Analysis of Trends in the Age-Specific Shape of Mortality Curves for Populations in the United States and Japan

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Abstract

Life expectancy at birth has improved dramatically over the course of the twentieth century. Over this period there has been a shift in that the highest improvements in mortality rates have been seen in progressively older ages. This paper discusses alternative ways of looking at this trend, fitting models to past trends, and projecting future mortality based on a forward projection of these trends, and calculates annuity values based on these projections.

We consider that the probability of age at death for very advanced ages can best be understood and projected in conjunction with the probability of deaths occurring at younger ages, as changes in mortality rates at younger ages will be correlated with the probability of deaths at very advanced ages.

The work of Jim Oeppen and James Vaupel (2002) influenced our thinking in this area. They concluded that the population life expectancy at birth of the country with the highest life expectancy has followed almost a straight line over the last 160 years with a rate of increase of 0.25 years per annum.

We took the view that in attempting to identify and project trends in mortality the analysis should, if possible, be based on those trends showing significant stability. This led us to investigate whether, in addition to the standard actuarial approach of considering curves of q_x, the following curves may provide a basis for modeling:

The curve of the probability of death at a specified age
The curve of the cumulative probability of death up to a specified age.

We therefore examined the progression over time of each of these curves.

We investigated age-specific mortality curves for population mortality experience for the United States and Japan over the last 40 years. Our investigation was based on population mortality experience from the Human Mortality Database. Japan was chosen as it has seen the highest population life expectancy of any developed nation over the last 20 years. Prior investigators of the Human Mortality Database have suggested that mortality improvements among the higher socioeconomic classes of the United States are not dissimilar to those seen among the general population of Japan.

We have developed models that attempt to relate the age-specific mortality curves for individual calendar years by means of time-dependent variables. The mortality curves that we considered for both males and females in both countries from life tables appropriate to individual calendar years are as follows:

Probability of death at specified age
Age-specific q_x
Cumulative probability of death.

We used these models to project future mortality rates in the United States and Japan. We further consider how these mortality rates may vary according to different future deterministic scenarios. We illustrate the impact of these mortality rates by the calculation of specimen annuity values and projections of age at death for different percentiles of the population.

We discuss the implications of the results for life expectancy and for population age structure. We conclude by providing commentary on the various views being expressed by experts in the fields of demography and medicine as to the likelihood of further improvements in life expectancy and the existence of limits to longevity.

Introduction

The twentieth century saw very dramatic reductions in mortality rates at all ages. In the first half of the century significant improvements in the treatment of infectious diseases resulted in the reductions being most significant for children and young adults. In contrast in the second half of the century, significant reductions in deaths associated with cardiovascular disease resulted in the main improvements being seen in those in their fifties and over.

The effect of these improvements has been that far more individuals are living to advanced ages than ever before. This has significant implications for society in general and governments in particular in terms of the work force and health and social costs. As experts in mortality, actuaries may be expected by nonactuaries to address fundamental questions about these trends. These could include the following:

- Are there appropriate models for projecting future changes in life expectancy?
- Is a model based on past trends in mortality consistent with expert opinion as to future changes in life expectancy from other fields such as medicine and demography?

In this paper we address the question by considering the time dependency of various measures of mortality. As regards the second question, we review these measures in the light of a number of different views that have been expounded on possible limits to life expectancy.

There is a long history of fitting models to mortality experience. In 1825 Gompertz, on examination of census data, noted an exponential rise in mortality rates after sexual maturity. He provided a physiological explanation for this observation as "the average exhaustion of man's power to avoid death gained in equal proportion in equal intervals of age." The formula he proposed was

$$\text{Force of mortality at age } x = Bc^{x}\ .$$

The relationship does not hold well at younger ages, and Makeham added a constant term in 1867 to reflect, in part, differences in cause of death between those due to accident and those due to disease. Heligman and Pollard further refined this process by proposing an eight-parameter model in 1980 that specifically modeled such features as childhood illnesses and mortality associated with pregnancy.

The Gompertz and Makeham formulas become increasingly inaccurate at very advanced ages. Logistic formulas have been suggested by Kannisto (1994), Beard (1971), and Perks (1932) that slow the rate of increase in mortality at older ages. It has been suggested that mortality experience at very old ages might point to the existence of a plateau in the rate of mortality, although there are significant problems with credibility of data at these ages.

For actuaries a particular interest of mortality models is the extent that they provide a methodology to project future mortality rates from past experience. The multifactorial nature of improvements in mortality rates means that past trends in mortality may not necessarily provide a good forecast of future trends. However, the future projection of past trends provides a basis of projection that actuaries should consider, whereas selecting between conflicting medical theories is not an actuarial skill. Therefore, where coefficients in appropriate mortality models have shown historical time dependency, such mortality models provide a methodology for projecting future mortality rates that, in our view, should be considered by actuaries.

Approach to Modeling

The trend in future longevity is unknowable. In practice it will be depend upon future medical advances. Some of these can be predicted from work already done, but significant work needs to be done to quantify its likely age-related impact. Other medical advances may not yet be foreseen. On the other side of the coin there are factors that may increase mortality such as the emergence of virulent strains of influenza or the spread of antibiotic-resistant bacteria.

These medical advances, or reverses, may themselves be subject to overarching constraints such as a maximum limit to human life. A later section of this paper discusses some of the arguments for and against such a limit.

Given these uncertainties, projecting future trends in mortality, very particularly at the high ages that are the focus of this symposium, is fraught with difficulty. Unfortunately for actuaries, many nonactuaries seem to think that actuaries should be able to solve this problem.

In the authors' view there are three basic ways of thinking about this:

a. The analysis of past trends and their forward projection, without taking into account medical data
b. Projections based on medical data at the specific condition level
c. Projections based on overarching medical constraints, most obviously, an upper limit to human lifespan, although others may perhaps be envisaged.

Approach (a) is the approach that comes most naturally to actuaries, and the detailed analysis in this paper provides examples of it. Two of the authors (Humble and Ryan) have done work involving a combination of approaches (a) and (b) and see considerable merit in, and scope to extend, this approach. A predictive model based solely on medical data, that is, (b) in isolation, is possible in principle, but the authors regard it as being, at least, several years away in practice.

Option (c) is clearly very important or rather may be very important (a clear demonstration that the limit to human lifespan was, say, 1,000 may be interesting but would have no discernible impact on calculating capital values of annuities at the present time). The authors do not consider, however, that the case for such a limit has been made with sufficient clarity to include it as a constraint in modeling at this time.

Methodology

Data Sources

We used the Human Mortality Database (HMD; www.mortality.org) as our source for age-specific mortality rates for both sexes and for both countries. The HMD represents a collaborative project between the Department of Demography at the University of California, Berkeley, United States, and the Max Planck Institute for Demographic Research in Rostock, Germany. The HMD was based on the Berkeley Mortality Database as founded by John Wilmoth in 1997 and was strongly influenced by the Kannisto-Thatcher Database on Old Age Mortality as founded by Vaino Kannisto and Roger Thatcher in 1993.

The HMD contains information on 22 countries, consisting of raw data information on births, deaths, population size, and exposure to risk, together with detailed descriptions on the sources of the data that we have summarized in the following paragraphs. The goal of the project is to adopt uniform procedures for each country in the collection and verification of data and in calculating death rates and life tables. The HMD notes that in particular the issue of age exaggeration is addressed in part by the derivation of population estimates at older ages through the death counts themselves, employing extinct cohort methods, as age reporting in death registration systems is assumed to be more reliable than from census counts or official population estimates.

Data on U.S. population size were taken from the 10-year population censuses conducted by the U.S. Census Bureau between 1960 and 2000. Census counts were used as basis for annual and monthly population estimates for intercensal and postcensal periods, as reported in *Current Population Reports.*

Data on U.S. deaths are provided by the National Center for Health Statistics (NCHS) from individual death records as coded from death certificates. The latter data are available in a detailed format to participating organisations through the Inter-University Consortium for Political and Social Science Research, with less detailed summaries being produced in periodical publications from the NCHS.

Data on Japanese population were taken from population censuses conducted every five years by the Statistics Bureau, Management and Coordination Agency between 1960 and 2000. The Statistics Bureau also produces annual postcensal and intercensal population estimates that are published in "Annual Reports on Current Population Estimates."

Data on Japanese deaths and births over this period were taken from annual publications by the Ministry of Health, Labour and Welfare, Division of Health and Welfare Statistics.

Mortality Curve Models

The investigation period is defined as 1960 to 1998. For each calendar year in the investigation period, we derived the following mortality measures for each sex and for each country:

q_x at each age
Cumulative probability of death at each age
Probability of death at specified age.

We used the statistical package SPSS to investigate and select the most appropriate mathematical model from those considered to each of these three mortality measures as applied to each calendar year in the investigation period.

For each mortality measure, we determined the degree of time dependency to the coefficient values as taken from successive fittings of the most appropriate mathematical model over the investigation period.

Forward Projections of Mortality and Life Expectancy

We used the time-dependency relationships identified in the mathematical models in respect of cumulative probability of death and of age-specific mortality rates to determine age-specific mortality rates in future years. We then calculated annuities in arrears for selected issue ages, assuming these mortality rates, annual payments, and an interest rate of 4.25 percent per annum. This interest rate may not be appropriate for both countries considered, but it was felt that a single interest rate would more clearly demonstrate differences in mortality experience between the two countries.

We used the time-dependency relationships identified in the mathematical models in respect of probability of death at each age to determine probability of death at each age in future years. This is simply an alternative method of analyzing the effect of future age-specific mortality rates, but does provide a direct presentation of the population age at death structure in the future.

We also considered an alternative methodology consisting of age at death for different percentiles in the population. We considered decennial percentiles from the 10th to 90th and then individual percentiles up to the 99th. We investigated the pattern of age at death for each of the percentiles considered over the investigation period as forming a basis for forward projection.

Weibull Distribution

The Weibull distribution was developed by Dr. Waloddi Weibull in 1937 and was first introduced in 1951 by his paper "A Statistical Distribution Function of Wide Applicability." It has since been used in statistical analysis. The Weibull distribution is widely used in reliability and life data analysis due to its versatility.

The Weibull distribution is determined by two parameters, c (the scale parameter) and ◎ (the shape parameter). Compared to the exponential distribution (special case of Weibull where ◎◎= 1, i.e., constant), the Weibull distribution can have a failure rate (of a life) that varies. This makes the distribution more suitable for models of mortality.

The shape of the Weibull distribution resembles the normal distribution with a right-skew and a tail that is lighter than other distributions. These characteristics of the Weibull distribution have allowed us to fit the distribution to the probability of death with some success.

Use of the Weibull Distribution

The probability density function (equation 1) and the probability distribution function (equation 2) of the Weibull distribution both contain two parameters, c and γ (Miller, 1999):

$$f(x) = c\gamma x^{\gamma-1} \exp(-cx^{\gamma}), \quad (1)$$
$$F(x) = 1 - \exp(-cx^{\gamma}). \quad (2)$$

The "Method of Moments" estimation (Hossack, 1999), equating the actual mean and variance of the data set to the theoretical mean and variance of the distribution in question, was not appropriate in this instance as both the mean and variance of the Weibull distribution contain both of the parameters, equations (3) and (4):

$$E(x) = \Gamma(1+1/\gamma)/\, c^{1/\gamma}, \quad (3)$$
$$V(x) = \Gamma(1+2/\gamma)/\, c^{2/\gamma} - \{\Gamma(1+1/\gamma)/\, c^{1/\gamma}\}^2. \quad (4)$$

Instead, in order to fit the Weibull model, we have used the "Method of Percentiles" (Klugman, 1998) to estimate the parameters of the distribution.

The Method of Percentiles enables two equations with two unknowns to be written. We have equated the probability distribution function at the percentile ages to the corresponding percentile. We have used the 50th percentile and the 95th percentile to fit the curve, as we were most concerned with fitting the latter part of the ages versus actual age of death curve.

The following equations have been used to estimate the parameters:

$$F(x) = 0.50, \quad (5)$$

where x is the actual age of the data set corresponding to the 50th percentile and where $F(x)$ is the value of the probability distribution function of the Weibull distribution, and

$$F(x) = 0.95, \quad (6)$$

where x is the actual age of the data set corresponding to the 95th percentile and where $F(x)$ is the value of the probability distribution function of the Weibull distribution.

By ratioing equation (5) to equation (6), the parameter c disappears, and parameter γ can be estimated. Using parameter γ, one can then deduce the parameter c.

In our investigations of probability of death at a specified age in conjunction with the Weibull distribution, we have defined parameter α to be equal to parameter γ, and parameter β to be $c^{-1/\gamma}$.

Principal Results: United States

Age-Specific q_x

Logistic Model

The logistic model is of the form

$$q_x = \frac{1}{\gamma + \alpha\beta^x}$$, where γ is a time-independent constant.

We fitted curves of this type to the actual mortality experience for each of the calendar years from 1960 to 1998. Figures 1–5 provide a demonstration of the appropriateness of the fit for females for years 1960, 1970, 1980, 1990, and 1998, respectively.

Figure 1
U.S. Females: Actual q_x as against Logistic Distribution for Year 1960

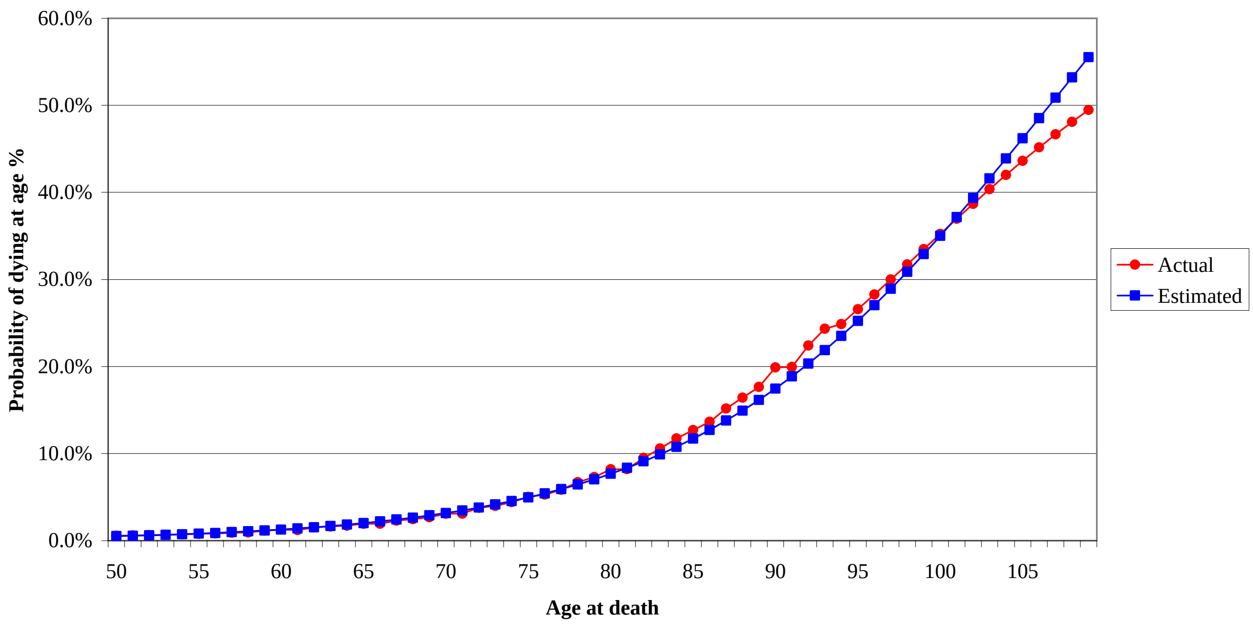


Figure 2
U.S. Females: Actual q_x as against Logistic Distribution for Year 1970

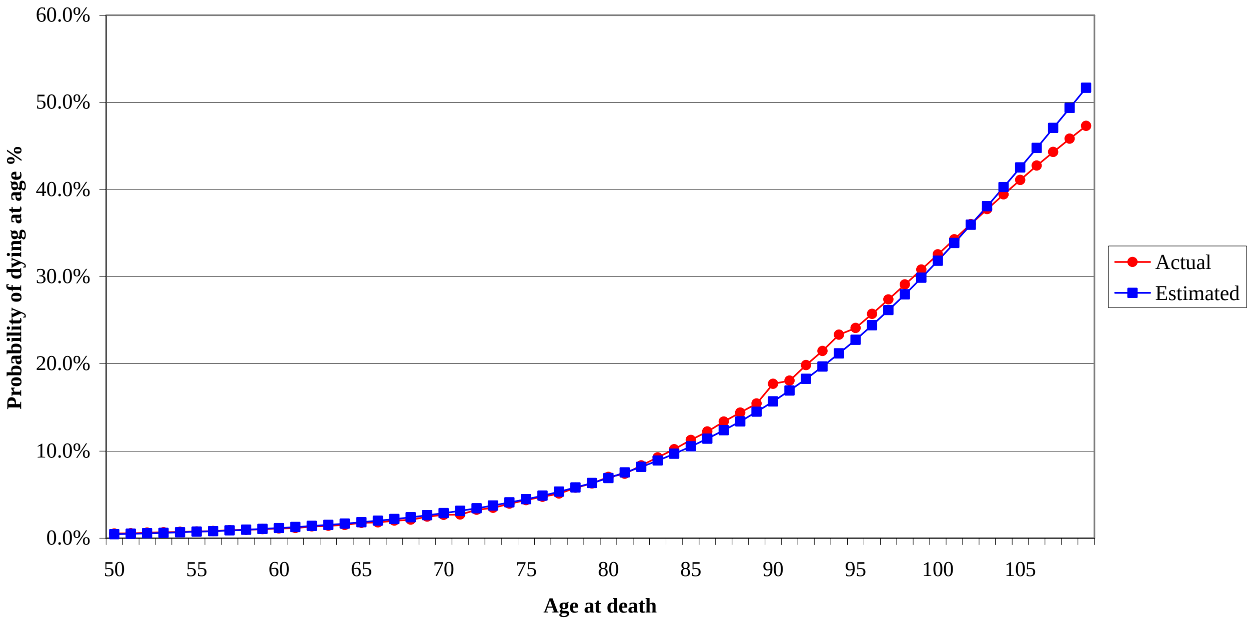


Figure 3
U.S. Females: Actual q_x as against Logistic Distribution for Year 1980

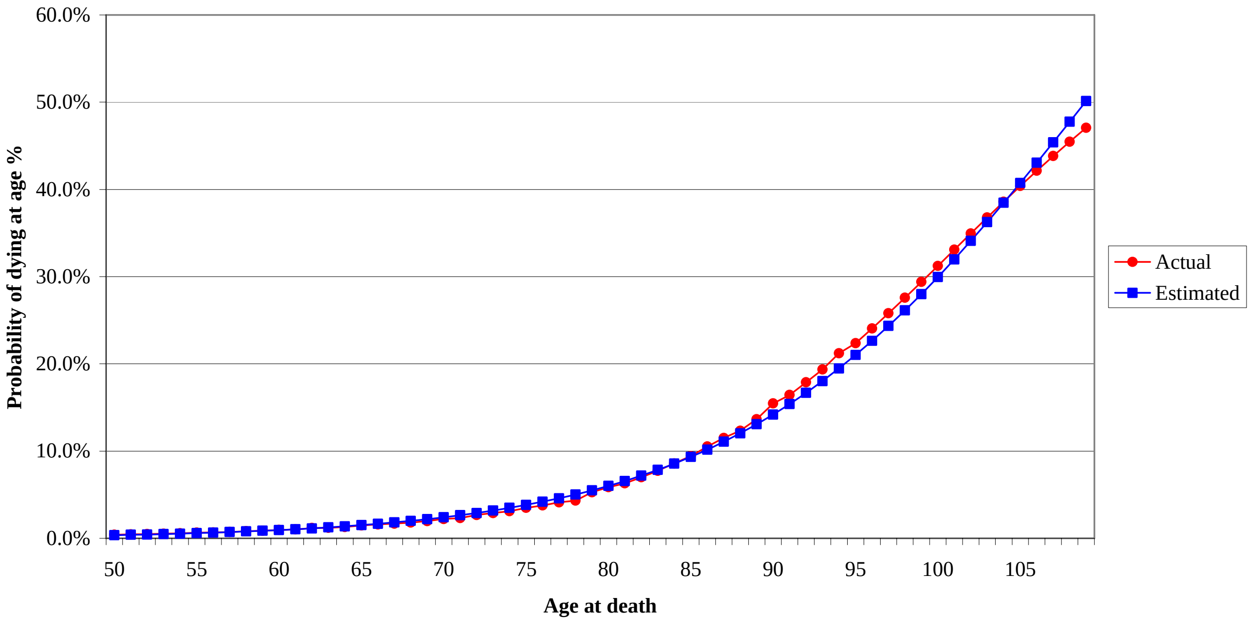


Figure 4
U.S. Females - Actual q_x as against Logistic Distribution for Year 1990

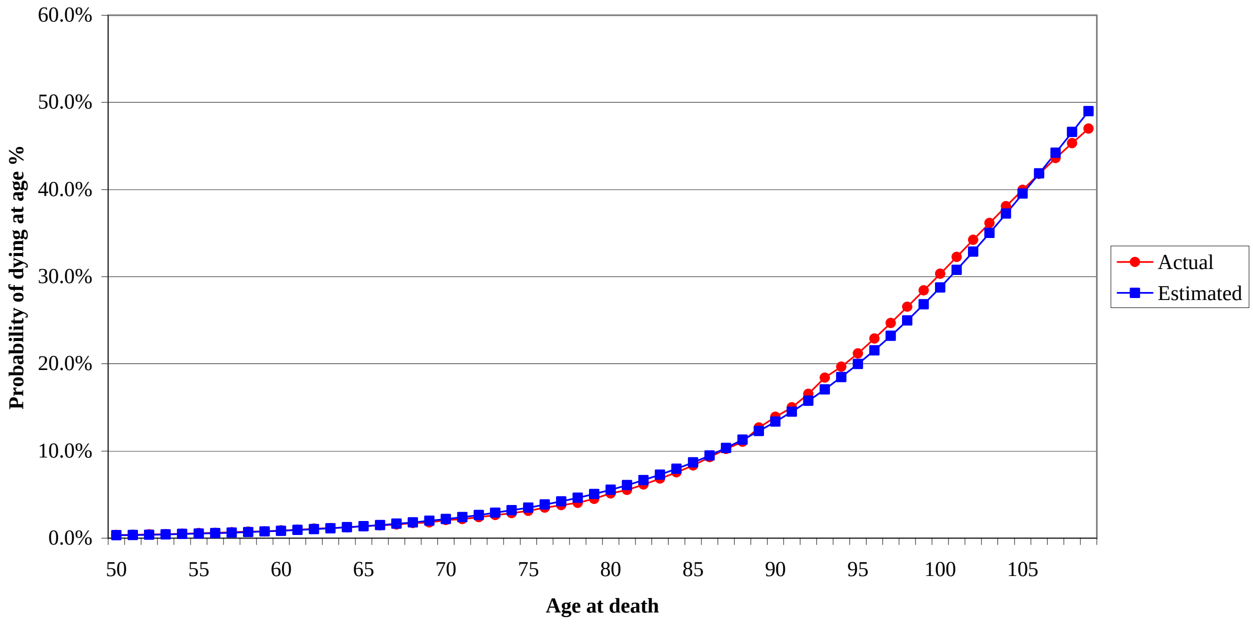


Figure 5
U.S. Females - Actual q_x as against Logistic Distribution for Year 1998

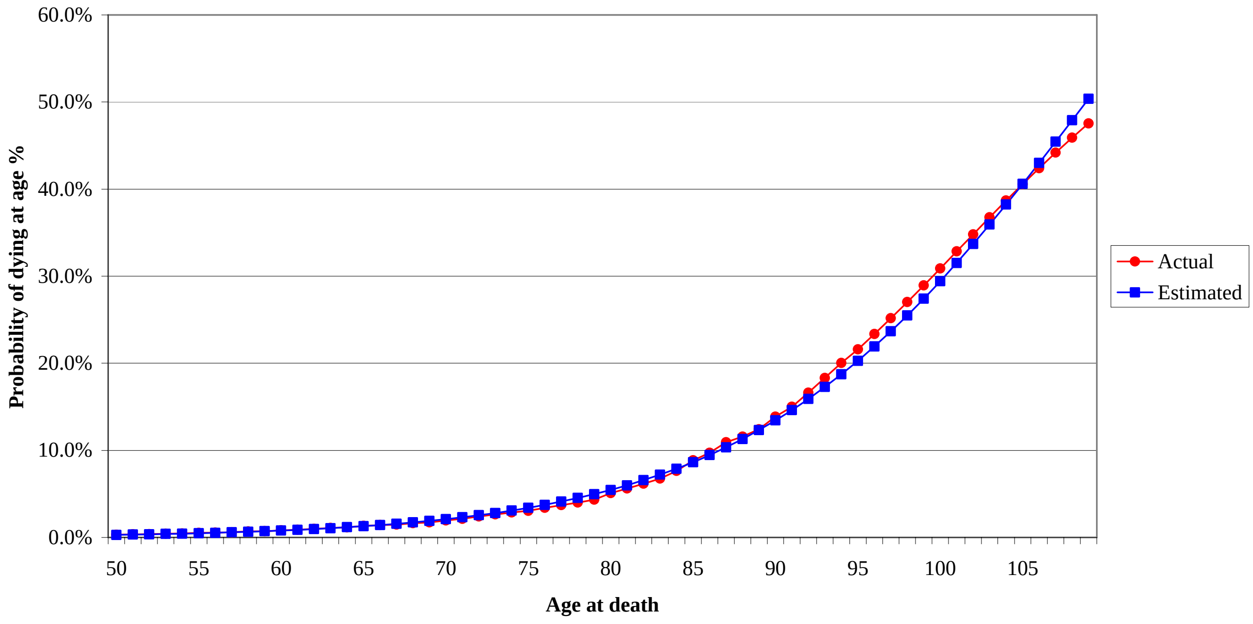


Table 1 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 1
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (U.S. Females)

	1960	**1970**	**1980**	**1990**	**1998**
Parameter α	21210	21208	31138	37634	46969
Parameter β	0.911	0.912	0.909	0.908	0.906
R^2 value	0.997	0.998	0.998	0.998	0.999

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter α = 19,851 $\circledcirc$ exp(0.0215 $\circledcirc$ (Calendar Year – 1959)); R^2 value: 0.970,
Parameter β = −0.000122 $\circledcirc$ (Calendar Year – 1959) + 0.91168; R^2 value: 0.806.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 2.

