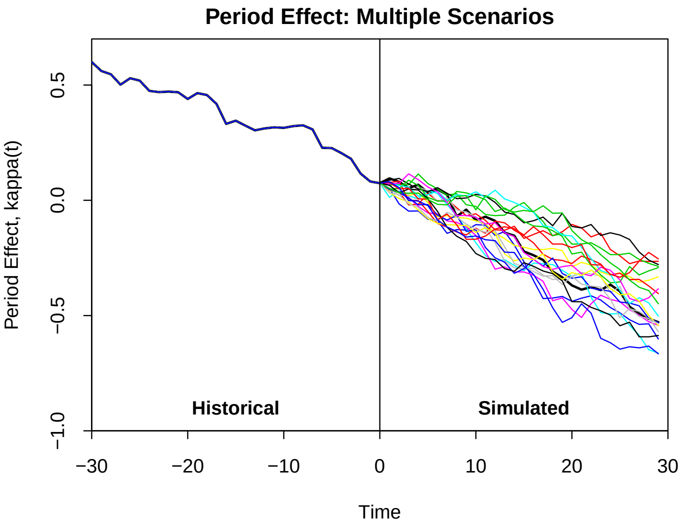
$+$ variations for some financial applications.

Applications: Scenario Generation, $\kappa(t)$

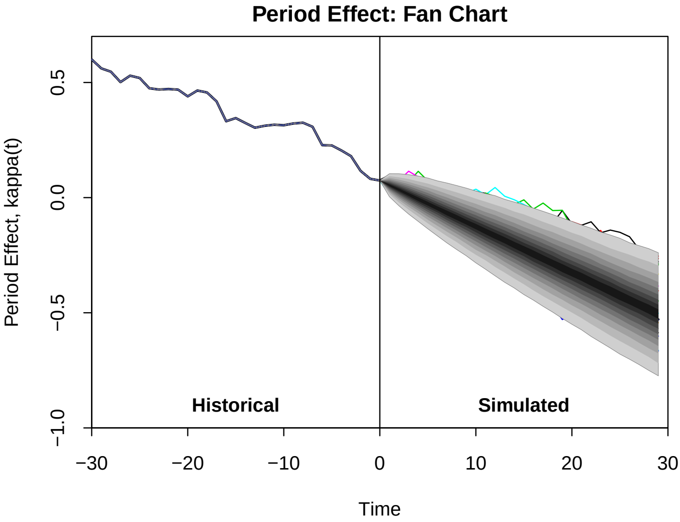


$\kappa(t)$: Generate scenario 1

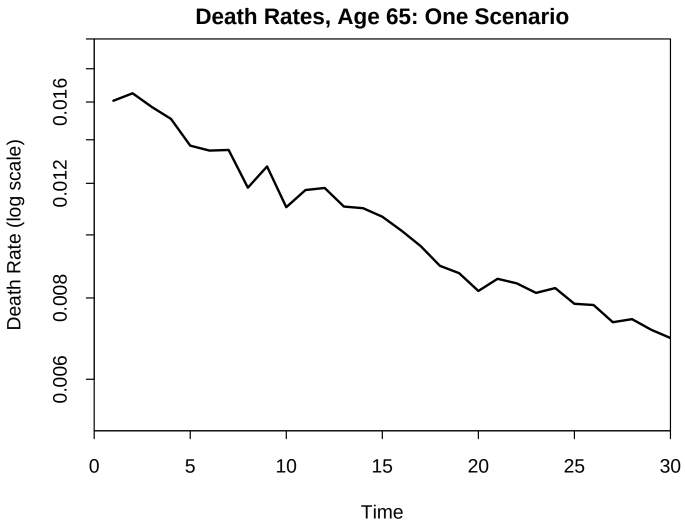
Applications: Scenario Generation, $\kappa(t)$



Applications: Scenario Generation, $\kappa(t)$



Applications: Scenario Generation, Future $m(t, x)$



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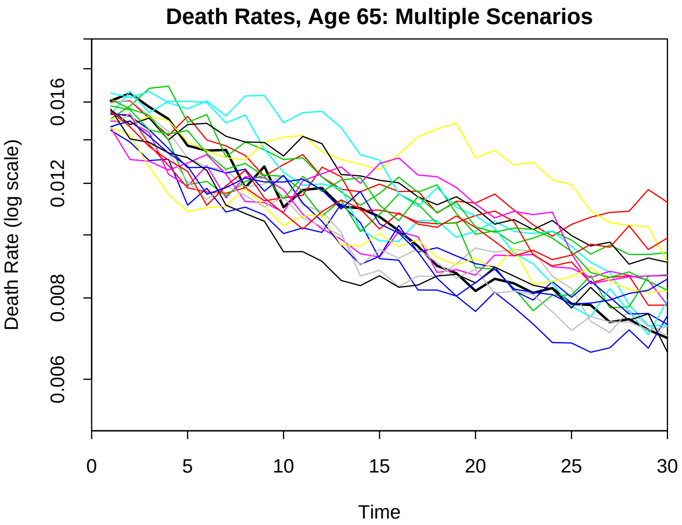

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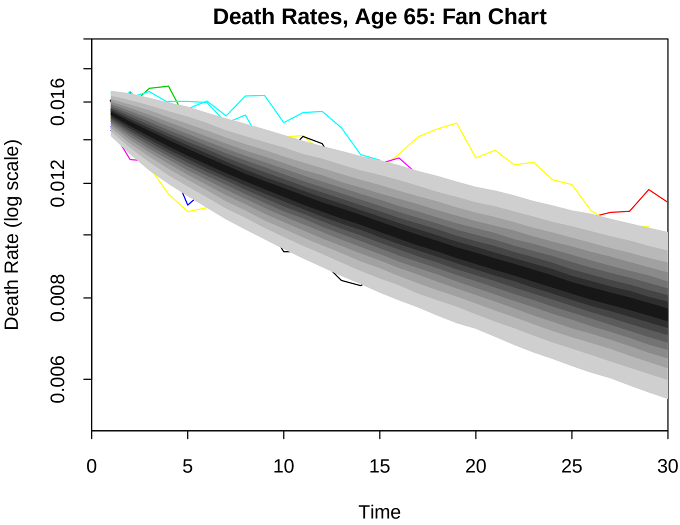
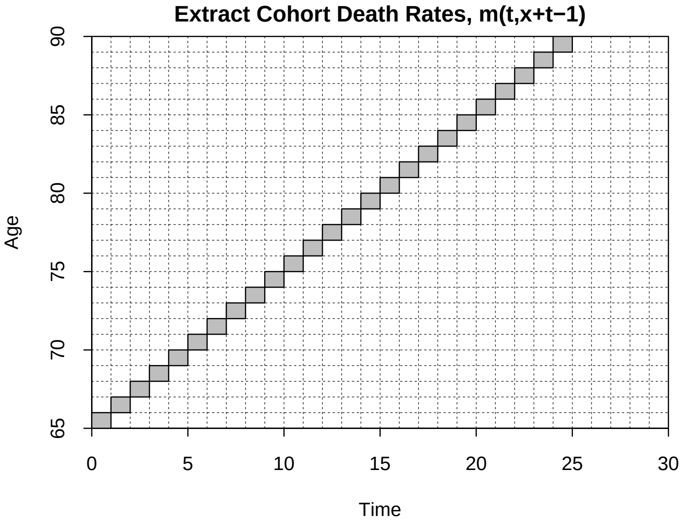


Annuity Pricing Requires Cohort Rates



Annuity valuation $\Rightarrow$ follow cohorts

$m(0, x) \to m(1, x+1) \to m(2, x+2) \ldots$

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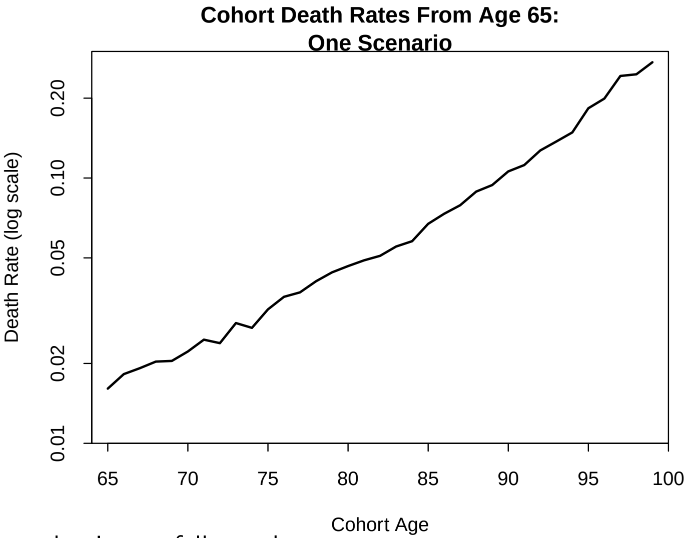

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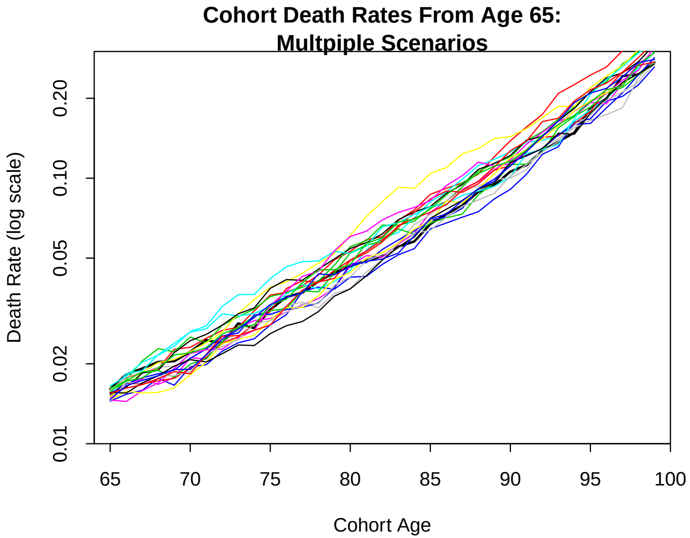