TABLE 2
Annuities Payable in Arrears on Different Mortality Bases (U.S. Females)

					Age				
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) This basis	14.31	12.63	10.84	9.01	7.21	5.54	4.07	2.86	1.91
(2) Population mortality	13.51	11.92	10.21	8.45	6.63	4.94	3.59	2.50	1.73
((1) / (2)) – 1	5.9%	6.0%	6.2%	6.6%	8.7%	12.0%	13.3%	14.4%	10.5%

Figures 6–10 provide a demonstration of the appropriateness of the fit for males for years 1960, 1970, 1980, 1990, and 1998, respectively. These figures are based on far fewer lives at high ages.

Figure 6
U.S. Males: Actual q_x as against Logistic Distribution for Year 1960

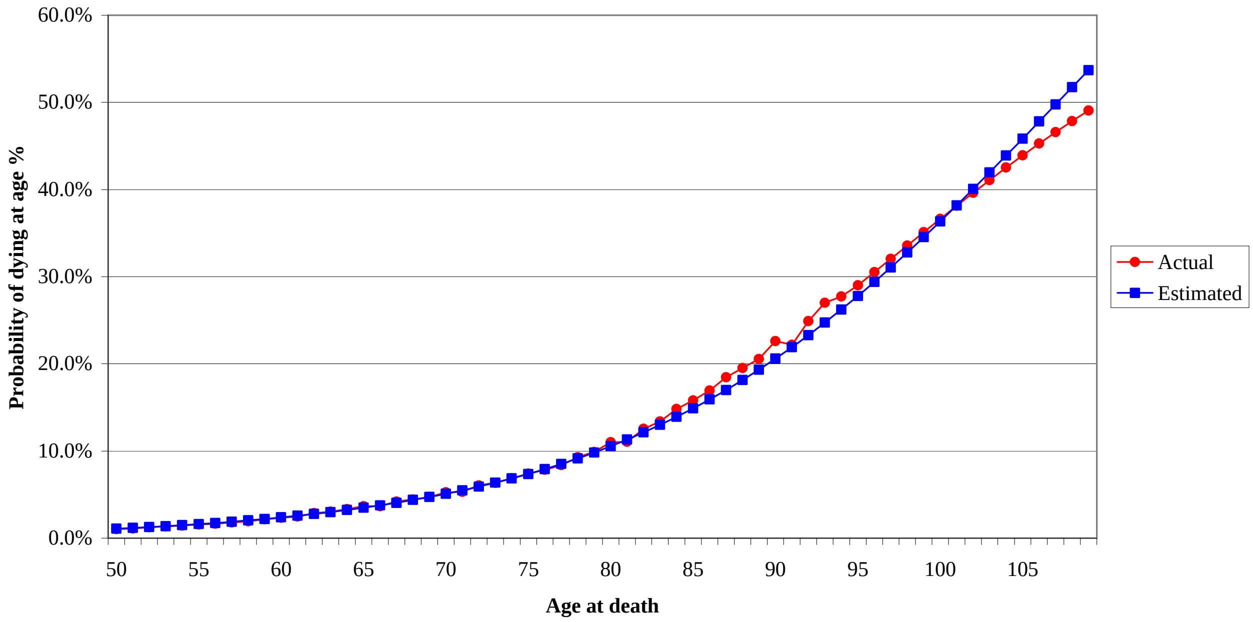


Figure 7
U.S. Males: Actual q_x as against Logistic Distribution for Year 1970

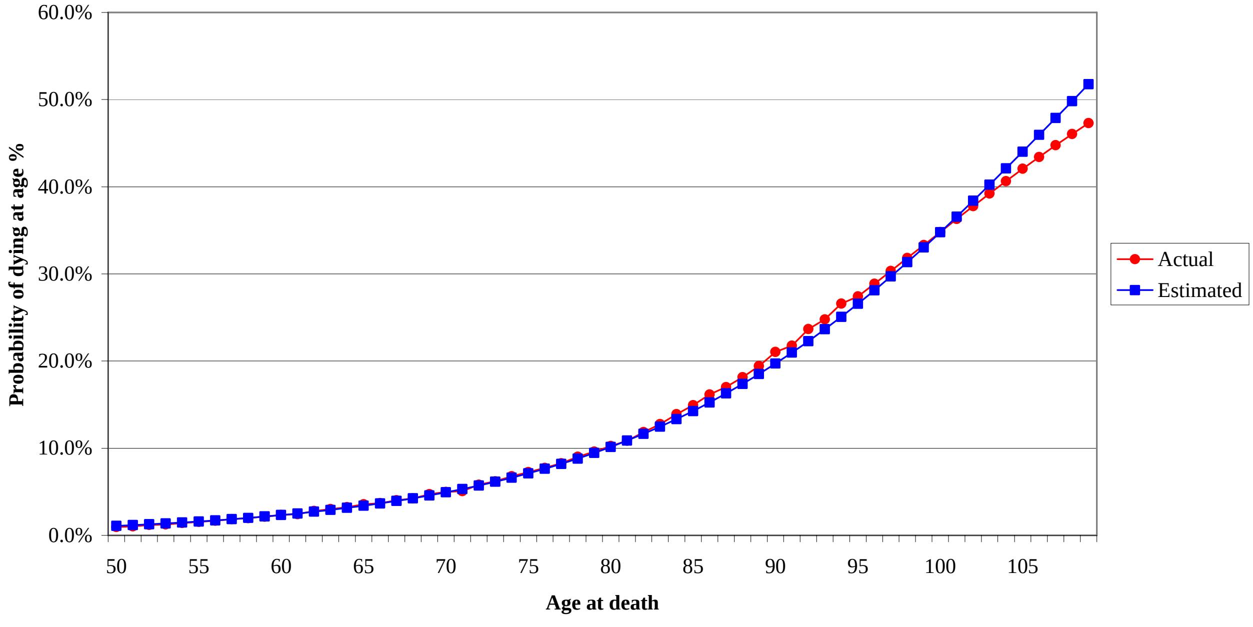


Figure 8
U.S. Males: Actual q_x as against Logistic Distribution for Year 1980

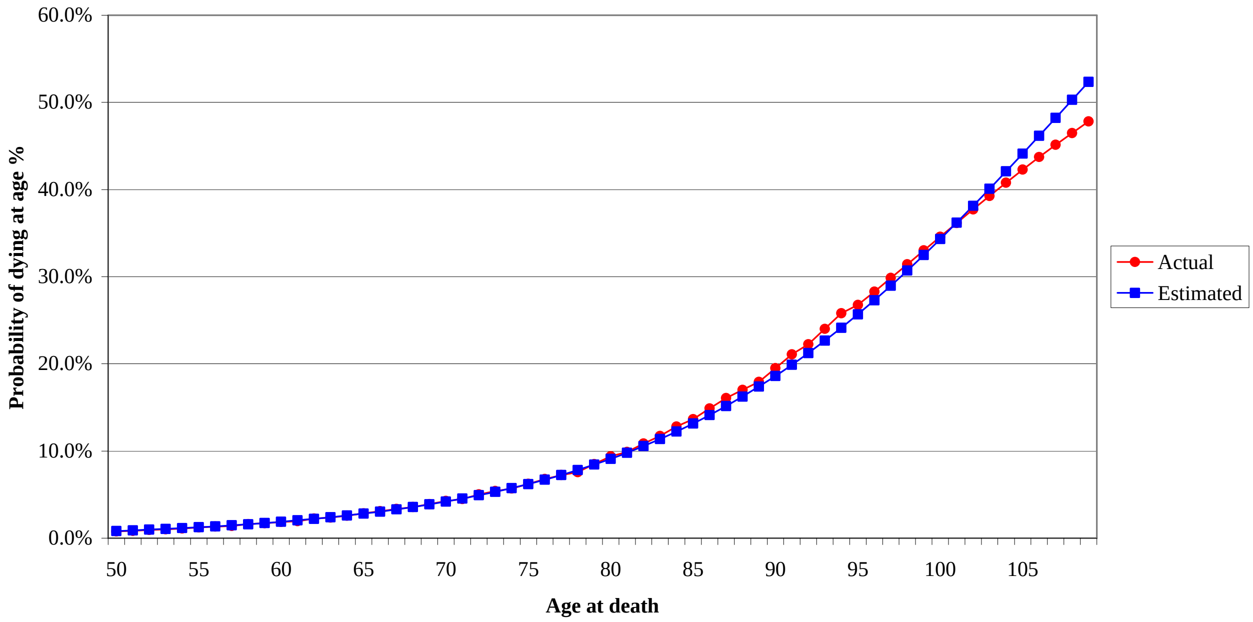


Figure 9
U.S. Males: Actual q_x as against Logistic Distribution for Year 1990

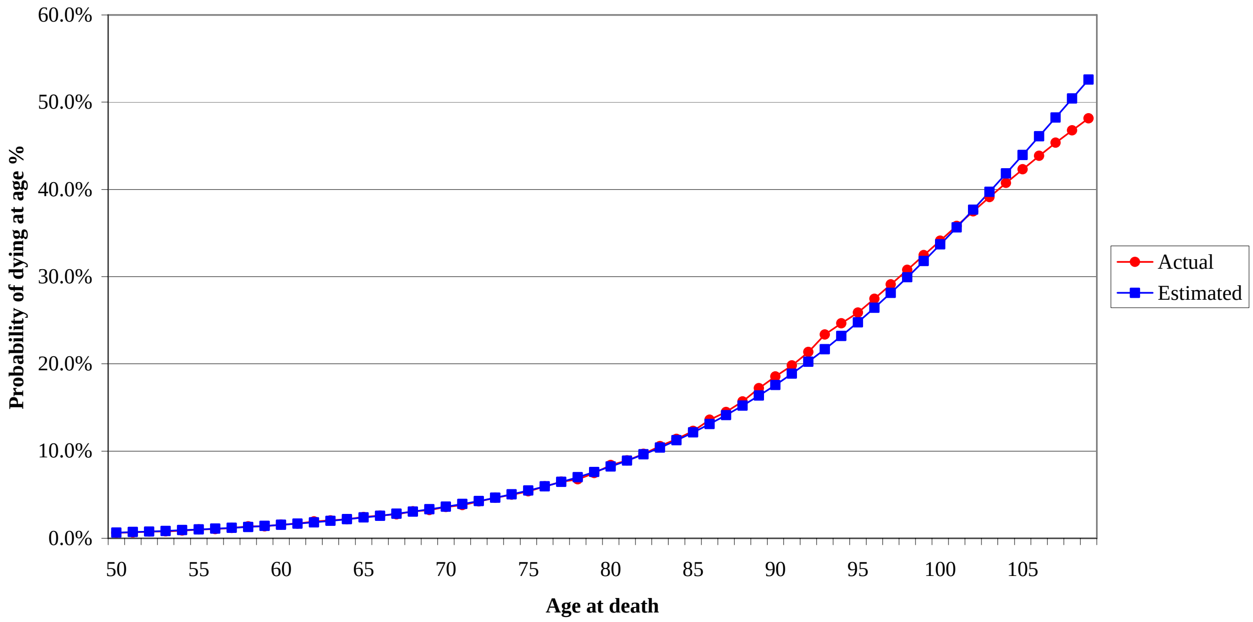


Figure 10
U.S. Males: Actual q_x as against Logistic Distribution for Year 1998

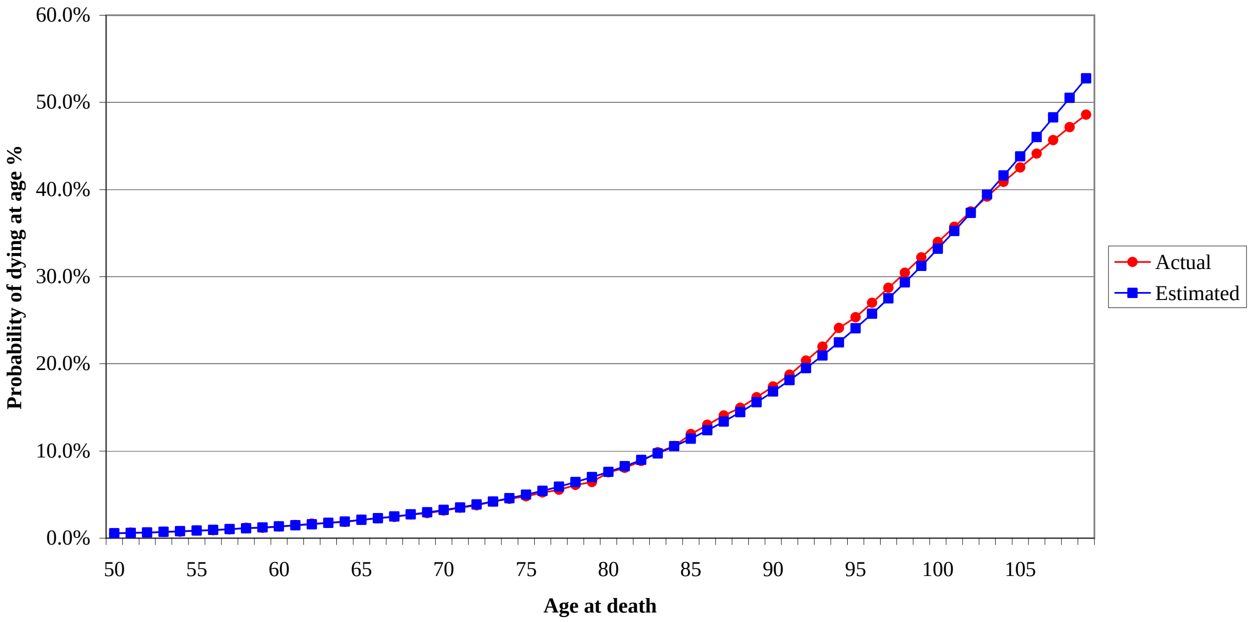


Table 3 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 3
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (U.S. Males)

	1960	1970	1980	1990	1998
Parameter α	4644	4412	7415	11534	16178
Parameter β	0.924	0.925	0.921	0.917	0.914
R^2 value	0.998	0.998	0.999	0.999	0.999

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter α = 3685.3 ⊚ exp(0.0369 ⊚ (Calendar Year – 1959)); R^2 value: 0.929,
Parameter β = −0.000304 ⊚ (Calendar Year – 1959) + 0.92631; R^2 value: 0.877.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we

calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 4.

TABLE 4
Annuities Payable in Arrears on Different Mortality Bases
(U.S. Males)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) This basis	12.71	10.97	9.20	7.46	5.84	4.40	3.19	2.22	1.48
(2) Population mortality	12.01	10.40	8.76	7.16	5.55	4.15	3.05	2.17	1.56
((1) / (2)) − 1	5.8%	5.5%	5.0%	4.3%	5.2%	6.2%	4.6%	2.2%	−5.2%

Cumulative Probability of Death

Logistic Model

We next fitted a logistic model to the cumulative probability of death:

$$1 - \frac{lx}{lo} = \frac{1}{\gamma + \alpha\beta^{x}}$$, where γ is a time-independent constant.

We fitted curves of this type to the actual cumulative probability of death for each of the calendar years from 1960 to 1998. Figures 11–15 provide a demonstration of the appropriateness of the fit for females for years 1960, 1970, 1980, 1990, and 1998, respectively.

Figure 11
U.S. Females: Cumulative q_x as against Logistic Distribution for Year 1960

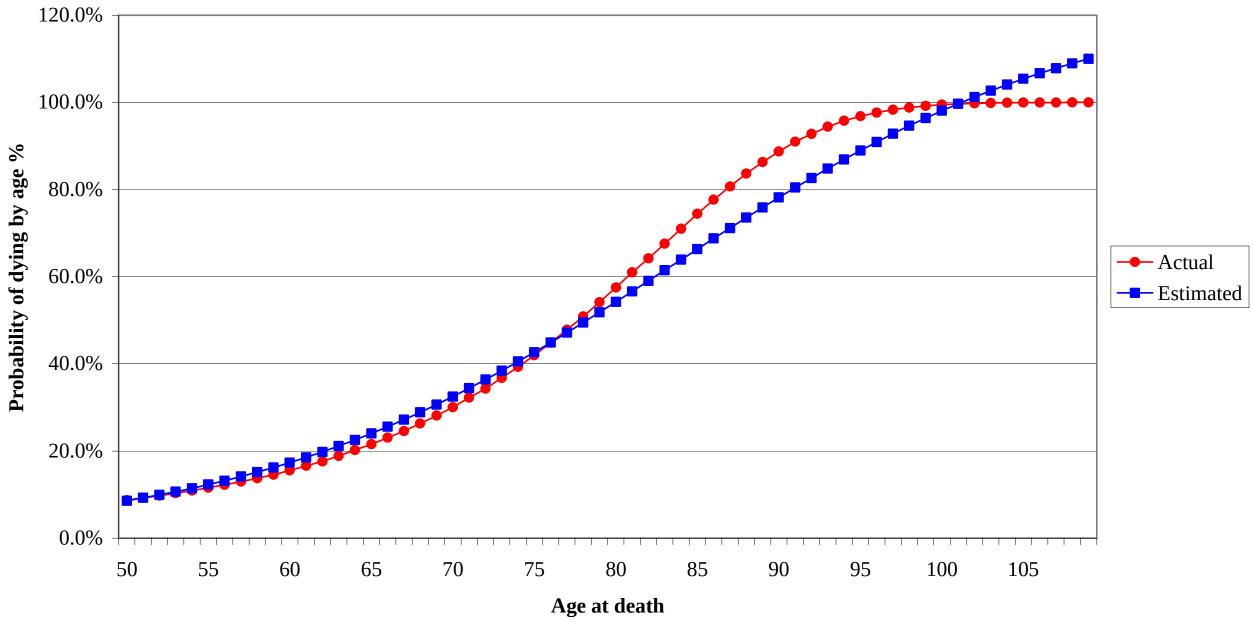


Figure 12
U.S. Females: Cumulative q_x as against Logistic Distribution for Year 1970

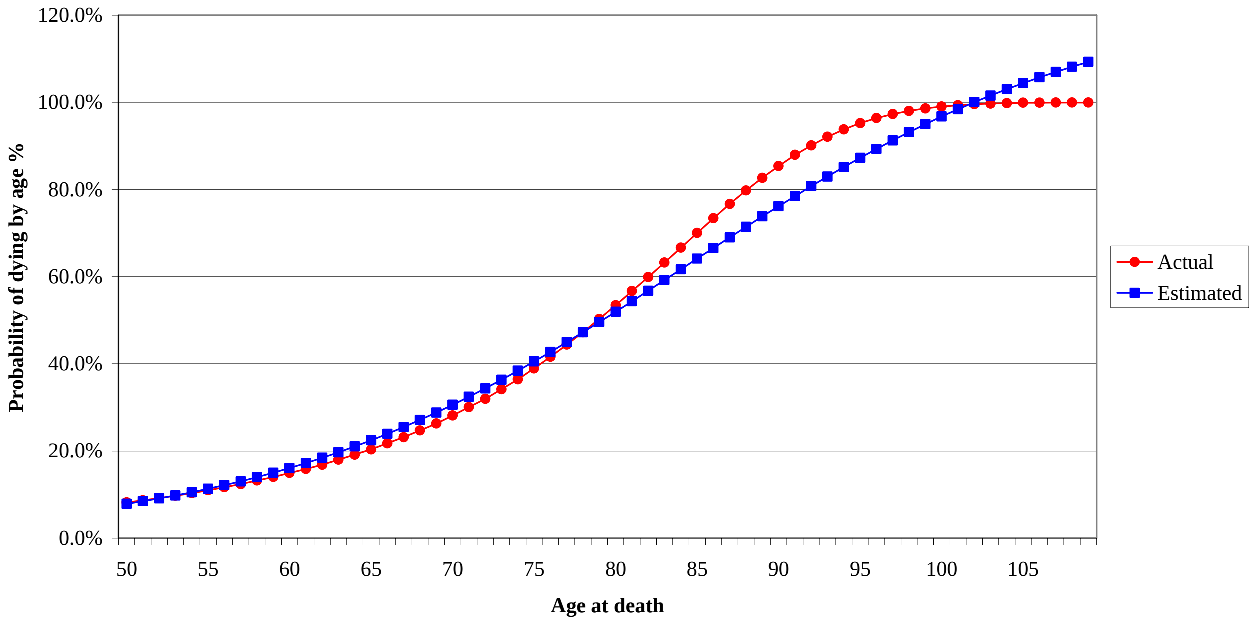


Figure 13
U.S. Females: Cumulative q_x as against Logistic Distribution for Year 1980

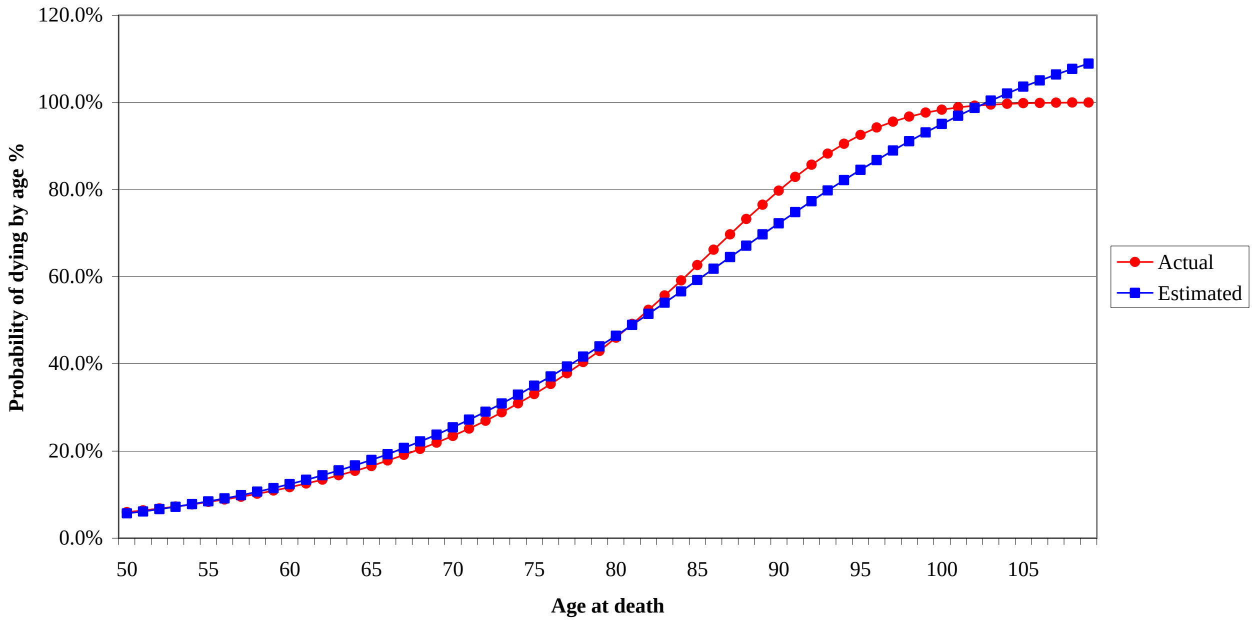


Figure 14
U.S. Females: Cumulative q_x as against Logistic Distribution for Year 1990

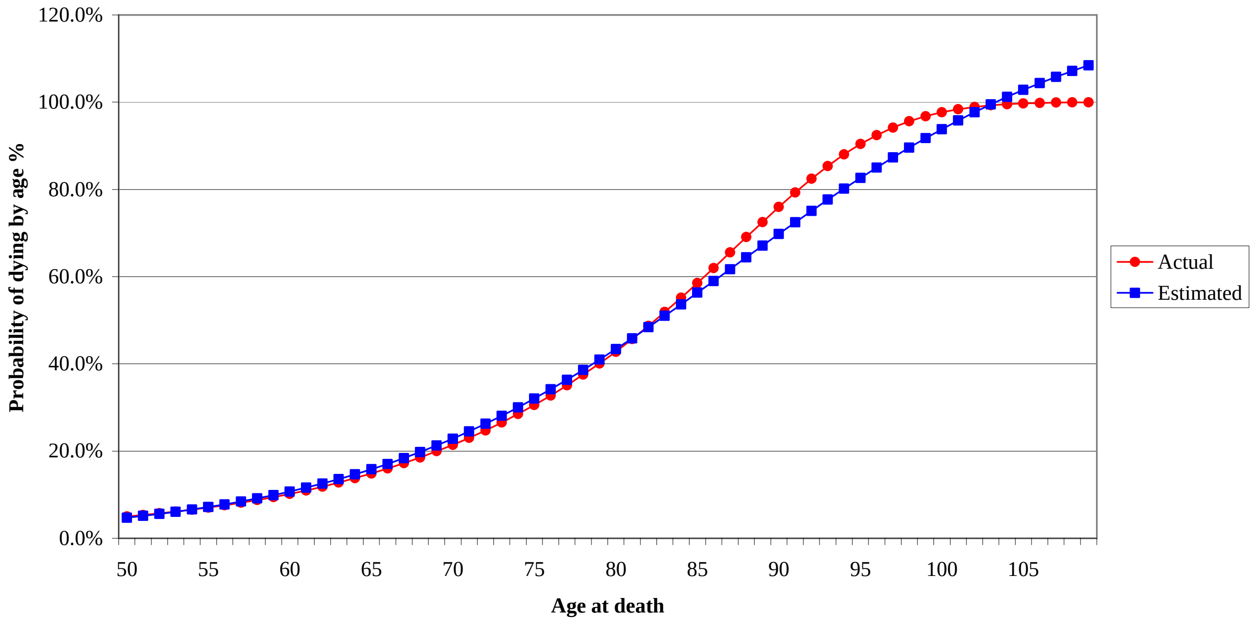


Figure 15
U.S. Females: Cumulative q_x as against Logistic Distribution for Year 1998

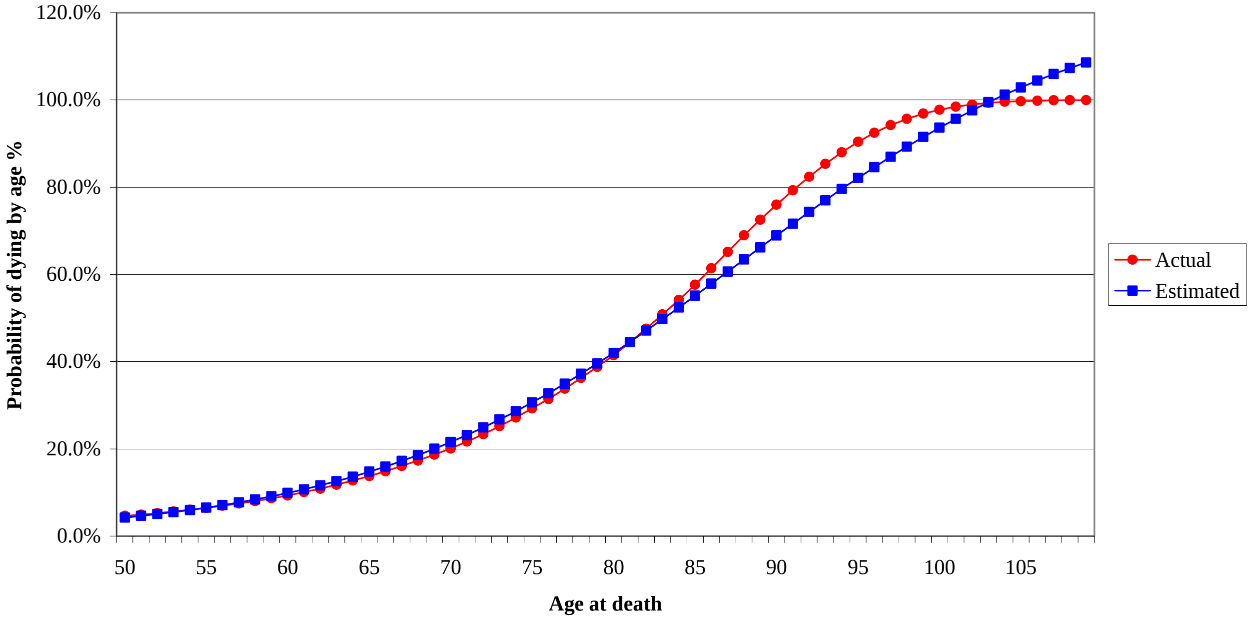


Table 5 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 5
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (U.S. Females)

	1960	1970	1980	1990	1998
Parameter α	495	563	1043	1426	1769
Parameter β	0.925	0.924	0.919	0.917	0.915
R^2 value	0.987	0.980	0.992	0.994	0.993

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter α = 452.77 ⊚ exp(0.0375 ⊚ (Calendar Year – 1959)); R^2 value: 0.962,
Parameter β = –0.000294 ⊚ (Calendar Year – 1959) + 0.92585; R^2 value: 0.961.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we

calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 6.

TABLE 6
Annuities Payable in Arrears on Different Mortality Bases
(U.S. Females)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) This basis	14.42	12.91	11.34	9.74	8.16	6.60	5.02	3.33	1.38
(2) Population mortality	13.51	11.92	10.21	8.45	6.63	4.94	3.59	2.50	1.73
((1) / (2)) −1	6.7%	8.3%	11.0%	15.3%	23.1%	33.6%	39.8%	33.3%	−20.4%

Figures 16–20 provide a demonstration of the appropriateness of the fit for males for years 1960, 1970, 1980, 1990, and 1998, respectively. These figures are based on far fewer lives at high ages.

Figure 16
U.S. Males: Cumulative q_x as against Logistic Distribution for Year 1960

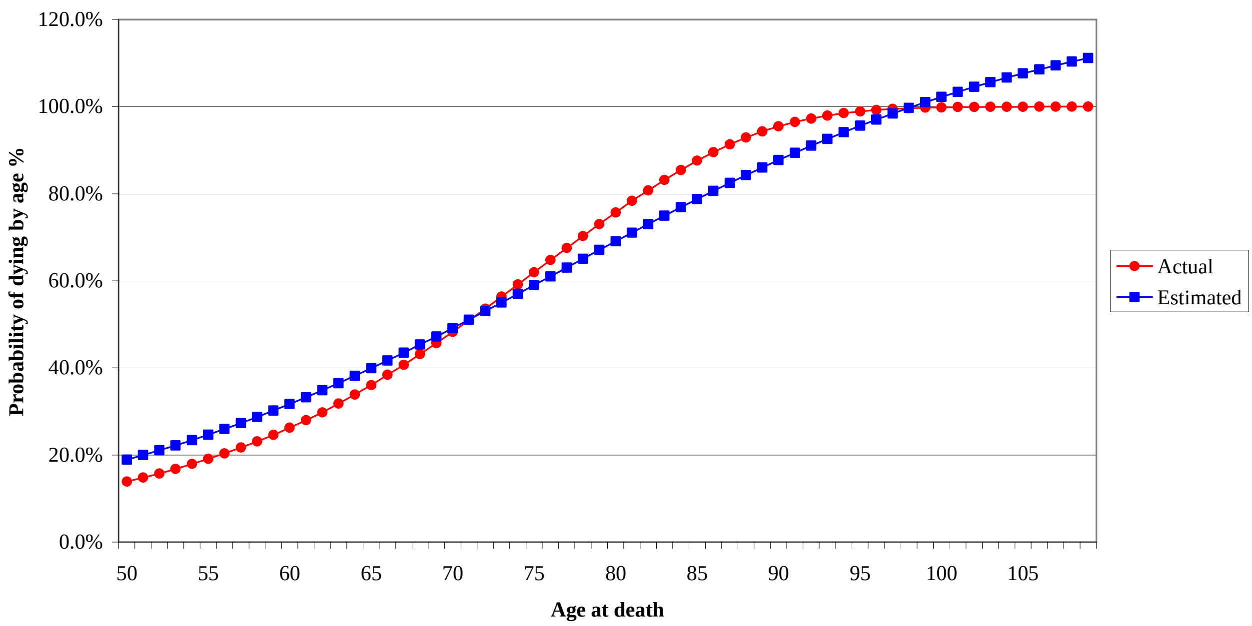


Figure 17
U.S. Males: Cumulative q_x as against Logistic Distribution for Year 1970

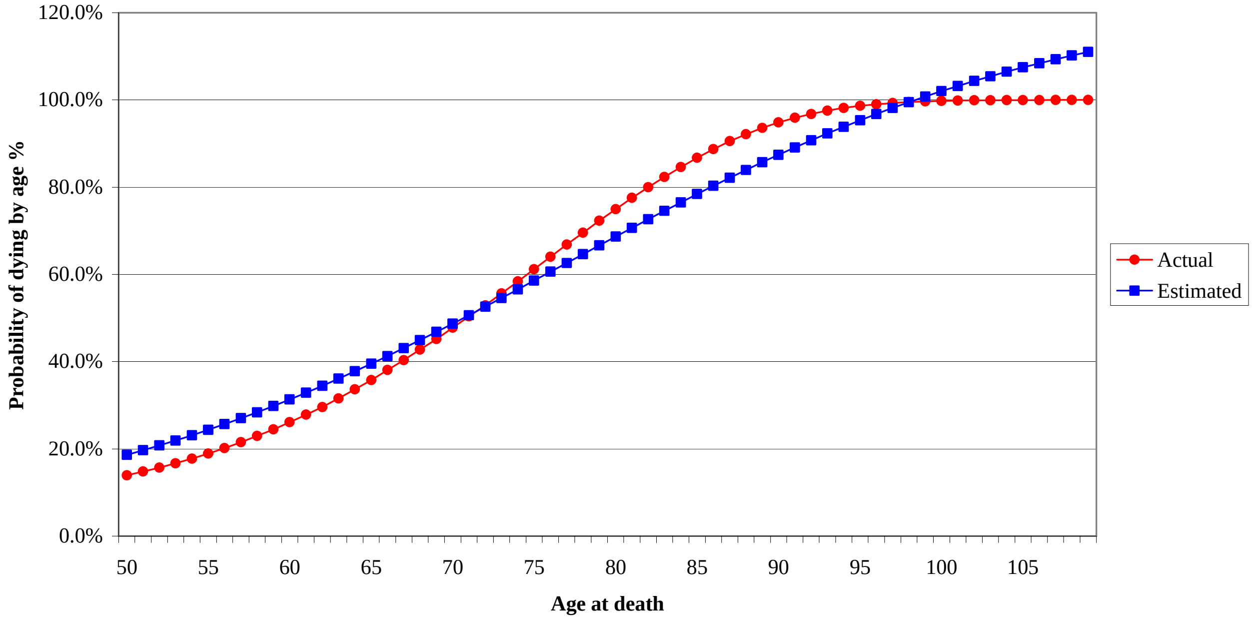


Figure 18
U.S. Males: Cumulative q_x as against Logistic Distribution for Year 1980

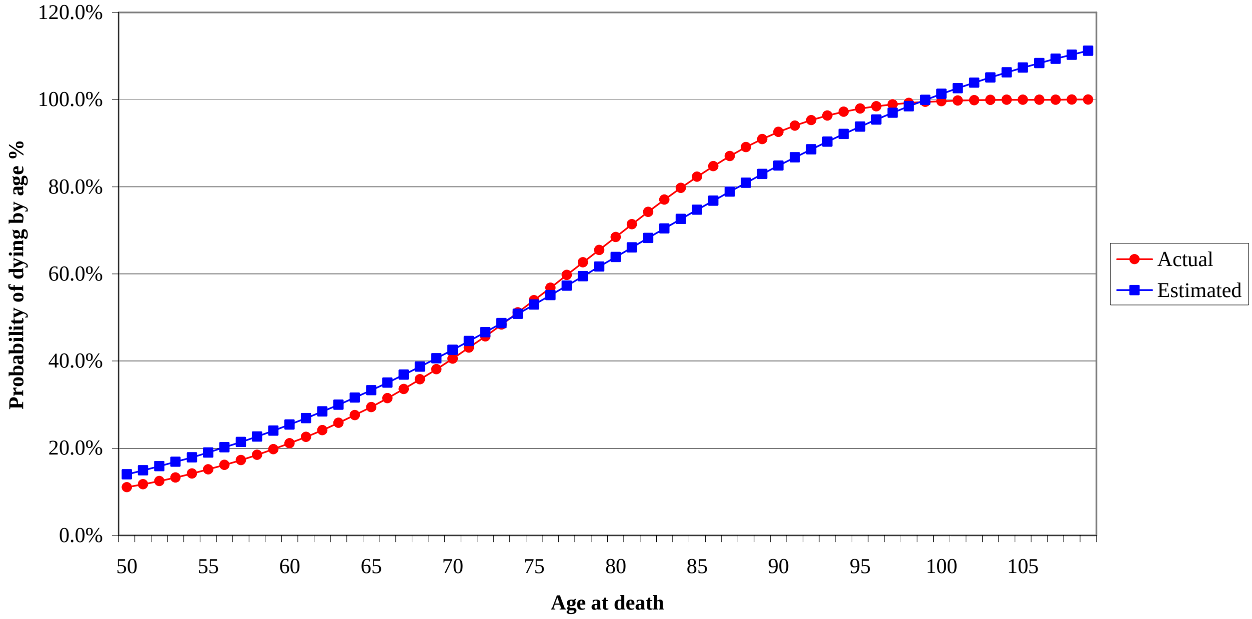


Figure 19
U.S. Males: Cumulative q_x as against Logistic Distribution for Year 1990

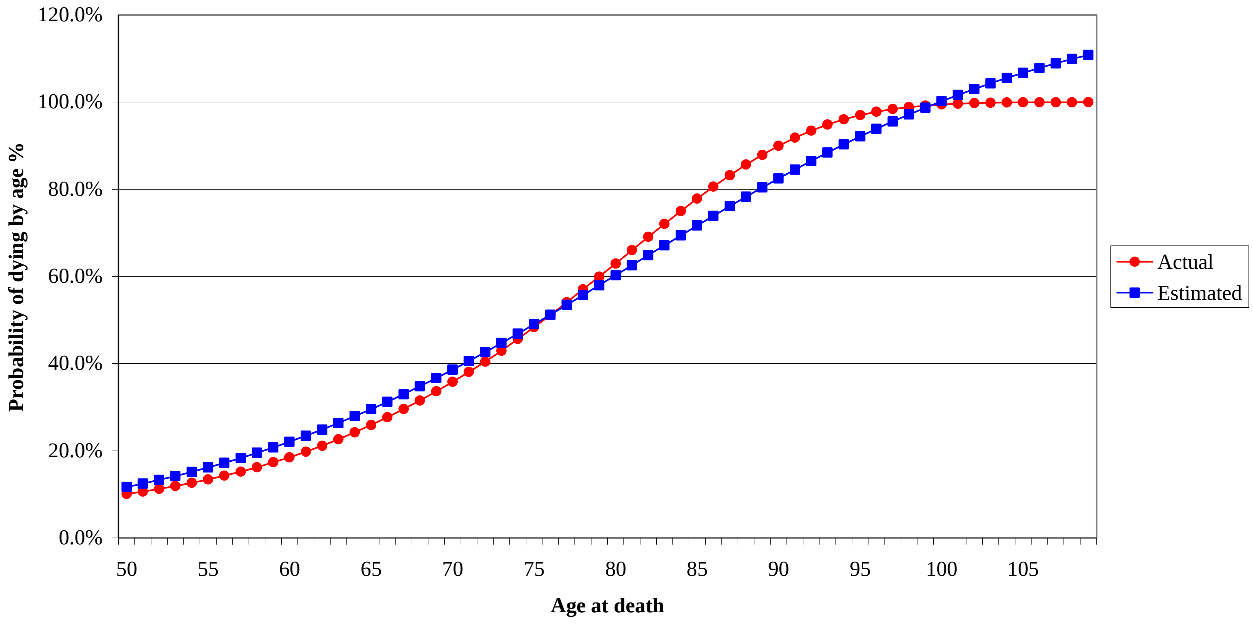


Figure 20
U.S. Males: Cumulative q_x as against Logistic Distribution for Year 1998

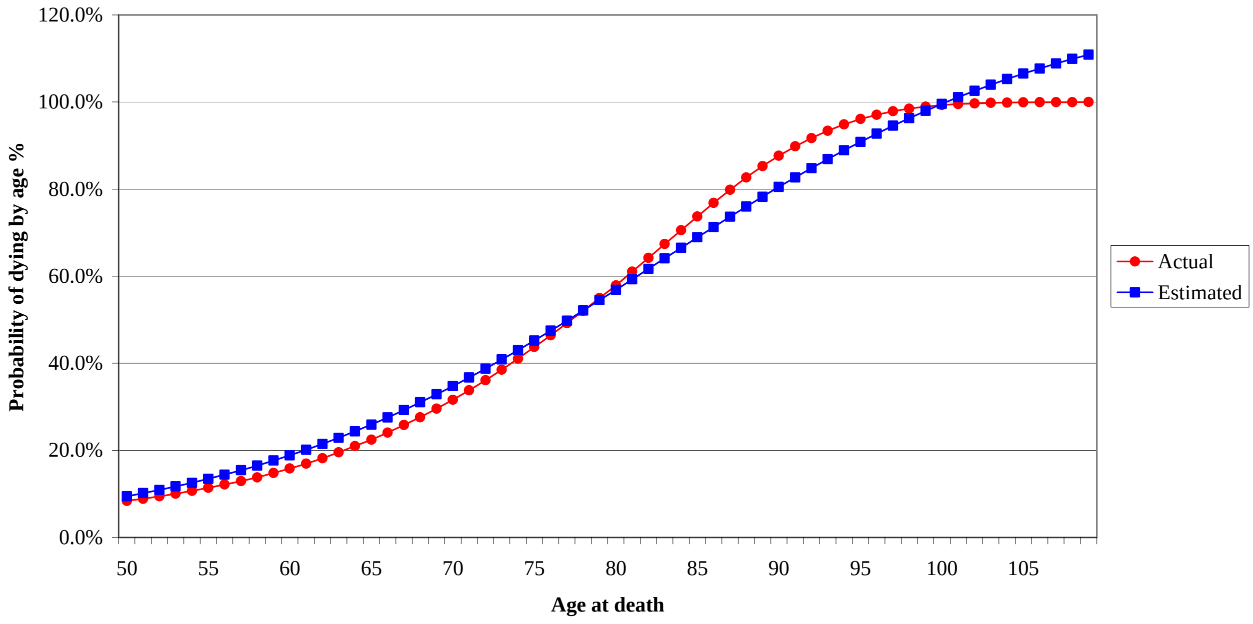


Table 7 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 7
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (U.S. Males)

	1960	1970	1980	1990	1998
Parameter α	113	116	214	306	465
Parameter β	0.938	0.937	0.932	0.929	0.926
R^2 value	0.939	0.943	0.955	0.966	0.971

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter $\alpha = 94.167 \circledcirc \exp(0.0393 \circledcirc (\text{Calendar Year} - 1959))$; R^2 value: 0.942,
Parameter $\beta = -0.00033 \circledcirc (\text{Calendar Year} - 1959) + 0.93899$; R^2 value: 0.933.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 8.

TABLE 8
Annuities Payable in Arrears on Different Mortality Bases (U.S. Males)

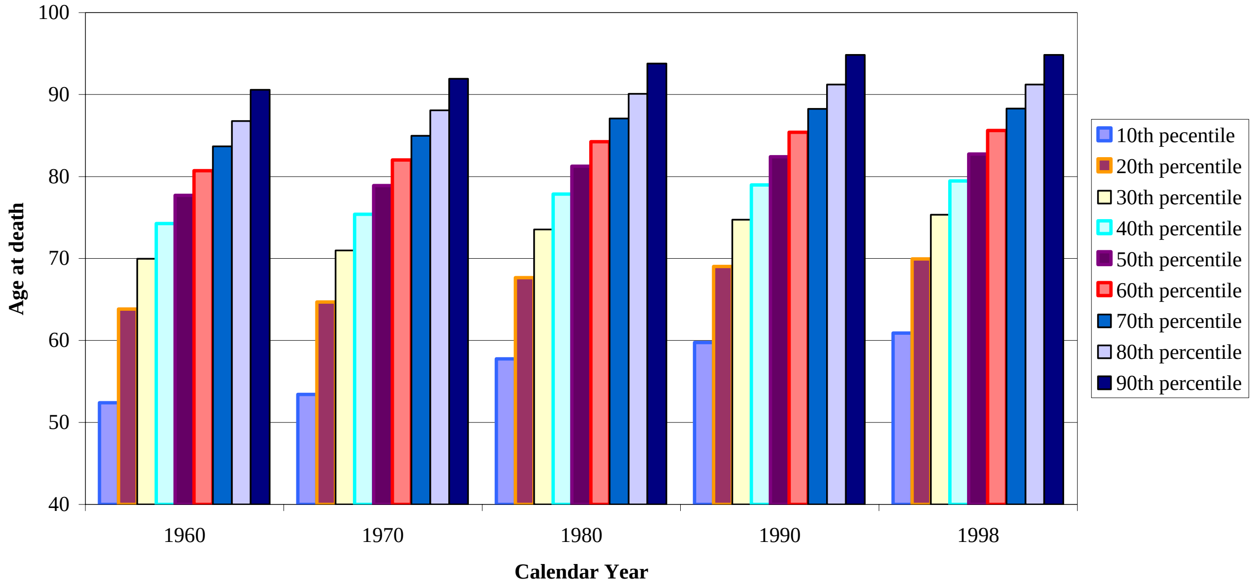
	Age								
	60	65	70	75	80	85	90	95	100
(1) This basis	13.04	11.62	10.17	8.73	7.28	5.80	4.20	2.40	0.30
(2) Population mortality	12.01	10.40	8.76	7.16	5.55	4.15	3.05	2.17	1.56
((1) / (2)) − 1	8.6%	11.7%	16.1%	21.9%	31.1%	39.8%	37.8%	10.7%	−80.7%

Probability of Death at Specified Age

Trends in Percentiles

We derived the age at death for decennial percentiles within the population for each of the calendar years from 1960 to 1998. We further derived age at death for individual percentiles for the same calendar years between the 91st and 99th percentiles. Figure 21 shows the results for females for years 1960, 1970, 1980, 1990, and 1998.