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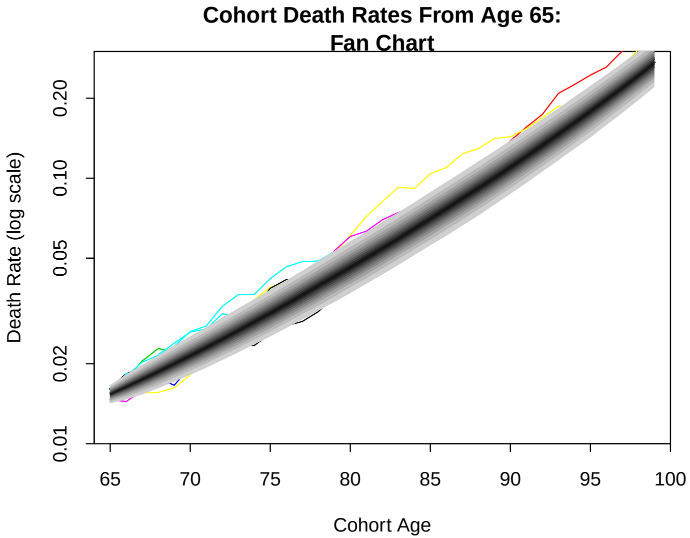

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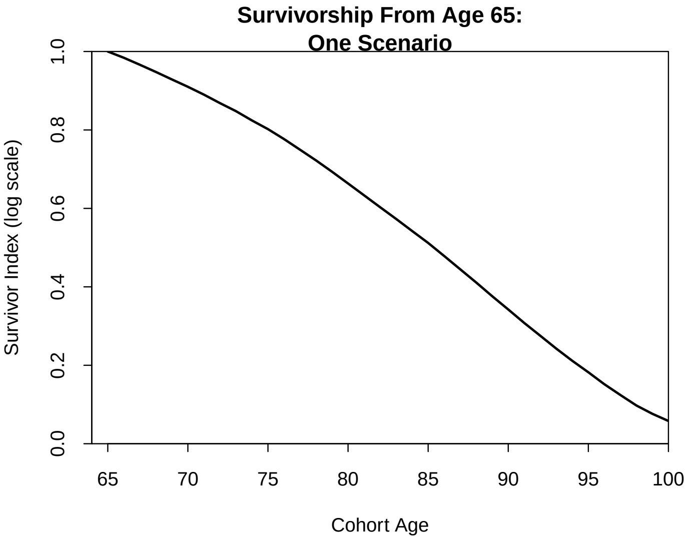
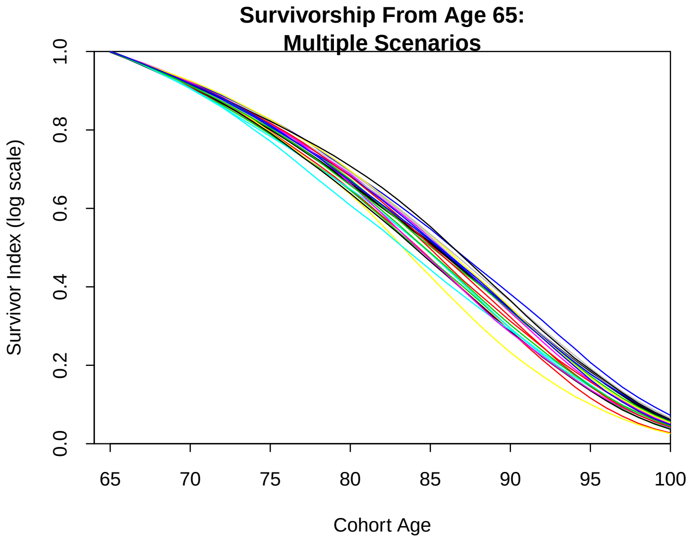
Cohort Survivor Index



Cohort death rates $\longrightarrow$ cohort survivorship

Cohort Survivor Index



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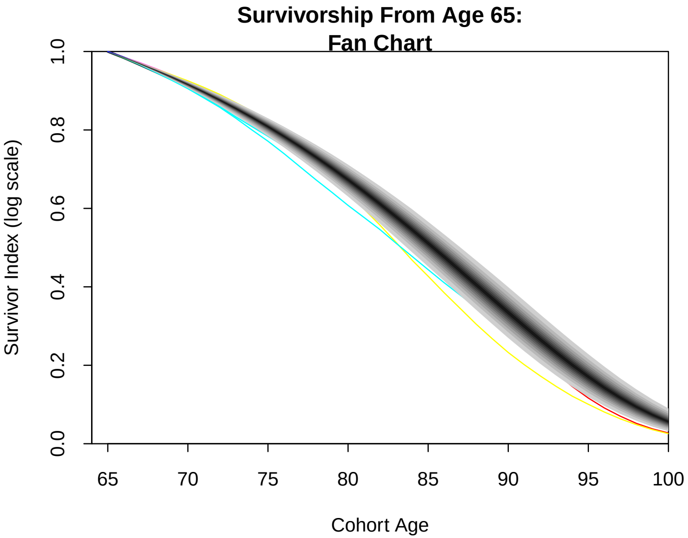
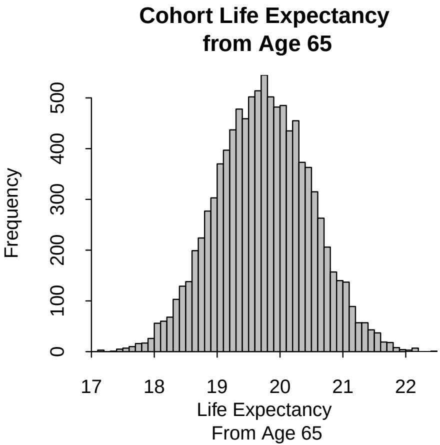
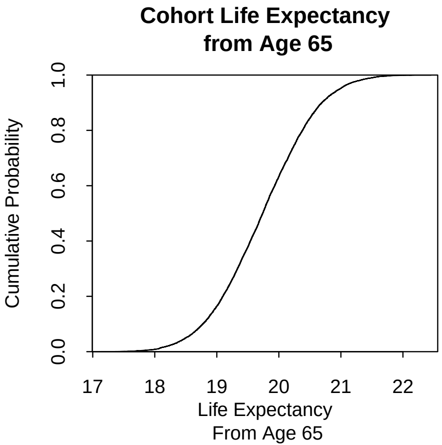