Figure 21
Age at Death from Life Tables for U.S. Females by Percentile for Selected Calendar Years



We determined a time-dependent relationship between age at death for each of the selected percentiles and calendar year. Table 9 sets out the parameters associated with these relationships, and the results of the goodness-of-fit test between the time-dependent relationship and the actual ages at death in each calendar year.

TABLE 9
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (U.S. Females)

	50th Percentile	90th Percentile	95th Percentile	99th Percentile
Intercept	77.5	90.4	92.4	93.9
Gradient	0.149	0.132	0.128	0.126
R^2 value	0.948	0.928		

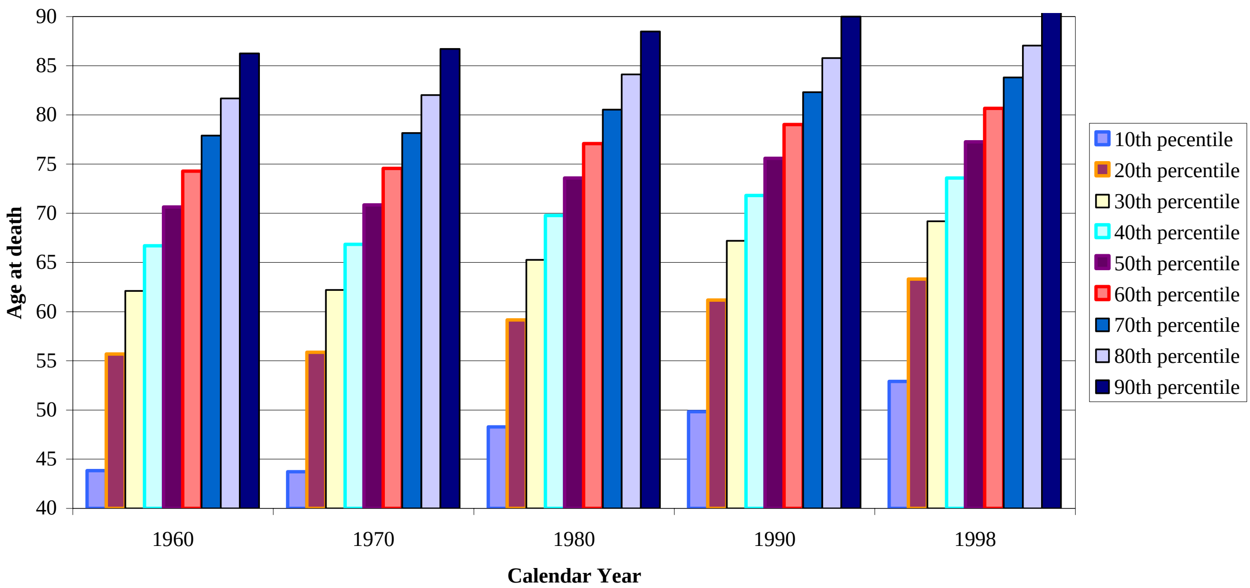
From this we projected the age at death for each of the percentiles set out above for selected future calendar years. The results are set out in Table 10.

TABLE 10
Age at Death for Specified Percentile in Selected Calendar Years
(U.S. Females)

Population Percentile	Calendar Year								
	1998	1999	2000	2005	2010	2015	2020	2025	2030
50th percentile	84.8	85.0	85.1	85.8	86.6	87.3	88.1	88.8	89.6
90th percentile	96.9	97.0	97.2	97.8	98.5	99.1	99.8	100.5	101.1
95th percentile	98.7	98.8	98.9	99.6	100.2	100.8	101.5	102.1	102.8
99th percentile	100.1	100.2	100.3	100.9	101.6	102.2	102.8	103.5	104.1

Figure 22 shows the results for males for years 1960, 1970, 1980, 1990, and 1998.

Figure 22
Age at Death from Life Tables for U.S. Males by Percentile for Selected Calendar Years



We determined a time-dependent relationship between age at death for each of the selected percentiles and calendar year. Table 11 sets out the parameters associated with these relationships, and the results of the goodness-of-fit test between the time-dependent relationship and the actual ages at death in each calendar year.

TABLE 11
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit
(U.S. Males)

	50th Percentile	90th Percentile	95th Percentile	99th Percentile
Intercept	69.4	85.4	87.8	89.7
Gradient	0.187	0.138	0.131	0.125
R^2 value	0.955	0.958		

From this we projected the age at death for each of the percentiles set out above for selected future calendar years. The results are set out in Table 12.

TABLE 12
Age at Death for Specified Percentile in Selected Calendar Years
(U.S. Males)

Population Percentile	Calendar Year								
	1998	1999	2000	2005	2010	2015	2020	2025	2030
50th percentile	78.6	78.8	79.0	79.9	80.9	81.8	82.7	83.7	84.6
90th percentile	92.2	92.3	92.5	93.2	93.9	94.6	95.2	95.9	96.6
95th percentile	94.2	94.3	94.5	95.1	95.8	96.4	97.1	97.7	98.4
99th percentile	95.8	95.9	96.1	96.7	97.3	97.9	98.5	99.2	99.8

Weibull Distribution

We also fitted a Weibull distribution to the probability of death at a given age with the formula

$$\text{Probability of death at a given age} = (\alpha/\beta^{\alpha}) \circledcirc (x^{\alpha-1}) \circledcirc\circledcirc\circledcirc \exp(-(x/\beta)^{\alpha}).$$

We fitted curves of this type to the actual cumulative probability of death for each of the calendar years from 1960 to 1998. Figures 23–27 provide a demonstration of the appropriateness of the fit for females for years 1960, 1970, 1980, 1990, and 1998, respectively.

Figure 23
U.S. Females: Age at Death as against Weibull Distribution for Year 1960

Figure 24
U.S. Females: Age at Death as against Weibull Distribution for Year 1970

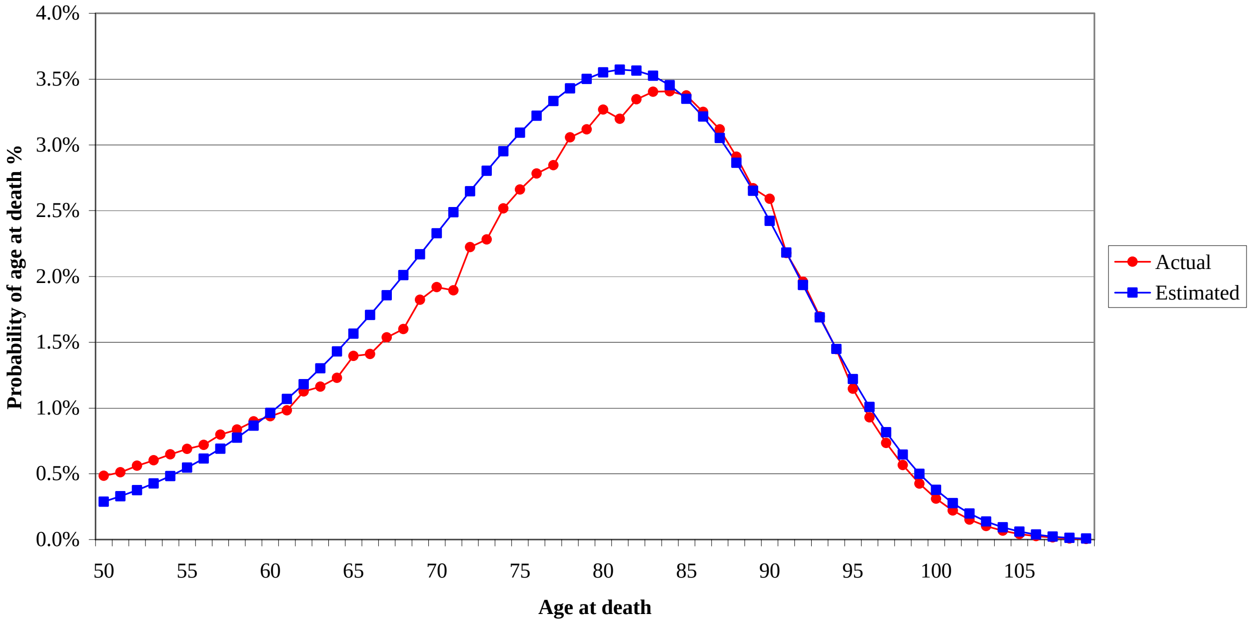


Figure 25
U.S. Females: Age at Death as against Weibull Distribution for Year 1980

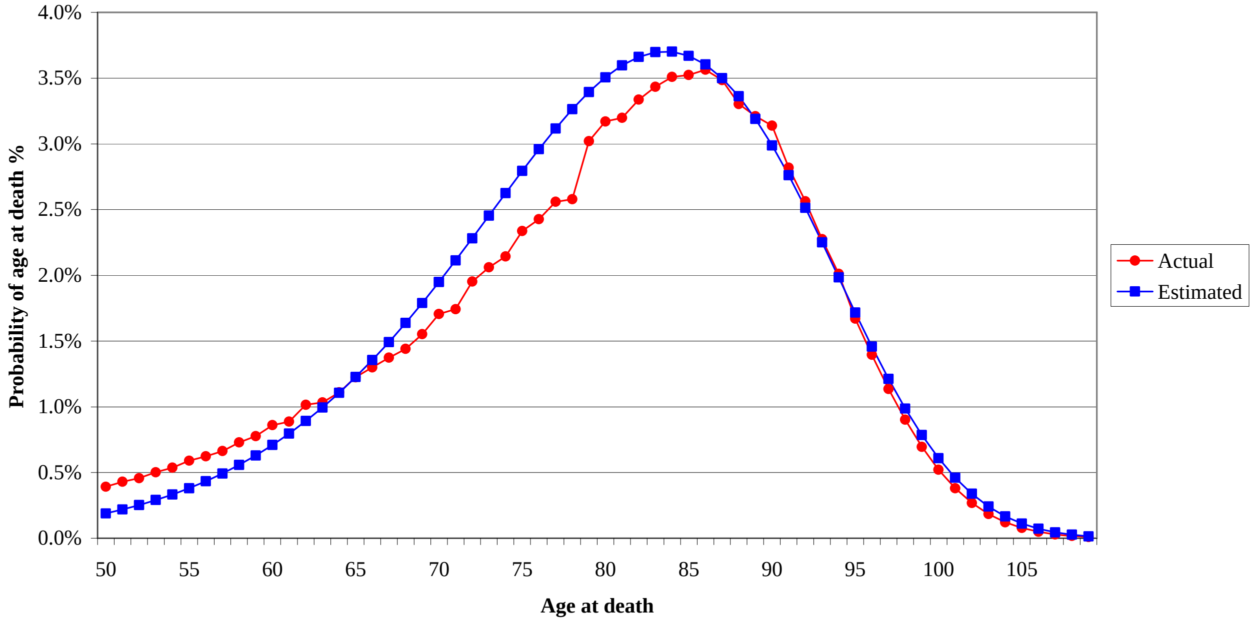


Figure 26
U.S. Females: Age at Death as against Weibull Distribution for Year 1990

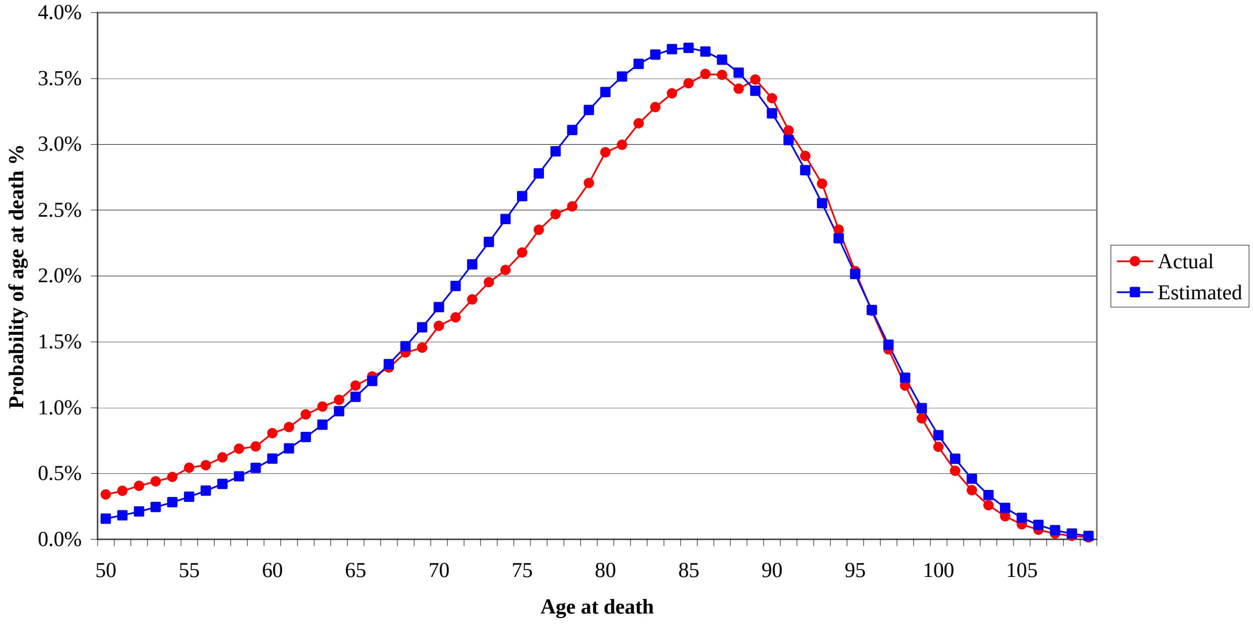


Figure 27
U.S. Females: Age at Death as against Weibull Distribution for Year 1998

Table 13 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the Weibull distribution for each of the selected years.

TABLE 13
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (U.S. Females)

	1960	1970	1980	1990	1998
Parameter α	7.94	7.96	8.48	8.66	8.91
Parameter β	81.38	82.62	84.85	85.97	86.23
R^2 value	0.991	0.989	0.989	0.989	0.991
Skewness	0.270	0.261	0.329	0.346	0.382

We derived the following time-dependent formulas for each of the parameters required by the Weibull distribution and provide details of the analysis of goodness of fit as follows:

Parameter $\alpha = 0.0259 \circledcirc$ (Calendar Year – 1959) + 7.8987; R^2 value: 0.959,
Parameter $\beta = 0.1444 \circledcirc$ (Calendar Year – 1959) + 81.493; R^2 value: 0.939.

From this we projected the age at death for selected percentiles for selected future calendar years. The results are set out in Table 14.

TABLE 14
Age at Death for Specified Percentile in Selected Calendar Years
(U.S. Females)

Population Percentile	Calendar Year								
	1998	1999	2000	2005	2010	2015	2020	2025	2030
50th percentile		84.3	84.4	85.1	85.9	86.6	87.3	88.1	88.8
90th percentile		96.3	96.4	97.1	97.8	98.4	99.1	99.7	100.4
95th percentile		99.2	99.3	99.9	100.6	101.2	101.8	102.4	103.0
99th percentile		104.0	104.1	104.7	105.2	105.8	106.3	106.9	107.4

Figures 28–32 provide a demonstration of the appropriateness of the fit for males for years 1960, 1970, 1980, 1990, and 1998, respectively. These figures are based on far fewer lives at high ages.