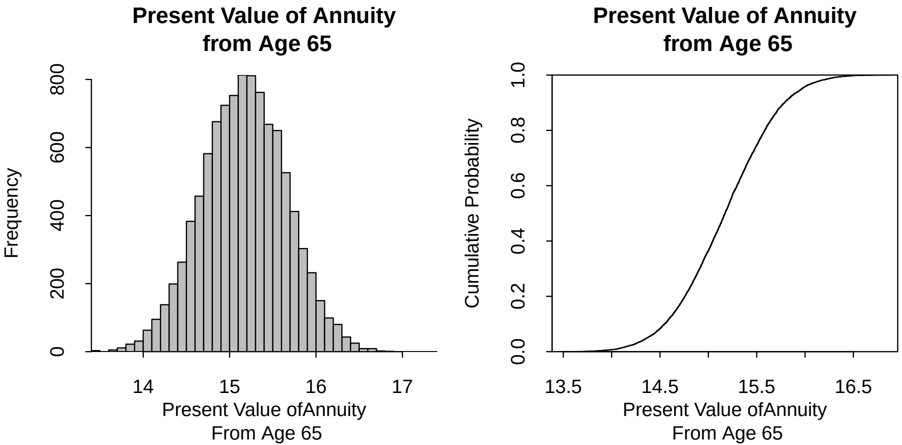




Cohort survivorship $\longrightarrow$ *ex post* cohort life expectancy

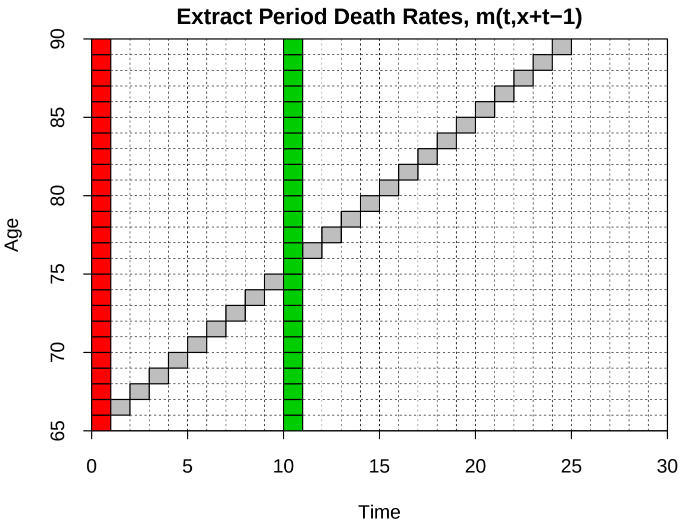
Equivalent to a continuous annuity with 0% interest

Annuity Reserving



- Annuity of 1 per annum payable annually in arrears
- Interest rate: 2%

A Real Example: US Male Period Life Expectancy



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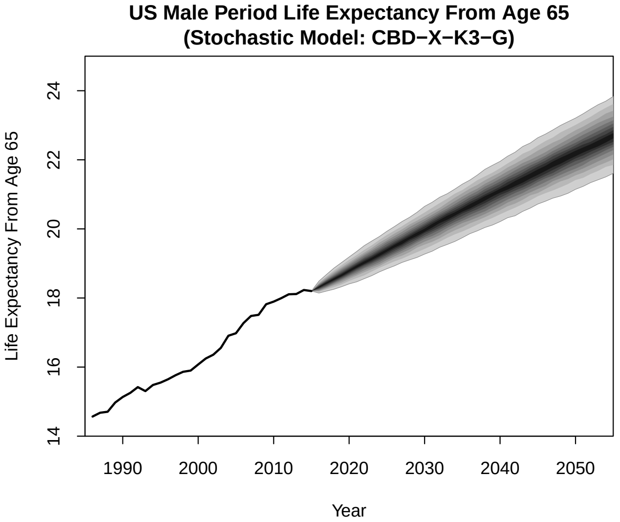

Mortality improvement rate $\approx 1.7\%$ p.a. at ages 65-85.