Figure 28
U.S. Males: Age at Death as against Weibull Distribution for Year 1960

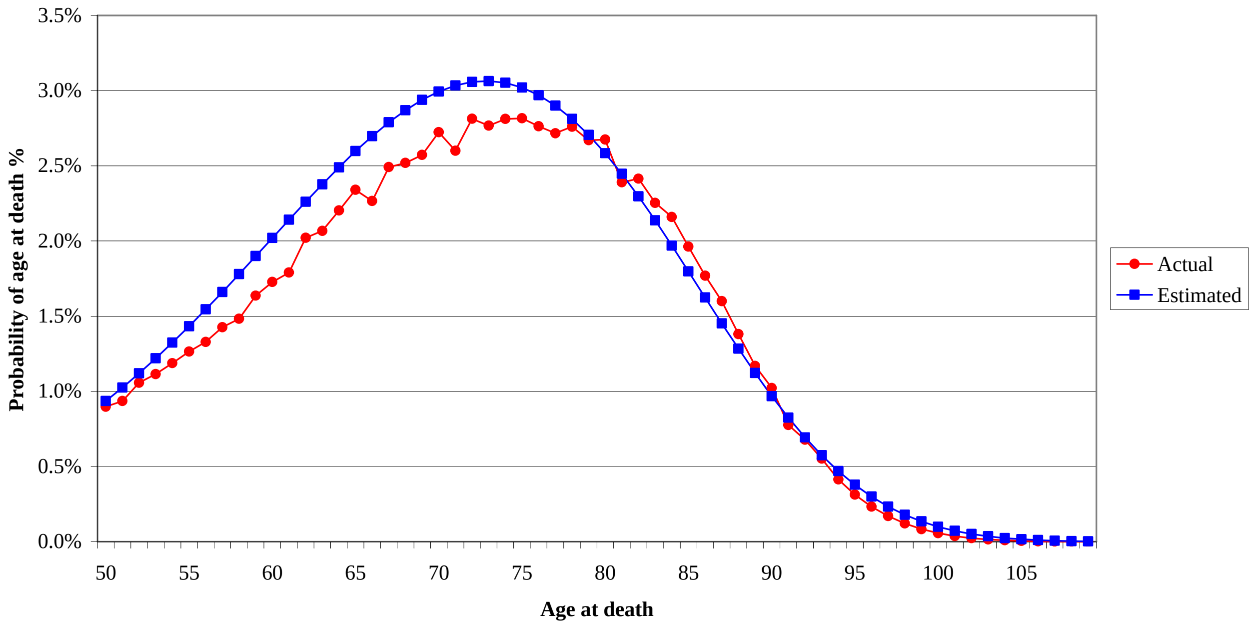


Figure 29
U.S. Males: Age at Death as against Weibull Distribution for Year 1970

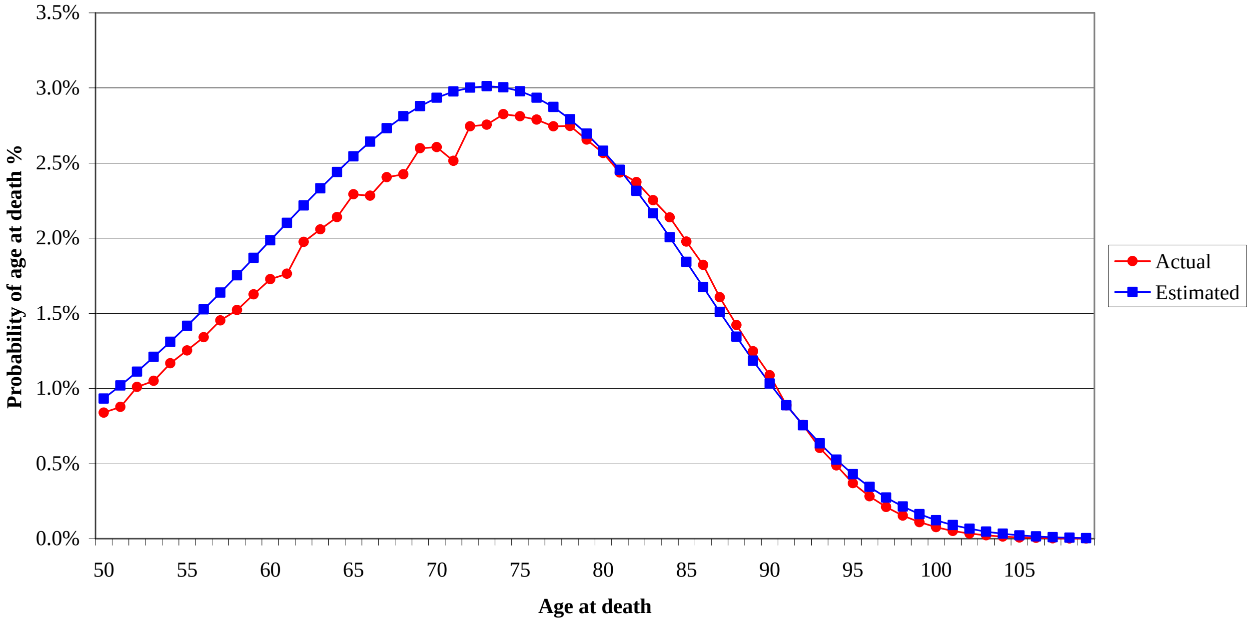


Figure 30
U.S. Males: Age at Death as against Weibull Distribution for Year 1980

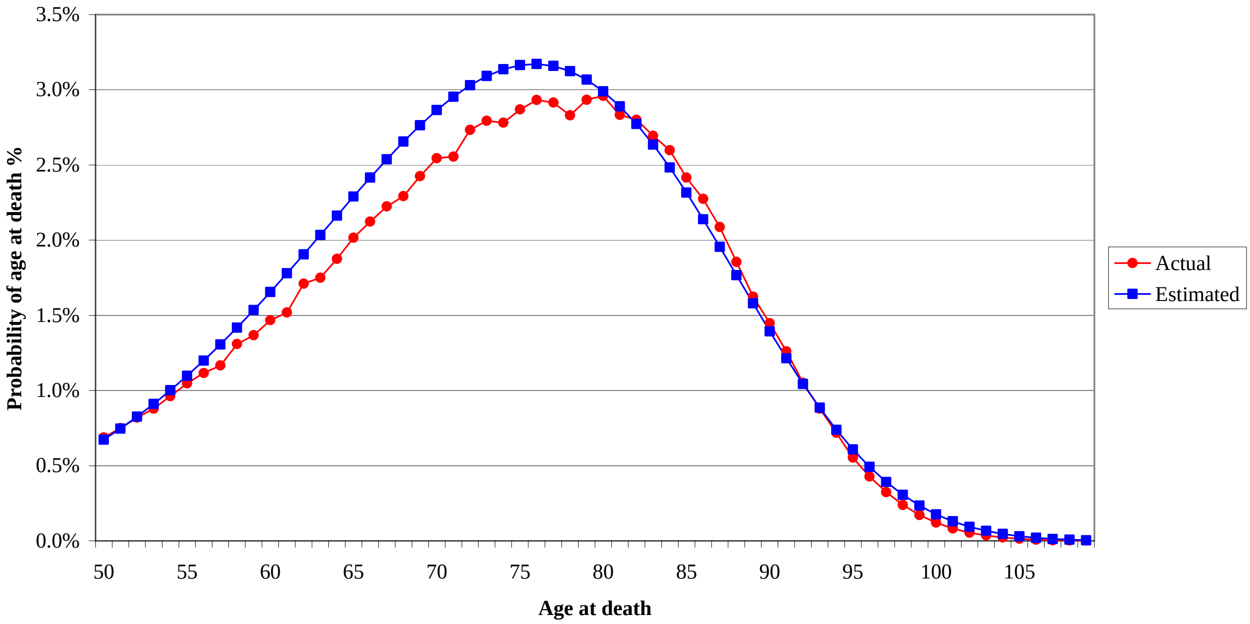


Figure 31
U.S. Males: Age at Death as against Weibull Distribution for Year 1990

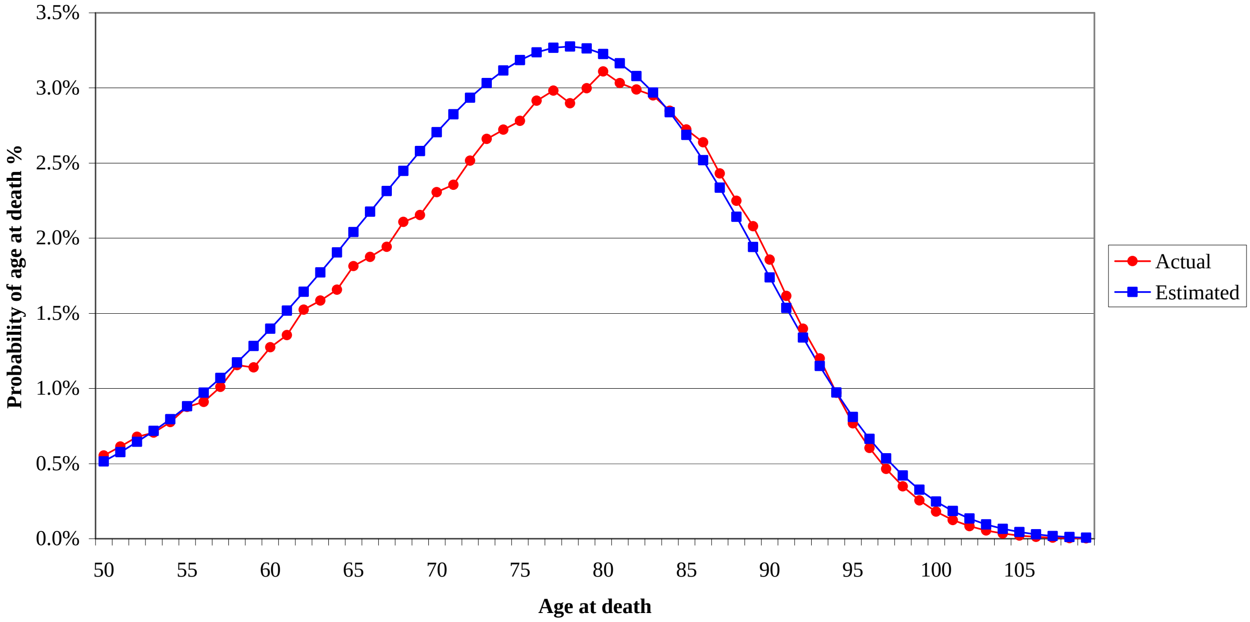


Figure 32
U.S. Males: Age at Death as against Weibull Distribution for Year 1998

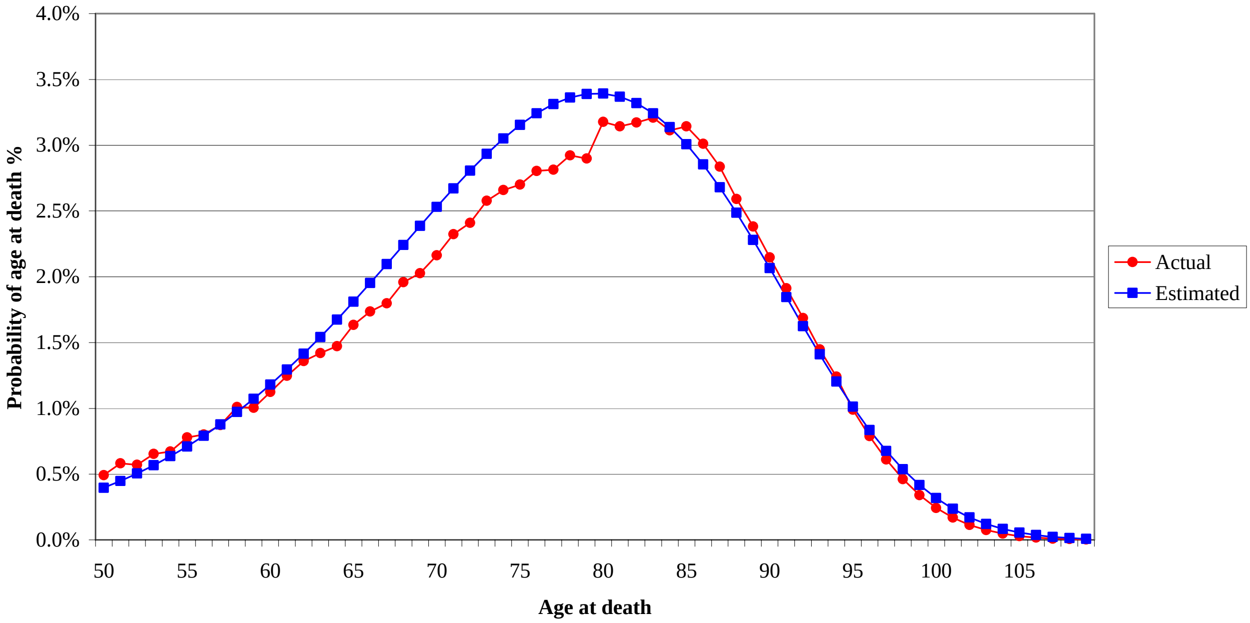


Table 15 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the Weibull distribution for each of the selected years.

TABLE 15
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit
(U.S. Males)

	1960	**1970**	**1980**	**1990**	**1998**
Parameter α	6.16	6.07	6.62	7.02	7.42
Parameter β	74.97	75.26	77.77	79.64	81.18
R^2 value	0.991	0.993	0.993	0.992	0.990
Skewness	–0.051	–0.080	0.021	0.096	0.169

We derived the following time-dependent formulas for each of the parameters required by the Weibull distribution and provide details of the analysis of goodness of fit as follows:

Parameter $\alpha = 0.036 \circledcirc (\text{Calendar Year} - 1959) + 5.9263$; R^2 value: 0.930,
Parameter $\beta = 0.1802 \circledcirc (\text{Calendar Year} - 1959) + 74.104$; R^2 value: 0.964.

From this we projected the age at death for selected percentiles for selected future calendar years. The results are set out in Table 16.

TABLE 16
Age at Death for Specified Percentile in Selected Calendar Years
(U.S. Males)

Population Percentile	**Calendar Year**								
	1998	**1999**	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**
50th percentile		77.9	78.1	79.0	79.9	80.9	81.8	82.8	83.7
90th percentile		91.6	91.7	92.5	93.2	94.0	94.8	95.6	96.4
95th percentile		94.9	95.0	95.7	96.4	97.2	97.9	98.6	99.4
99th percentile		100.5	100.7	101.3	101.9	102.5	103.1	103.8	104.4

Principal Results: Japan

Age-Specific q_x
Logistic Model

The logistic model is of the form

$$q_x = \frac{1}{\gamma + \alpha\beta^x}$$

, where γ is a time-independent constant.

We fitted curves of this type to the actual mortality experience for each of the calendar years from 1960 to 1998. Figures 33–37 provide a demonstration of the appropriateness of the fit for females for years 1960, 1970, 1980, 1990, and 1998, respectively.

Figure 33
Japanese Females: Actual q_x as against Logistic Distribution for Year 1960

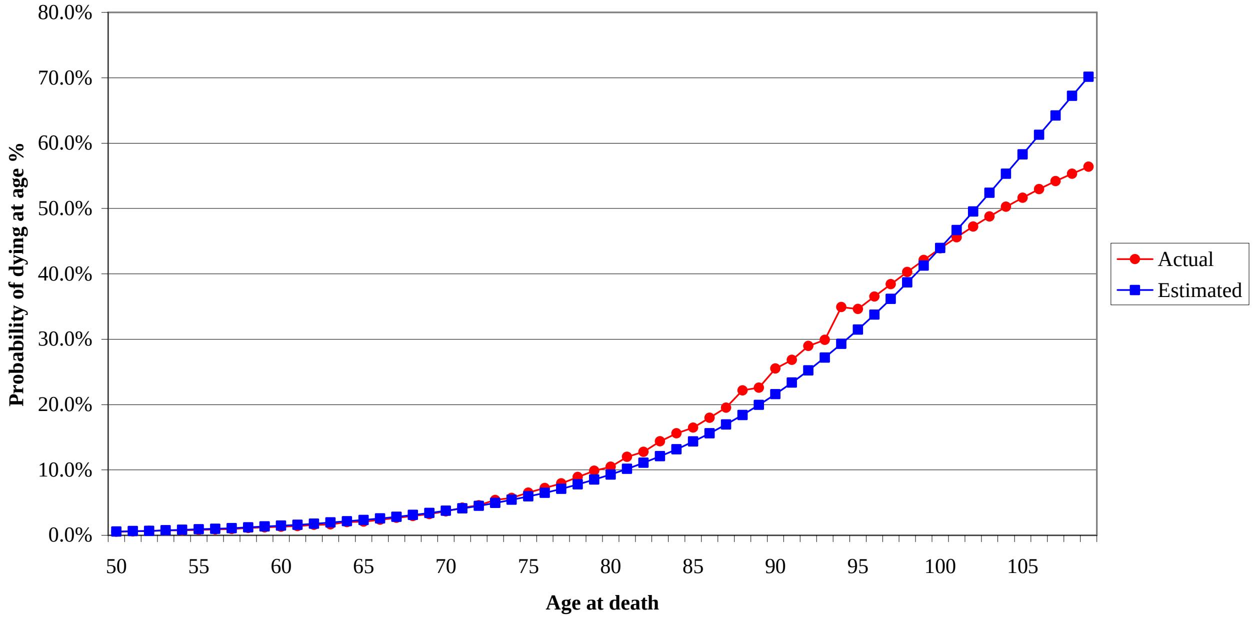


Figure 34
Japanese Females: Actual q_x as against Logistic Distribution for Year 1970

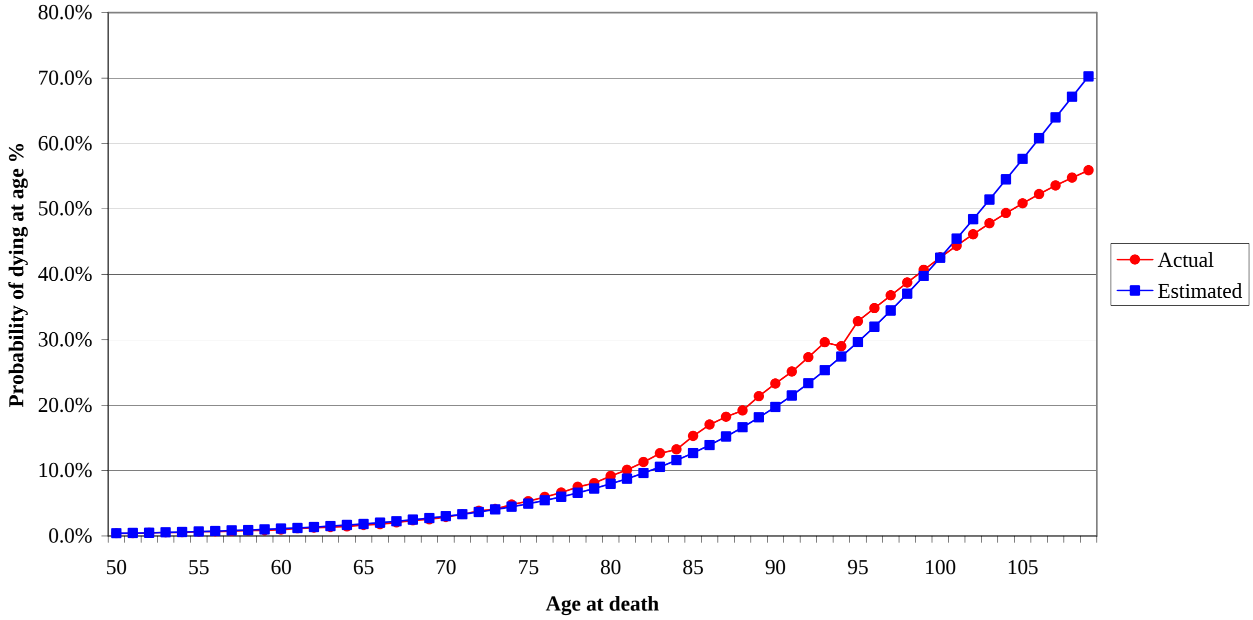


Figure 35
Japanese Females: Actual q_x as against Logistic Distribution for Year 1980

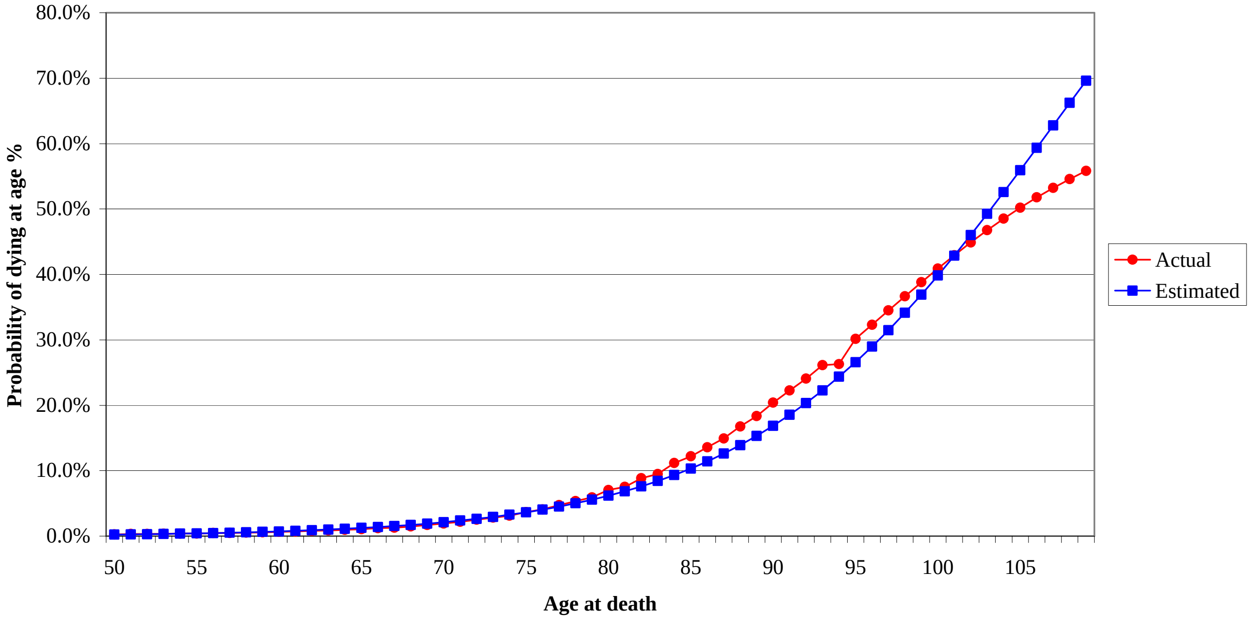


Figure 36
Japanese Females: Actual q_x as against Logistic Distribution for Year 1990

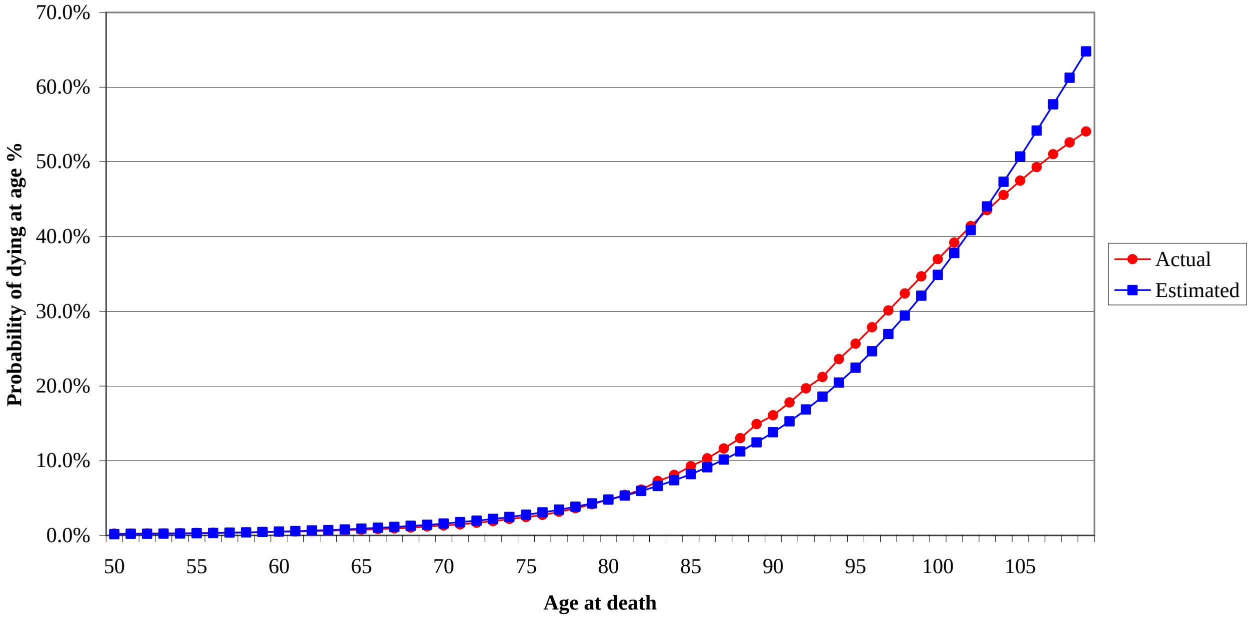


Figure 37
Japanese Females: Actual q_x as against Logistic Distribution for Year 1998

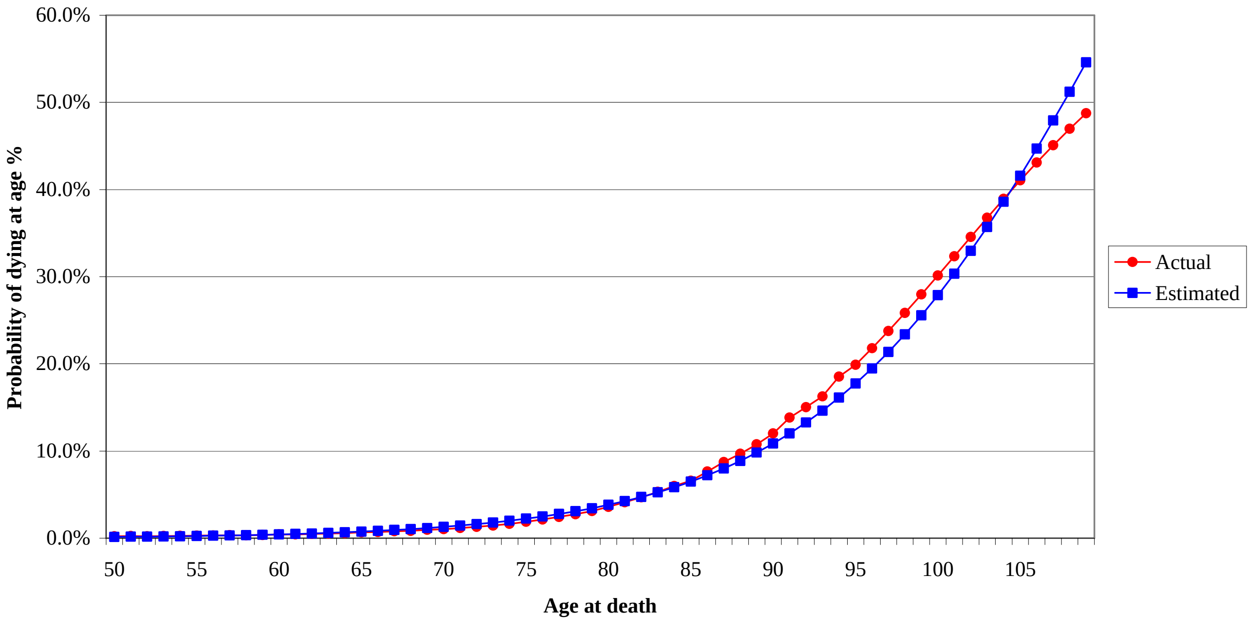


Table 17 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 17
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (Japanese Females)

	1960	1970	1980	1990	1998
Parameter α	20540	38735	100505	179851	173158
Parameter β	0.909	0.904	0.896	0.893	0.896
R^2 value	0.992	0.992	0.993	0.994	0.994

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter α = 22885 ◎ exp(0.0641 ◎ (Calendar Year – 1959)); R^2 value: 0.960,
Parameter β = –0.000438 ◎ (Calendar Year – 1959) + 0.90732; R^2 value: 0.888.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we

calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 18.

TABLE 18
Annuities Payable in Arrears on Different Mortality Bases
(Japanese Females)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) This basis	16.23	14.53	12.61	10.53	8.39	6.34	4.50	3.00	1.86
(2) Population mortality	14.86	13.25	11.44	9.44	7.40	5.49	3.86	2.64	1.75
((1) / (2)) − 1	9.2%	9.7%	10.3%	11.5%	13.4%	15.3%	16.5%	13.6%	6.0%

Figures 38–42 provide a demonstration of the appropriateness of the fit for males for years 1960, 1970, 1980, 1990, and 1998, respectively. These figures are based on far fewer lives at high ages.

Figure 38
Japanese Males: Actual q_x as against Logistic Distribution for Year 1960

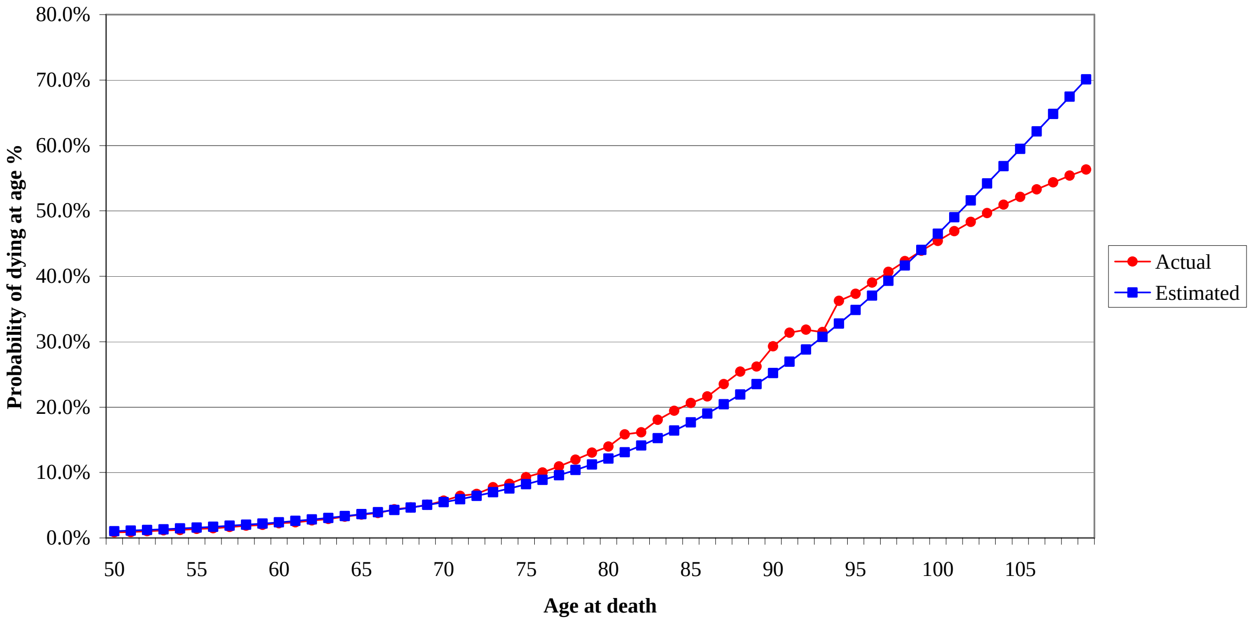


Figure 39
Japanese Males: Actual q_x as against Logistic Distribution for Year 1970

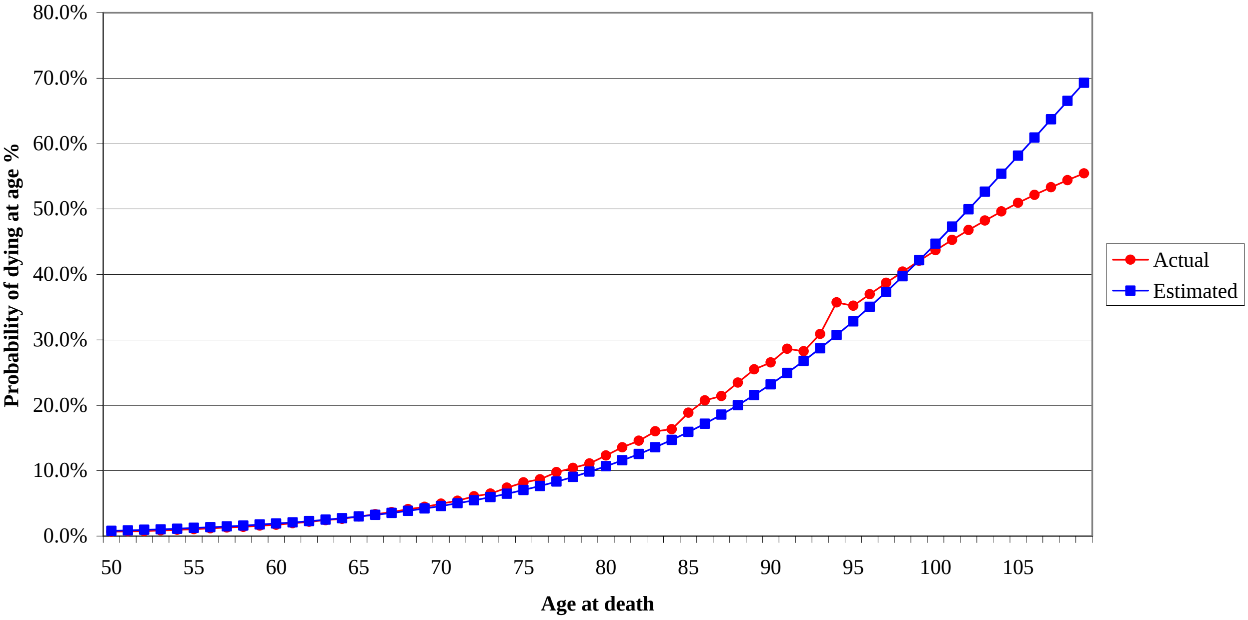


Figure 40
Japanese Males: Actual q_x as against Logistic Distribution for Year 1980

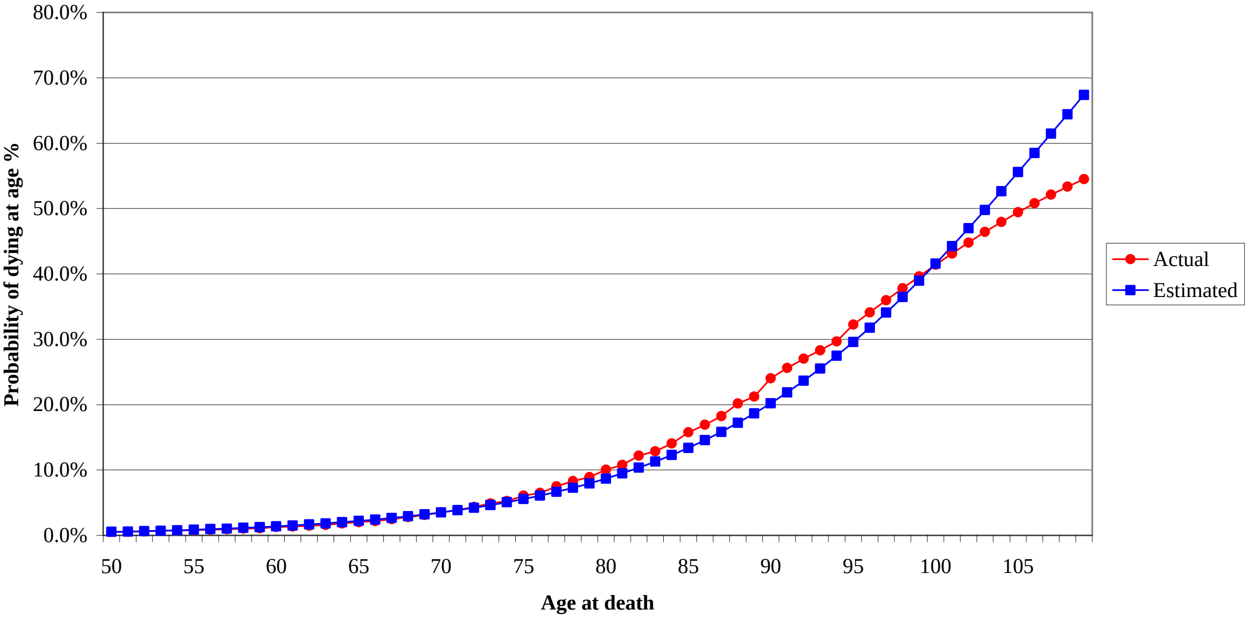


Figure 41
Japanese Males: Actual q_x as against Logistic Distribution for Year 1990

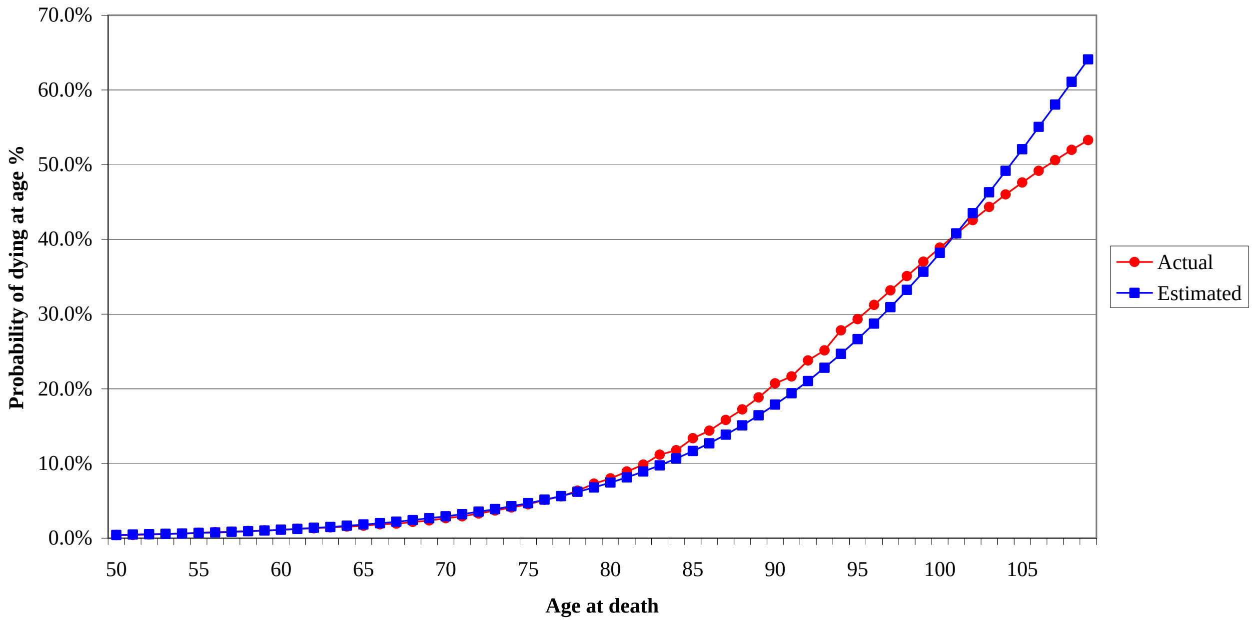


Figure 42
Japanese Males: Actual q_x as against Logistic Distribution for Year 1998

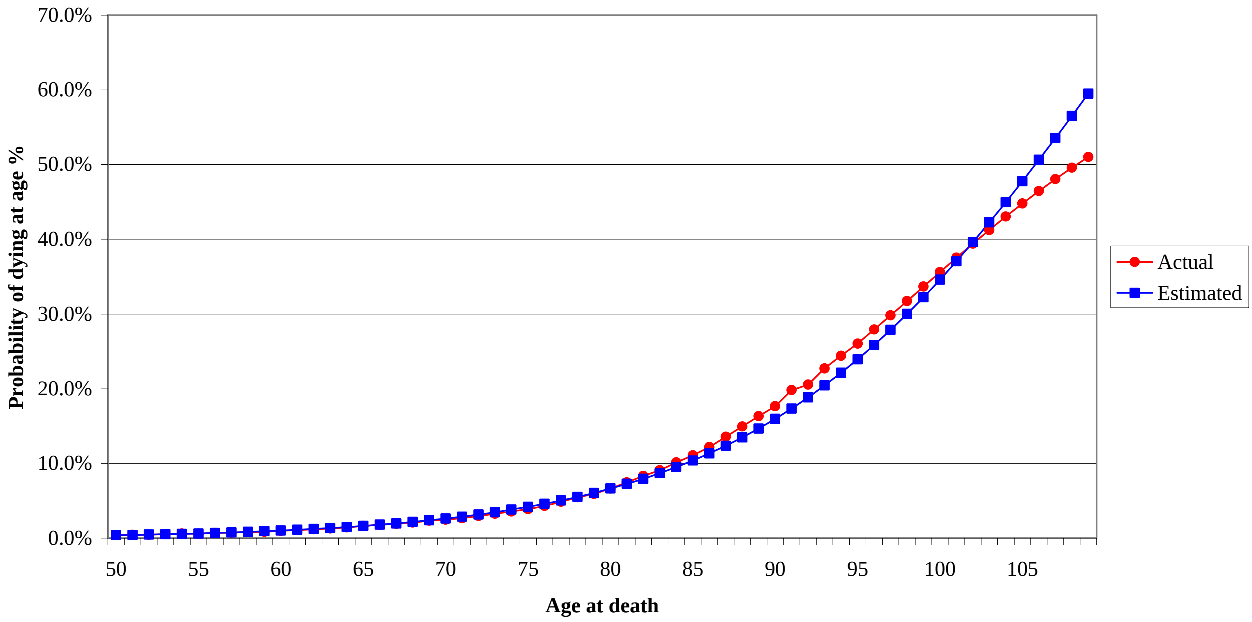


Table 19 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 19
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (Japanese Males)

	1960	1970	1980	1990	1998
Parameter α	6867	10824	21158	29541	31079
Parameter β	0.918	0.915	0.910	0.908	0.908
R^2 value	0.989	0.989	0.993	0.995	0.997

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter α = 7792.8 ⊚ exp(0.042 ⊚ (Calendar Year – 1959)); R^2 value: 0.952,
Parameter β = –0.00028 ⊚ (Calendar Year – 1959) + 0.91677; R^2 value: 0.876.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 20.

TABLE 20
Annuities Payable in Arrears on Different Mortality Bases (Japanese Males)

	Age								
	60	65	70	75	80	85	90	95	100
(1) This basis	14.05	12.26	10.37	8.45	6.62	4.95	3.54	2.40	1.55
(2) Population mortality	12.72	11.04	9.30	7.49	5.73	4.20	2.97	2.08	1.46
((1) / (2)) − 1	10.5%	11.1%	11.5%	12.8%	15.4%	17.9%	19.0%	15.3%	6.3%

Cumulative Probability of Death

Logistic Model

We next fitted a logistic model to the cumulative probability of death:

$$1 - \frac{\text{lx}}{\text{lo}} = \frac{1}{\gamma + \alpha\beta^{x}}$$, where γ is a time-independent constant.

We fitted curves of this type to the actual cumulative probability of death for each of the calendar years from 1960 to 1998. Figures 43–47 provide a demonstration of the appropriateness of the fit for females for years 1960, 1970, 1980, 1990, and 1998, respectively.

Figure 43
Japanese Females: Cumulative q_x as against Logistic Distribution for Year 1960

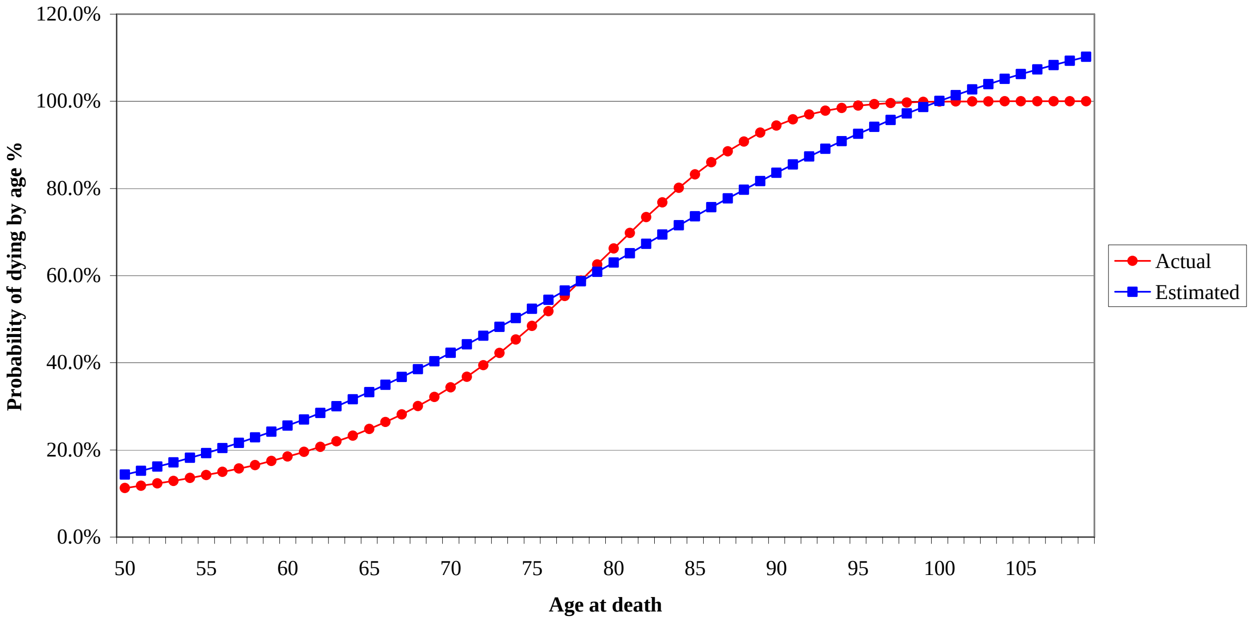


Figure 44
Japanese Females: Cumulative q_x as against Logistic Distribution for Year 1970

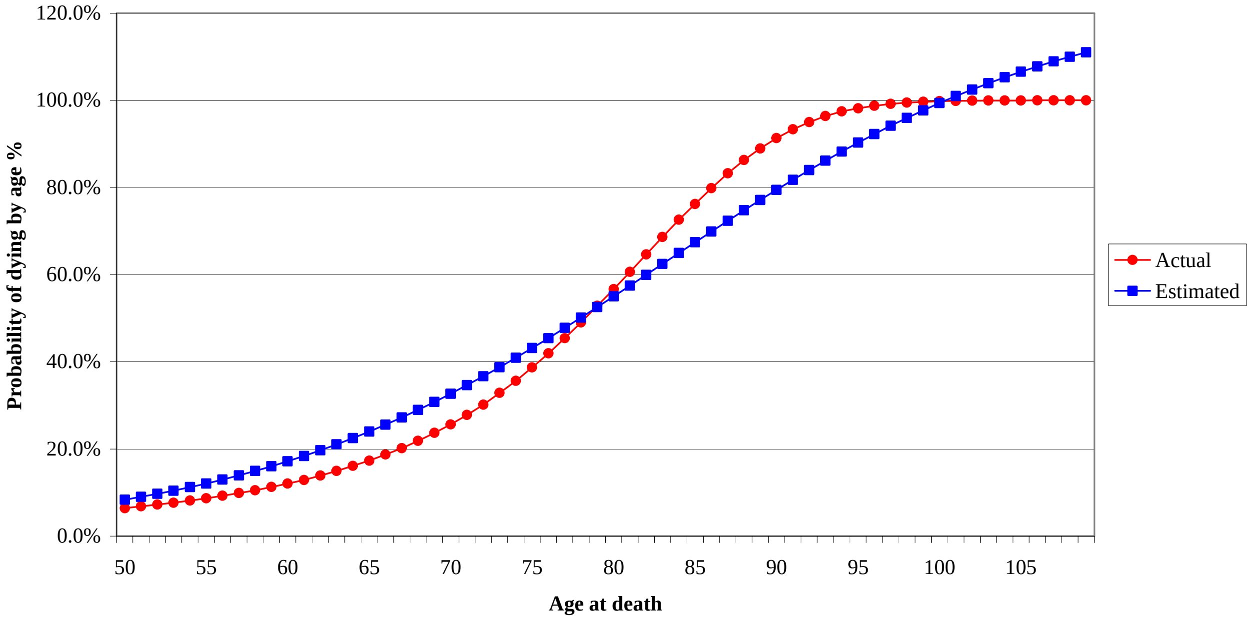


Figure 45

Japanese Females: Cumulative q_x as against Logistic Distribution for Year 1980

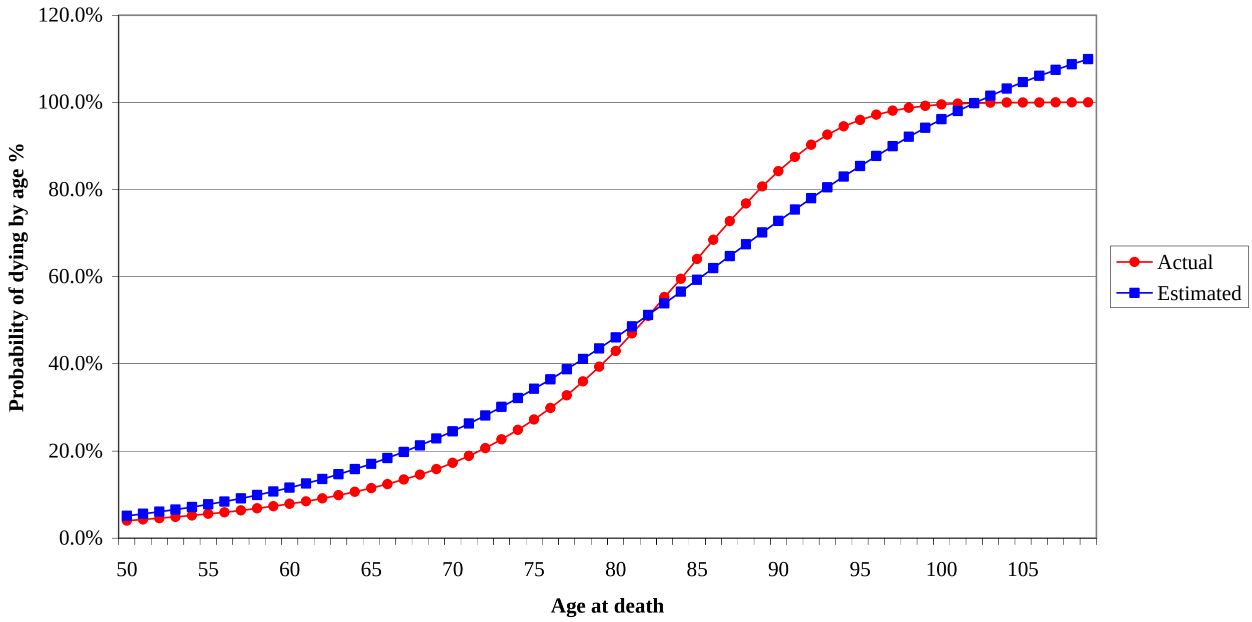


Figure 46

Japanese Females: Cumulative q_x as against Logistic Distribution for Year 1990

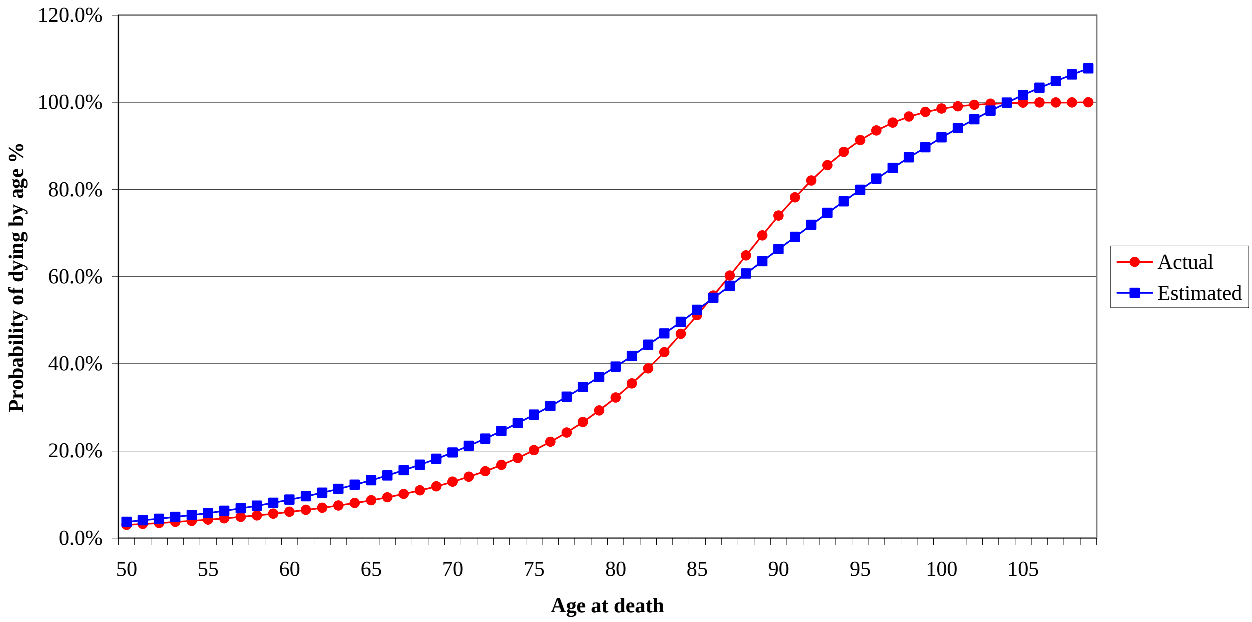


Figure 47
Japanese Females: Cumulative q_x as against Logistic Distribution for Year 1998

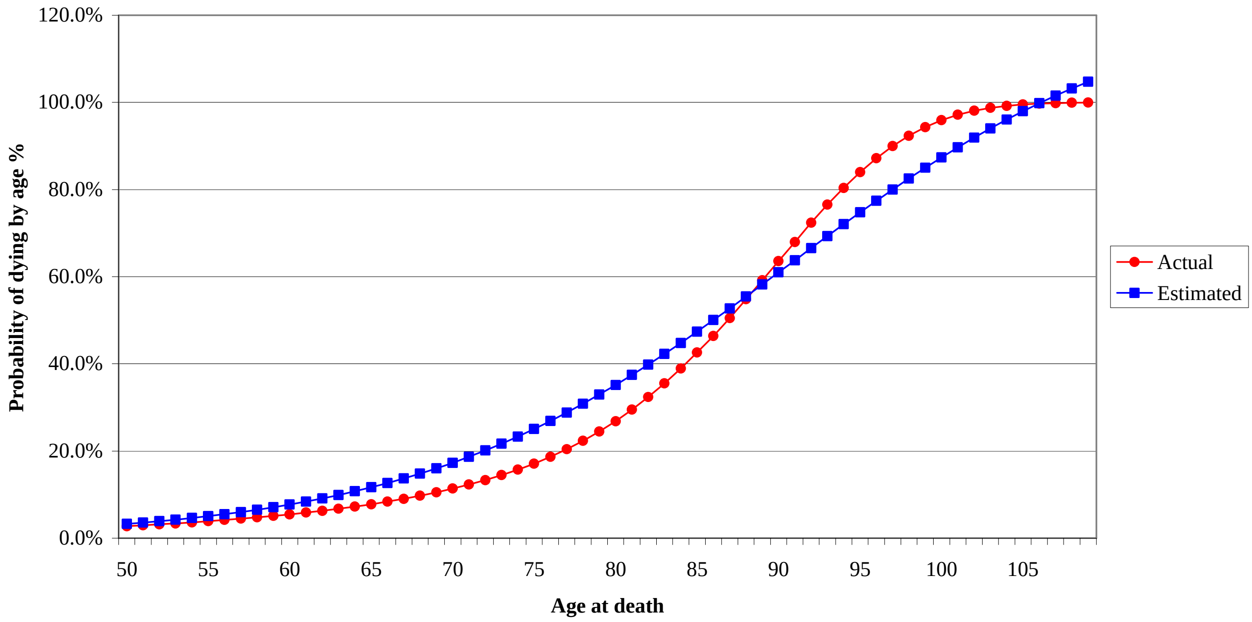


Table 21 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 21
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (Japanese Females)

	1960	1970	1980	1990	1998
Parameter α	311	984	2993	5757	6524
Parameter β	0.928	0.918	0.909	0.904	0.905
R^2 value	0.980	0.982	0.985	0.989	0.993

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter α = 454.44 ◎ exp(0.0825 ◎ (Calendar Year – 1959)); R^2 value: 0.957,
Parameter β = −0.00064 ◎ (Calendar Year – 1959) + 0.92423; R^2 value: 0.925.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we

calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 22.