How to incorporate Expert Judgement?

- E.g. CBD model $\Rightarrow$
  - $m^j_{CBD}(t,x)$ scenarios
  - $\bar{m}_{CBD}(t,x)$ central forecast
- Expert judgement $\Rightarrow$
  - $\hat{m}(t,x)$ (central) forecast

- Blending $\Rightarrow$ stochastic scenario j becomes

$$m^j(t,x) = \frac{m^j_{CBD}(t,x)}{\bar{m}_{CBD}(t,x)} \times \hat{m}(t,x)$$

- Fully stochastic $\Rightarrow$ full risk assessment

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How to incorporate Expert Judgement?

- A variation on this is required by UK life insurance regulators

- $\Rightarrow$ Don't ignore stochastic models simply because you disagree with the central forecast!

- Additionally: new approaches to bring the two together are being developed

Part 2: Key Drivers

Drill into the Detail of US Data

- Level of educational attainment $\Rightarrow$ predictor
- Individual cause of death $\Rightarrow$ outcome

- Beware of grade inflation
- Help to understand trends in national data and subpopulations (e.g. white collar pension plan)

Data Sources

- Total Exposures: Human Mortality Database (smoothed to mitigate anomalies)
- CDC deaths: cause of death + education (+ ethnic group)
- CPS survey data: education proportions

Research $\Rightarrow$

- smart synthesis of three data sources
- improved, less noisy, exposures by education level

Purpose of looking at cause of death data

- What are the key drivers of all-cause mortality?
- How are the key drivers changing over time?
- Which causes of death have high levels of inequality:
  - by education
  - other predictors
- Insight into mortality underpinning life insurance and pensions
- Insight into potential future mortality improvements
- Beware of
  - changes in ICD classification of deaths (e.g. 1999)
  - drift in how deaths are classified
  - changing education levels (grade inflation)

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Education Levels

Education	
Low education	Primary and lower secondary education
Medium education	Upper secondary education
High education	Tertiary education

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Cause of Death Groupings

1	Infectious diseases incl. tuberculosis	2	Cancer: mouth, gullet, stomach
3	Cancer: gut, rectum	4	Cancer: lung, larynx, ..
5	Cancer: breast	6	Cancer: uterus, cervix
7	Cancer: prostate, testicular	8	Cancer: bones, skin
9	Cancer: lymphatic, blood-forming tissue	10	Benign tumours
11	Diseases: blood	12	Diabetes
13	Mental illness	14	Meningitis + nervous system (Alzh.)
15	Blood pressure + rheumatic fever	16	Ischaemic heart diseases
17	Other heart diseases	18	Diseases: cerebrovascular
19	Diseases: circulatory	20	Diseases: lungs, breathing
21	Diseases: digestive	22	Diseases: urine, kidney,...
23	Diseases: skin, bone, tissue	24	Senility without mental illness
25	Road/other accidents	26	Other causes
27	Alcohol → liver disease	28	Suicide
29	Accidental Poisonings		

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US Education Data

- Males and Females (2)
- Single ages 55-75 (21)
- Single years 1989-2015 (27)
- Causes of death (29)
- Low, medium & high education level (3)

Note: HMD's *Human Cause of Death Database* $\Rightarrow$ All ages (5's), 1999-2015, No education

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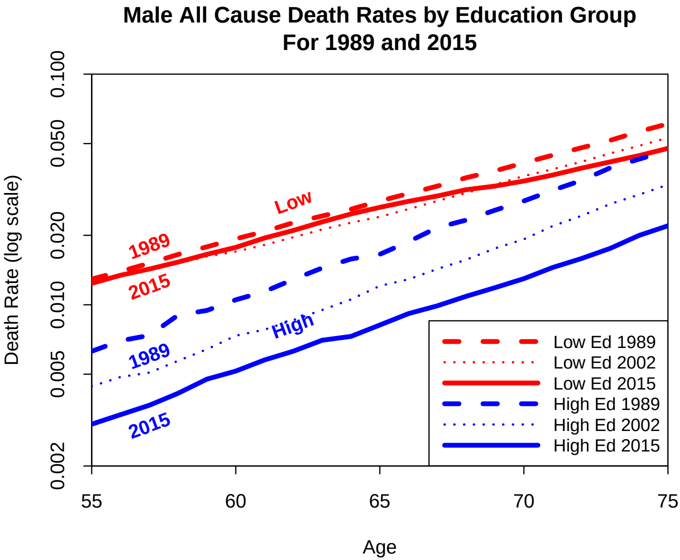
US Education Data: Growing Inequality, Males