TABLE 22
Annuities Payable in Arrears on Different Mortality Bases
(Japanese Females)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) This basis	15.88	14.26	12.50	10.66	8.81	7.03	5.31	3.58	1.66
(2) Population mortality	14.86	13.25	11.44	9.44	7.40	5.49	3.86	2.64	1.75
((1) / (2)) − 1	6.8%	7.7%	9.3%	12.9%	19.2%	27.9%	37.3%	35.7%	−5.3%

Figures 48–52 provide a demonstration of the appropriateness of the fit for males for years 1960, 1970, 1980, 1990, and 1998, respectively. These figures are based on far fewer lives at high ages.

Figure 48
Japanese Males: Cumulative q_x as against Logistic Distribution for Year 1960

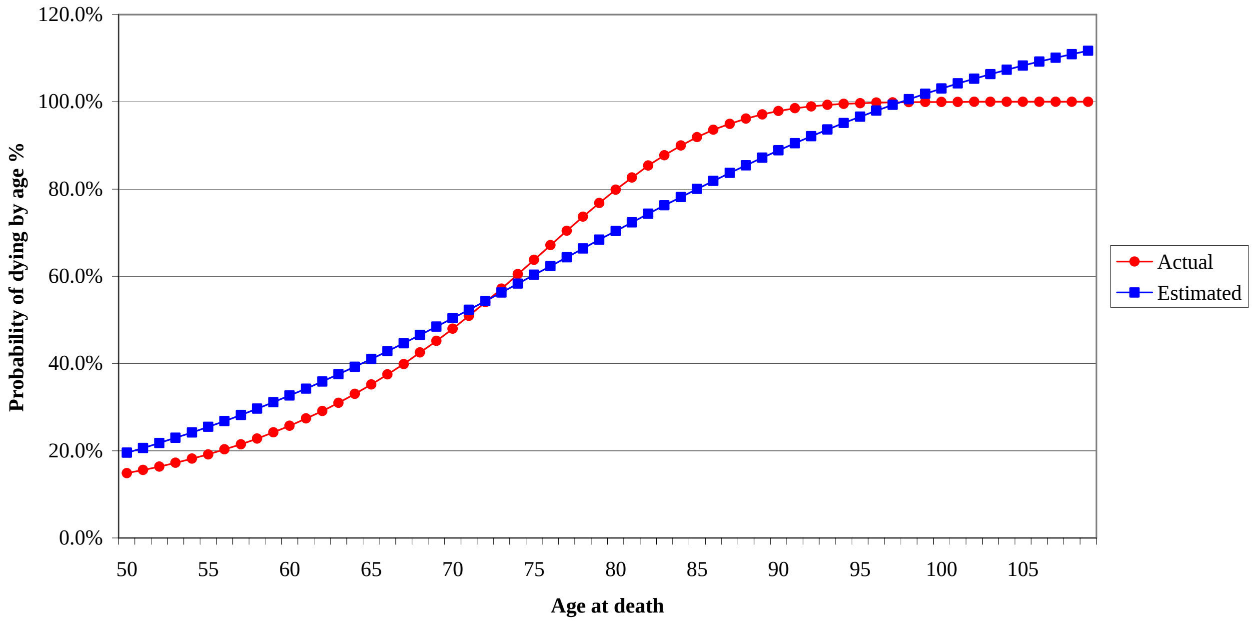


Figure 49
Japanese Males: Cumulative q_x as against Logistic Distribution for Year 1970

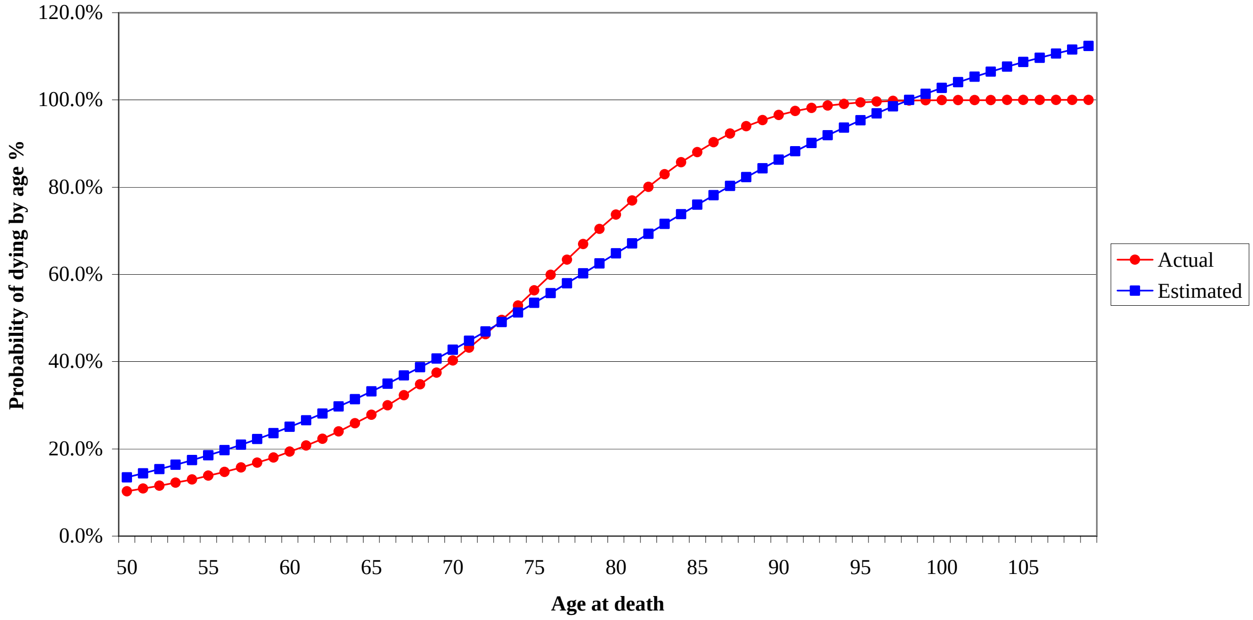


Figure 50
Japanese Males: Cumulative q_x as against Logistic Distribution for Year 1980

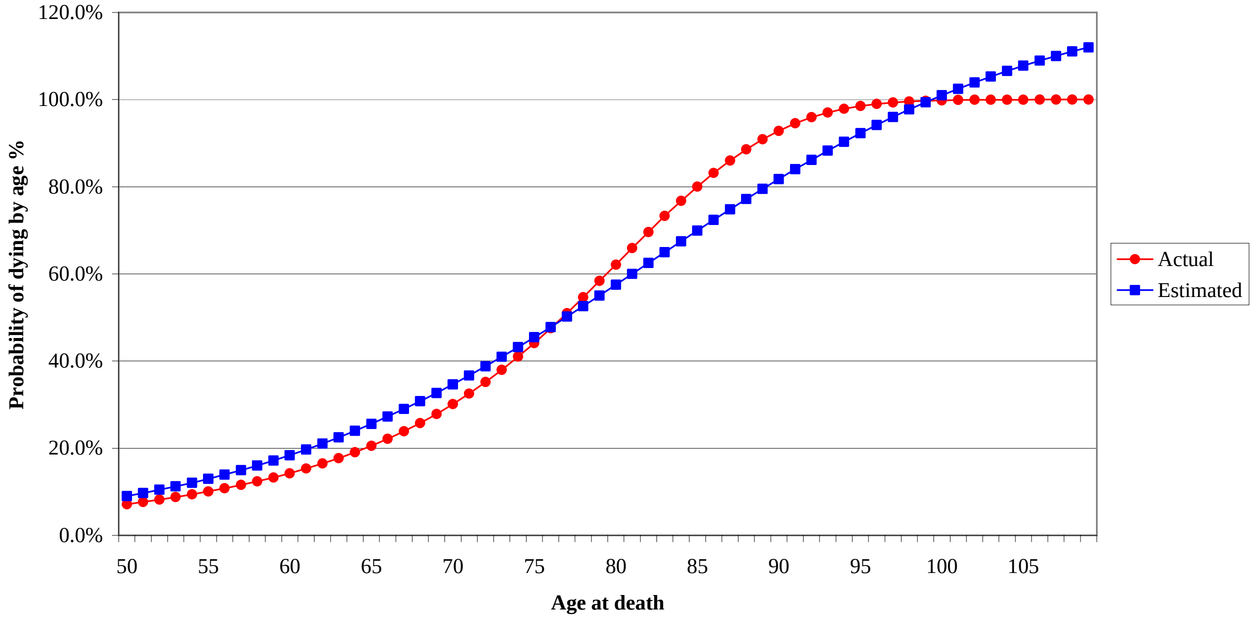


Figure 51

Japanese Males: Cumulative q_x as against Logistic Distribution for Year 1990

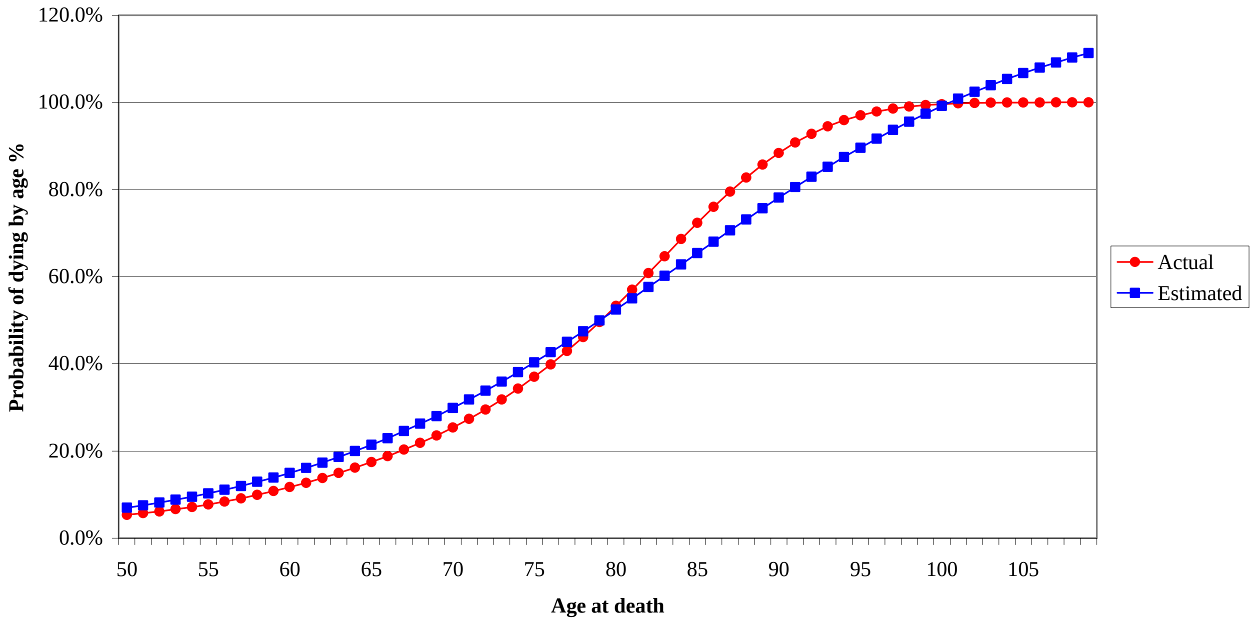


Figure 52

Japanese Males: Cumulative q_x as against Logistic Distribution for Year 1998

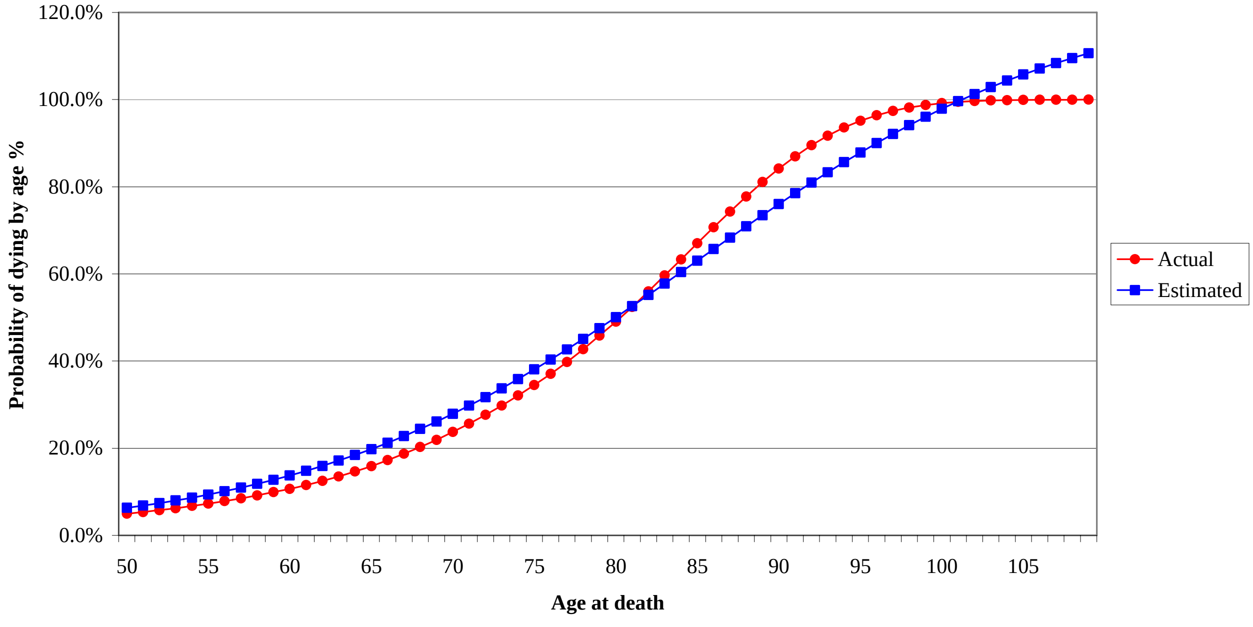


Table 23 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the logistic model for each of the selected years.

TABLE 23
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit
(Japanese Males)

	1960	**1970**	**1980**	**1990**	**1998**
Parameter α	115	261	638	1134	1343
Parameter β	0.937	0.929	0.922	0.917	0.916
R^2 value	0.971	0.974	0.981	0.985	0.989

We derived the following time-dependent formulas for each of the parameters required by the logistic model and provide details of the analysis of goodness of fit as follows:

Parameter $\alpha = 144.1 \circledcirc \exp(0.0673 \circledcirc (\text{Calendar Year} - 1959))$; R^2 value: 0.974,
Parameter $\beta = -0.000562 \circledcirc (\text{Calendar Year} - 1959) + 0.93442$; R^2 value: 0.966.

From this we projected q_x at each age and future calendar year-end and hence derived the capital value of an annuity as at December 31, 2004, payable annually in arrears with an interest rate of 4.25 percent per annum. For comparison we calculated an annuity based on population mortality in 1998 with no allowance for mortality improvements. The results are set out in Table 24.

TABLE 24
Annuities Payable in Arrears on Different Mortality Bases
(Japanese Males)

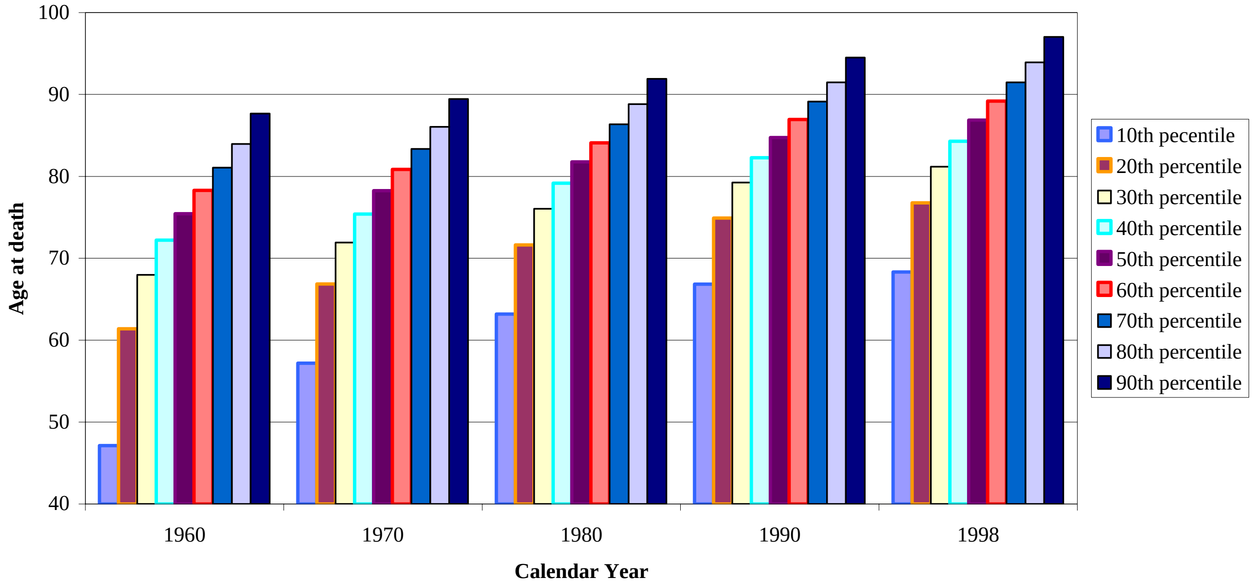
					Age				
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) This basis	14.13	12.52	10.86	9.19	7.56	5.97	4.34	2.55	0.44
(2) Population mortality	12.72	11.04	9.30	7.49	5.73	4.20	2.97	2.08	1.46
((1) / (2)) − 1	11.1%	13.4%	16.8%	22.7%	32.0%	42.0%	45.9%	22.5%	−69.6%

Probability of Death at Specified Age

Trends in Percentiles

We derived the age at death for decennial percentiles within the population for each of the calendar years from 1960 to 1998. We further derived age at death for individual percentiles for the same calendar years between the 91st and 99th percentiles. Figure 53 shows the results for females for years 1960, 1970, 1980, 1990, and 1998.

Figure 53
Age at Death from Life Tables for Japanese Females by Percentile for Selected Calendar Years



We determined a time-dependent relationship between age at death for each of the selected percentiles and calendar year. Table 25 sets out the parameters associated with these relationships and the results of the goodness-of-fit test between the time-dependent relationship and the actual ages at death in each calendar year.

TABLE 25
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (Japanese Females)

	50th Percentile	90th Percentile	95th Percentile	99th Percentile
Intercept	75.1	86.9	88.7	90.2
Gradient	0.301	0.242	0.234	0.227
R^2 value	0.996	0.990		

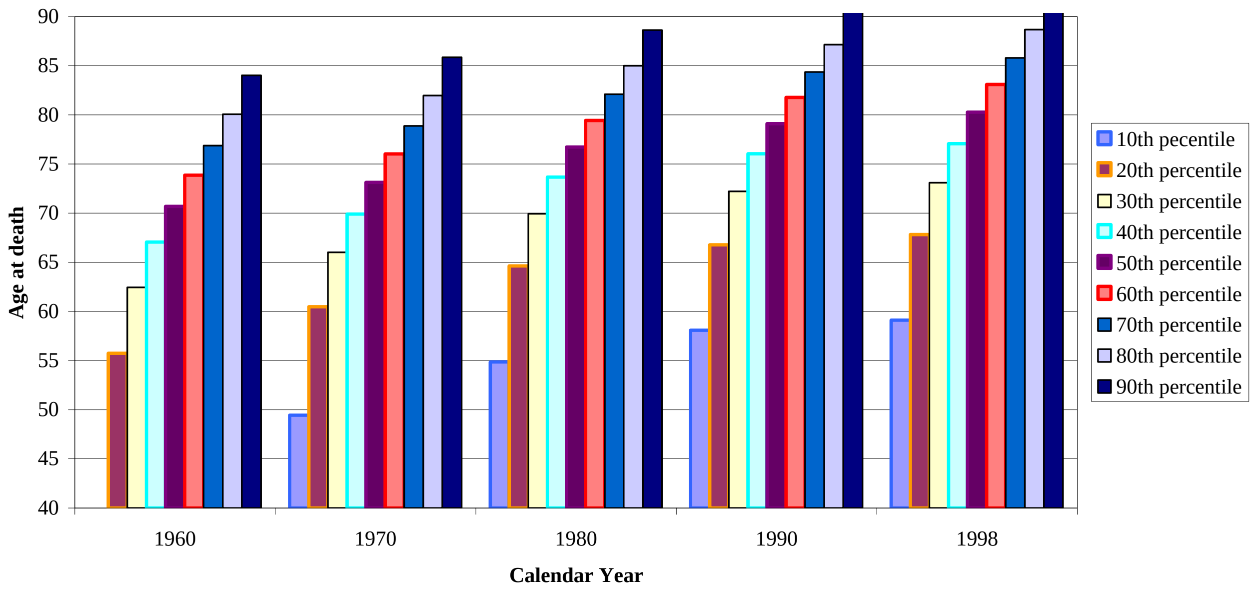
From this we projected the age at death for each of the percentiles set out above for selected future calendar years. The results are set out in Table 26.

TABLE 26
Age at Death for Specified Percentile in Selected Calendar Years (Japanese Females)

Population Percentile	Calendar Year								
	1998	**1999**	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**
50th percentile	89.8	90.1	90.4	91.9	93.5	94.9	96.5	98.0	99.5
90th percentile	98.8	99.0	99.3	100.5	101.7	102.9	104.1	105.3	106.5
95th percentile	100.2	100.4	100.7	101.8	103.0	104.2	105.3	106.5	107.7
99th percentile	101.3	101.5	101.8	102.9	104.0	105.2	106.3	107.5	108.6

Figure 54 shows the results for males for years 1960, 1970, 1980, 1990, and 1998.

Figure 54
Age at Death from Life Tables for Japanese Males by Percentile for Selected Calendar Years



We determined a time-dependent relationship between age at death for each of the selected percentiles and calendar year. Table 27 sets out the parameters associated with these relationships, and the results of the goodness-of-fit test between the time-dependent relationship and the actual ages at death in each calendar year.