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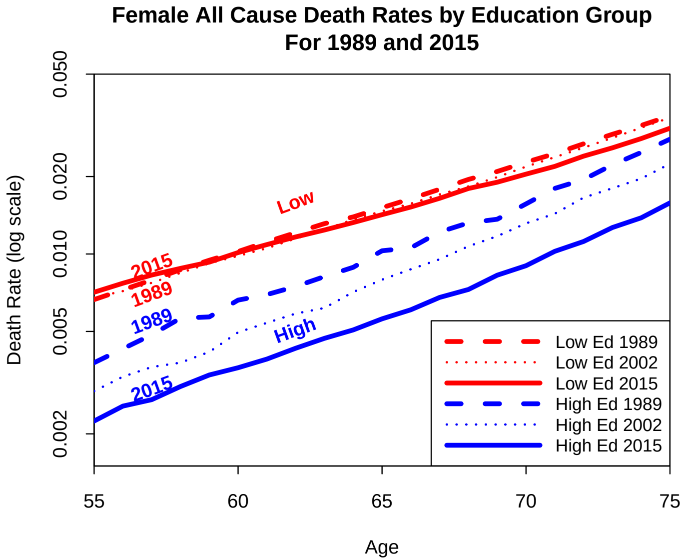
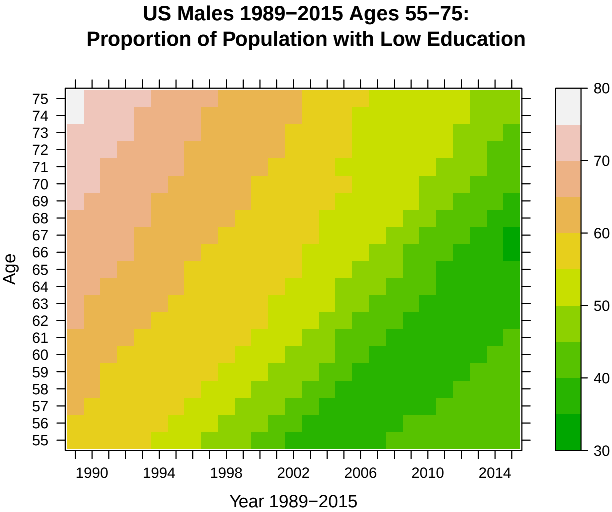
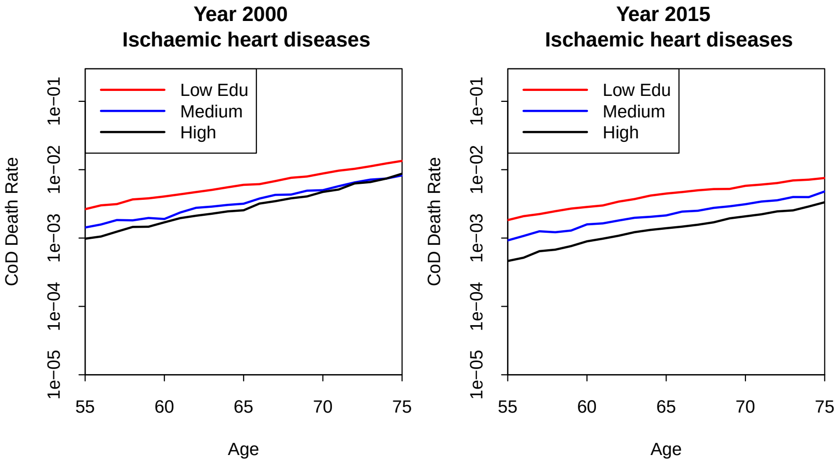


Proportion of Males with Low Education



Cohort diagonals ⇒ *falling* percentage

US Education Data: CoD Death Rates



Widening gap

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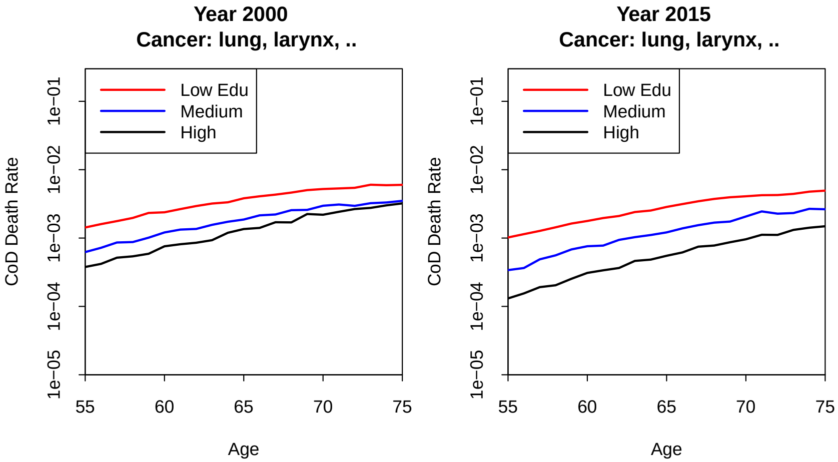

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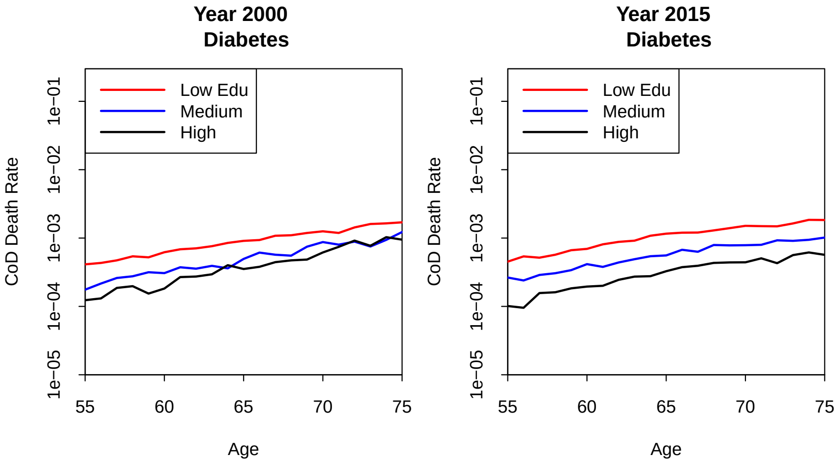
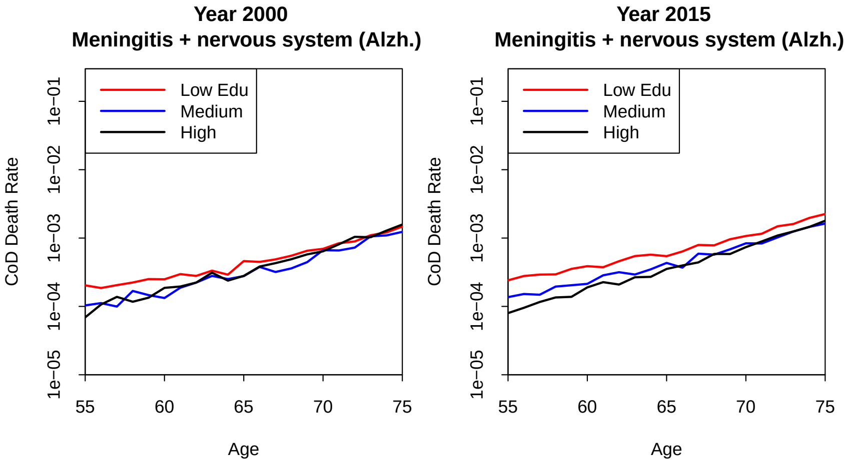


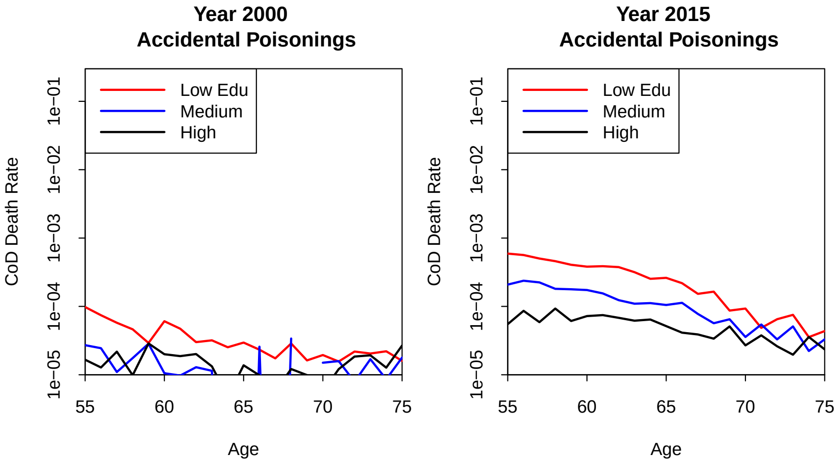
Widening gap; Mixed improvements

US Education Data: CoD Death Rates



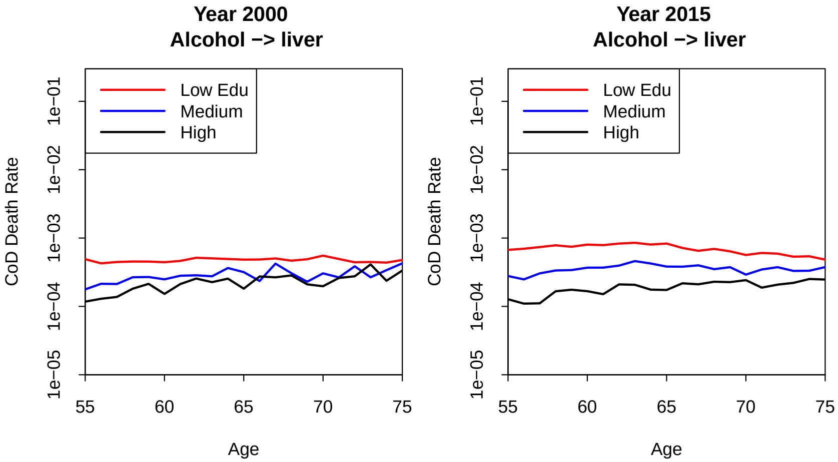
Widening gap; almost no improvements

US Education Data: CoD Death Rates



Case & Deaton (2015) $\Rightarrow$ Accidental poisoning $\nearrow$

US Education Data: CoD Death Rates



Widening gap

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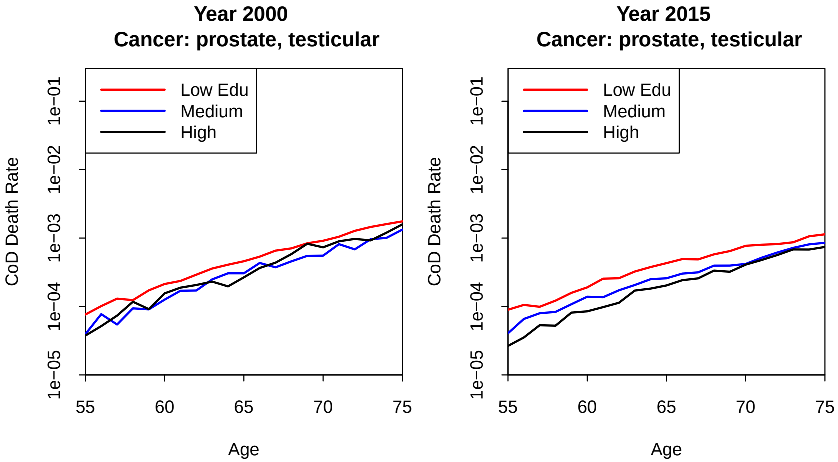

Denmark $\Rightarrow$ almost NO gap by education;

Denmark $\Rightarrow$ small gap by affluence; smaller than US by education

Cause of Death Data: Health Inequalities

- Some causes of death have no obvious link to lifestyle/affluence/education
  e.g. Prostate Cancer
  CancerUK:*Prostate cancer is not clearly linked to any preventable risk factors.*
- But education level ⇒ inequalities
- Possible explanations (a very non-expert view)
  - *onset* is not dependent on lifestyle/affluence/education
  - BUT lower educated ⇒
    ??? poorer health insurance coverage
    ??? later diagnosis
    ??? engage less well with treatment process
    ??? lower quality housing/diet etc.

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US Males: Low versus High Education

Do Low and High education groups have the same CoD rate?

- Four $\times$ 5-year age groups
- 29 causes of death
- Signs Test (count low edu. $>$ high edu. mort.)
- $29 \times 4 = 116$ individual tests
- Reject equality hypothesis *in all but one test*
- Accept H_0 ($p = 0.08$) for only one pairing: Meningitis $+$ nervous system (Alzh.), 70-74
- Most p-values $< 10^{-6}$

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Summary

- Future work
  - Analysis of sub-national datasets
  - e.g. SoA Group and Individual Annuity data
  - e.g. individual pension plan data
  - Multiple population modelling

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Thank You!

Questions?

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Discussion Point

- Medicare kicks in after age 65
- But no obvious impact on inequality gap
- Although inequality gap naturally narrows with age

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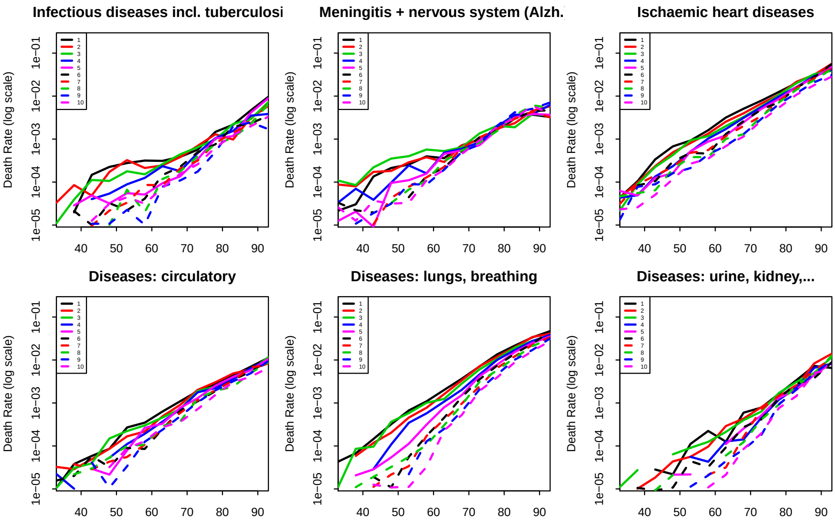
CoD Death Rates: Different Shapes & Patterns



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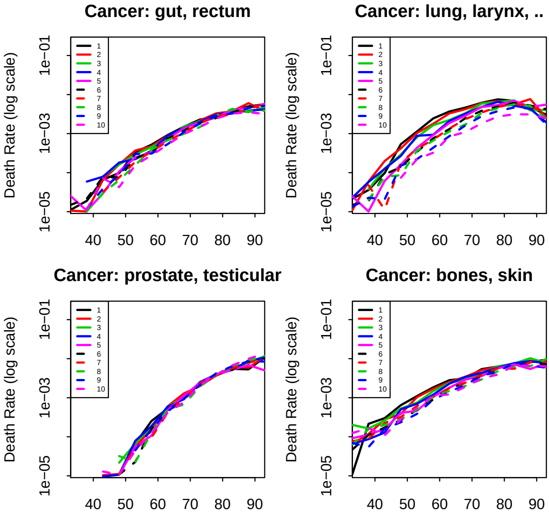

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Shapes: Conclusions

- Typically:
  - Non-cancerous diseases $\Rightarrow$ approximately exponential growth
  - Neoplasms (cancers) $\Rightarrow$ subexponential ??? polynomial
- What does this reveal about different disease mechanisms?

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