TABLE 27
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (Japanese Males)

	50th Percentile	90th Percentile	95th Percentile	99th Percentile
Intercept	70.6	83.7	85.7	87.2
Gradient	0.264	0.220	0.214	0.210
R^2 value	0.981	0.983		

From this we projected the age at death for each of the percentiles set out above for selected future calendar years. The results are set out in Table 28.

TABLE 28
Age at Death for Specified Percentile in Selected Calendar Years (Japanese Males)

Population Percentile	Calendar Year								
	1998	**1999**	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**
50th percentile	83.5	83.8	84.0	85.3	86.7	88.0	89.3	90.6	91.9
90th percentile	94.5	94.7	94.9	96.0	97.1	98.2	99.3	100.4	101.5
95th percentile	96.2	96.4	96.6	97.7	98.7	99.8	100.9	102.0	103.3
99th percentile	97.5	97.7	97.9	99.0	100.0	101.1	102.1	103.2	104.2

Weibull Distribution

We also fitted a Weibull distribution to the probability of death at a given age with the formula

Probability of death at a given age = (α/β^{α})◎◎ $(x^{\alpha-1})$◎◎◎exp$(-(x/\beta)^{\alpha})$.

We fitted curves of this type to the actual cumulative probability of death for each of the calendar years from 1960 to 1998. Figures 55–59 provide a demonstration of the appropriateness of the fit for females for years 1960, 1970, 1980, 1990, and 1998, respectively.

Figure 55
Japanese Females: Age at Death as against Weibull Distribution for Year 1960

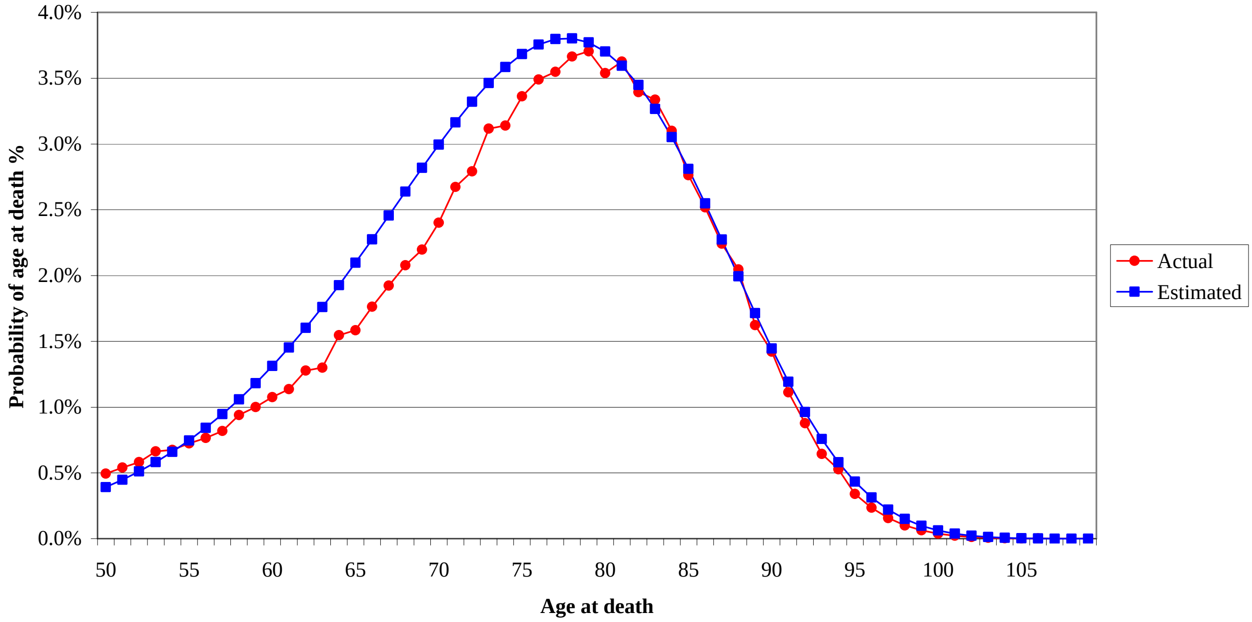


Figure 56
Japanese Females: Age at Death as against Weibull Distribution for Year 1970

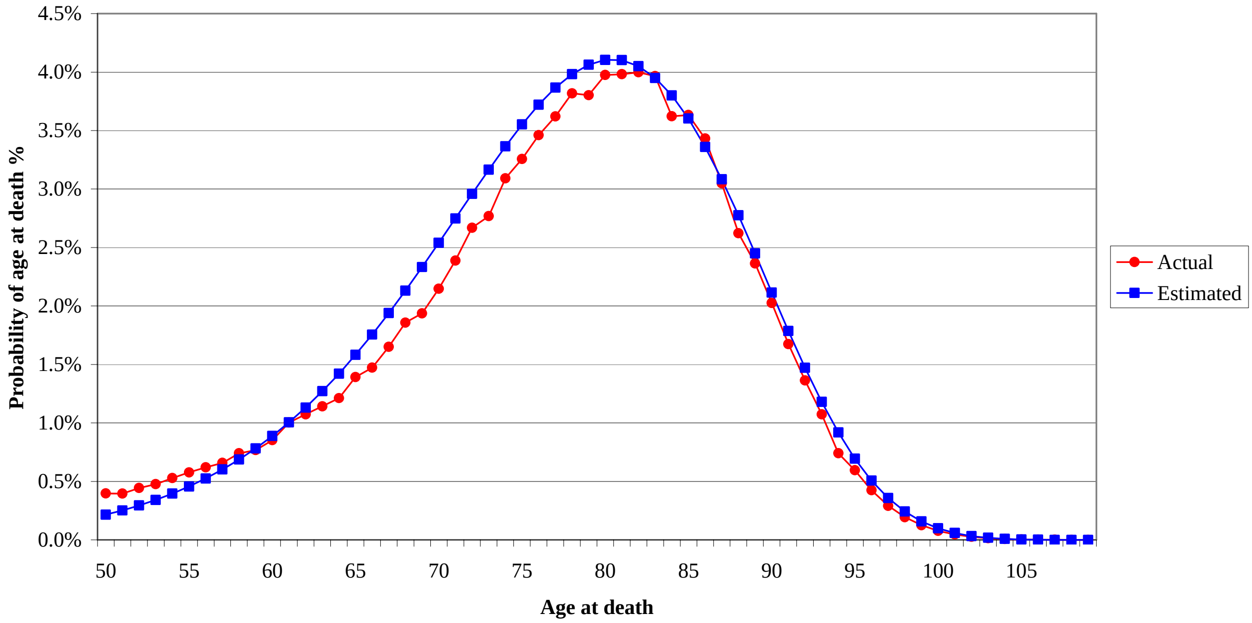


Figure 57
Japanese Females: Age at Death as against Weibull Distribution for Year 1980

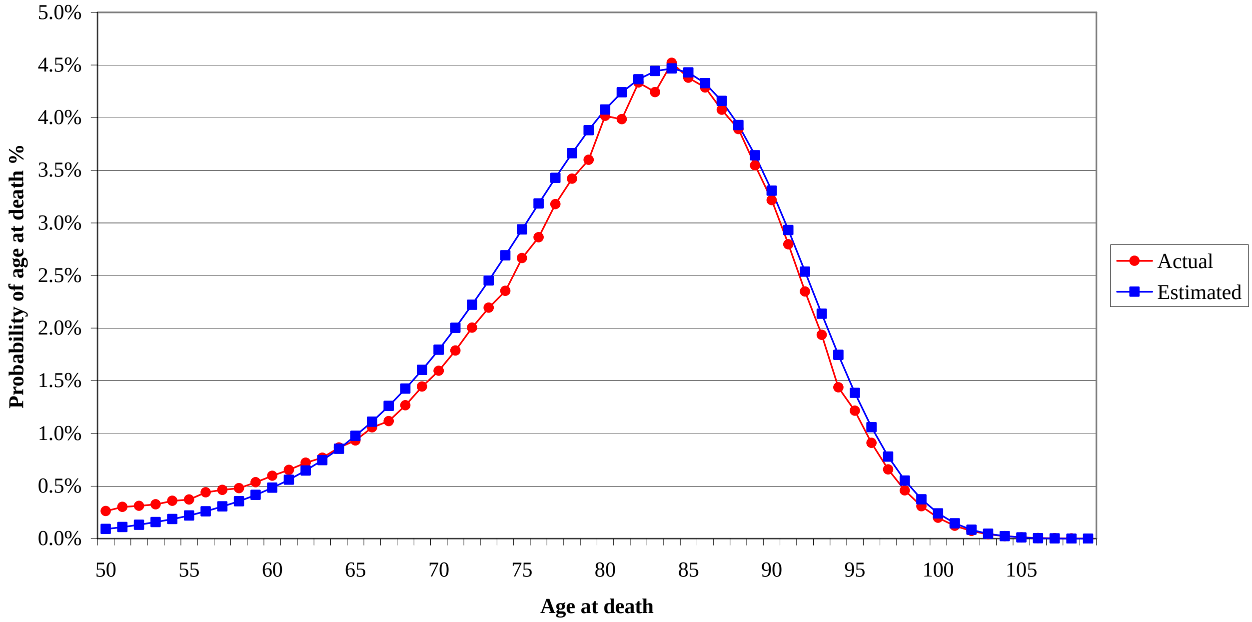


Figure 58
Japanese Females: Age at Death as against Weibull Distribution for Year 1980

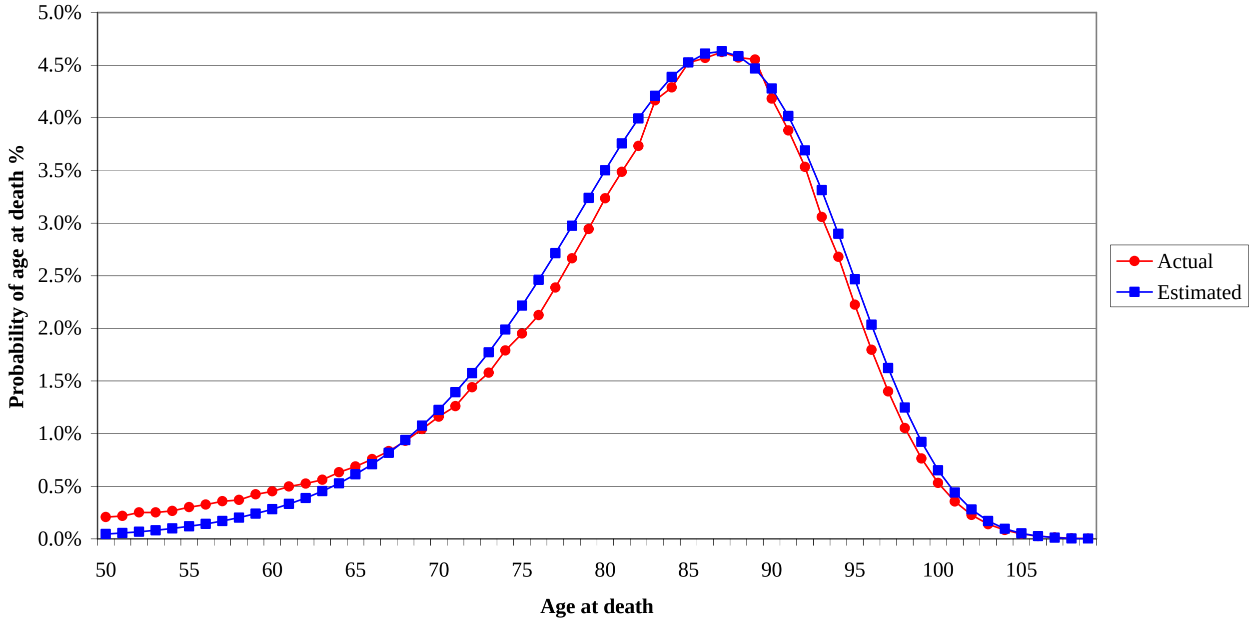


Figure 59
Japanese Females: Age at Death as against Weibull Distribution for Year 1998

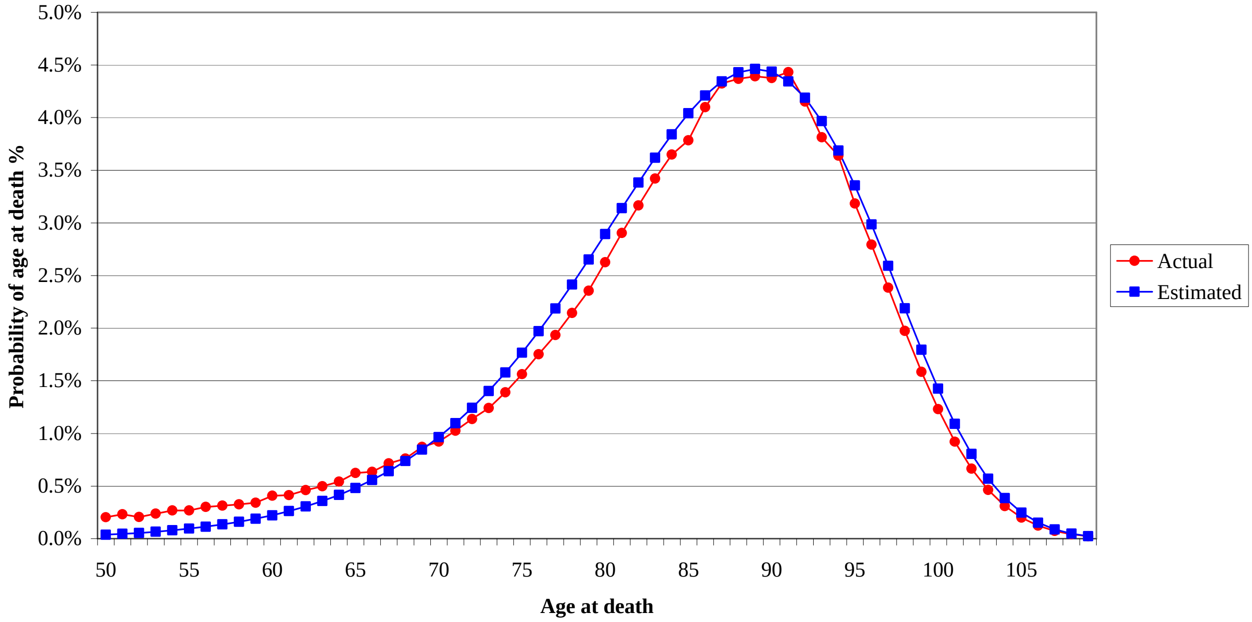


Table 29 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the Weibull distribution for each of the selected years.

TABLE 29
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (Japanese Females)

	1960	**1970**	**1980**	**1990**	**1998**
Parameter α	8.10	9.04	10.24	10.99	10.85
Parameter β	78.95	81.48	84.74	87.61	89.86
R^2 value	0.990	0.996	0.997	0.996	0.996
Skewness	0.330	0.464	0.597	0.655	0.604

We derived the following time-dependent formulas for each of the parameters required by the Weibull distribution and provide details of the analysis of goodness of fit as follows:

Parameter α = 0.0787 ⊚ (Calendar Year – 1959) + 8.3996; R^2 value: 0.957,
Parameter β = 0.2867 ⊚ (Calendar Year – 1959) + 79.054; R^2 value: 0.996.

From this we projected the age at death for selected percentiles for selected future calendar years. The results are set out in Table 30.

TABLE 30
Age at Death for Specified Percentile in Selected Calendar Years
(Japanese Females)

Population Percentile	Calendar Year								
	1998	**1999**	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**
50th percentile		88.2	88.5	90.0	91.4	92.9	94.3	95.7	97.1
90th percentile		97.8	98.1	99.4	100.7	102.0	103.3	104.5	105.6
95th percentile		100.0	100.3	101.6	102.8	104.1	105.3	106.4	107.5
99th percentile		103.8	104.0	105.2	106.4	107.5	108.5	109.3	109.9

Figures 60–64 provide a demonstration of the appropriateness of the fit for males for years 1960, 1970, 1980, 1990, and 1998, respectively. These figures are based on much fewer lives at high ages.

Figure 60
Japanese Males: Age at Death as against Weibull Distribution for Year 1960

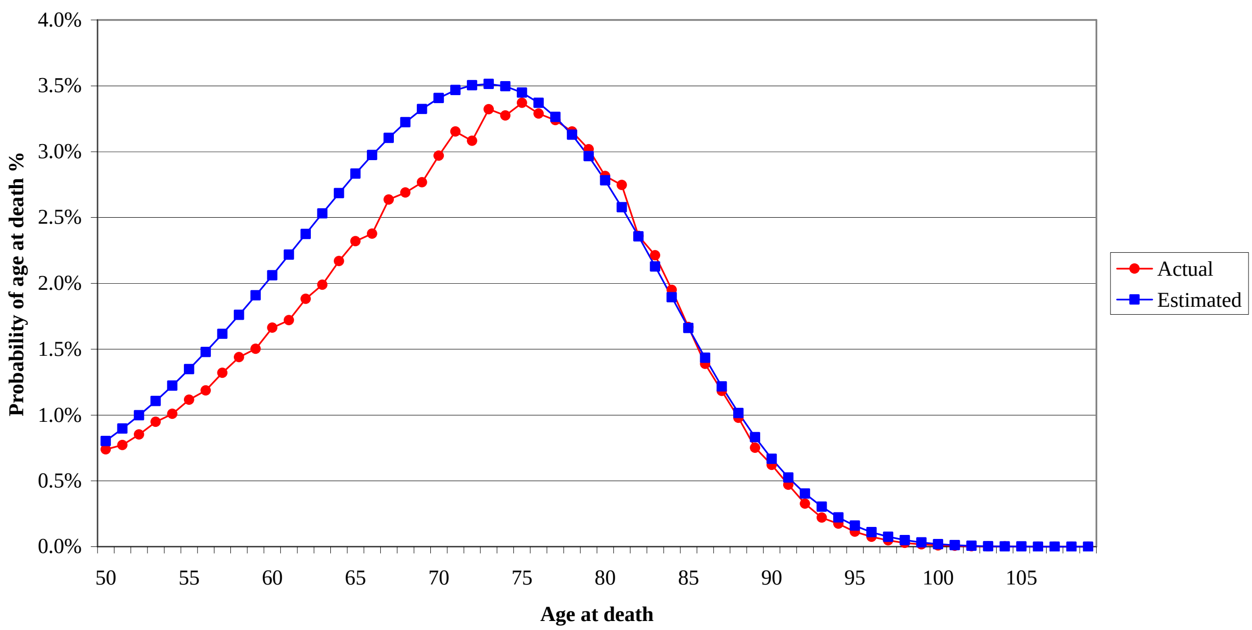


Figure 61
Japanese Males: Age at Death as against Weibull Distribution for Year 1970

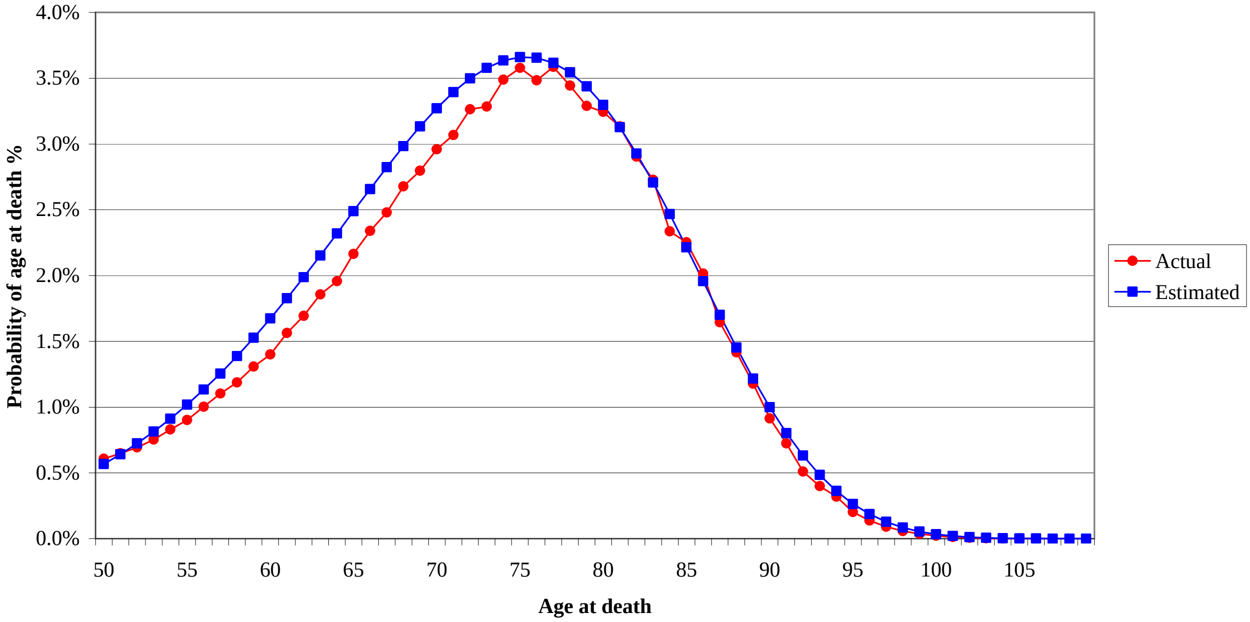


Figure 62
Japanese Males: Age at Death as against Weibull Distribution for Year 1980

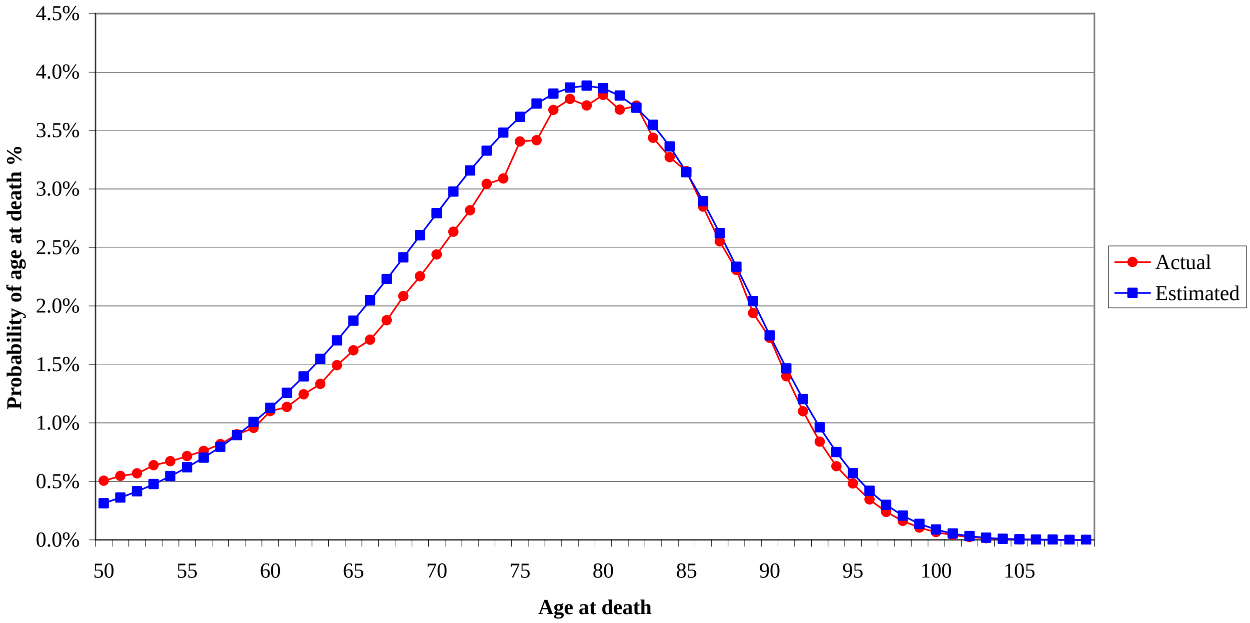


Figure 63
Japanese Males: Age at Death as against Weibull Distribution for Year 1990

Figure 64
Japanese Males: Age at Death as against Weibull Distribution for Year 1998

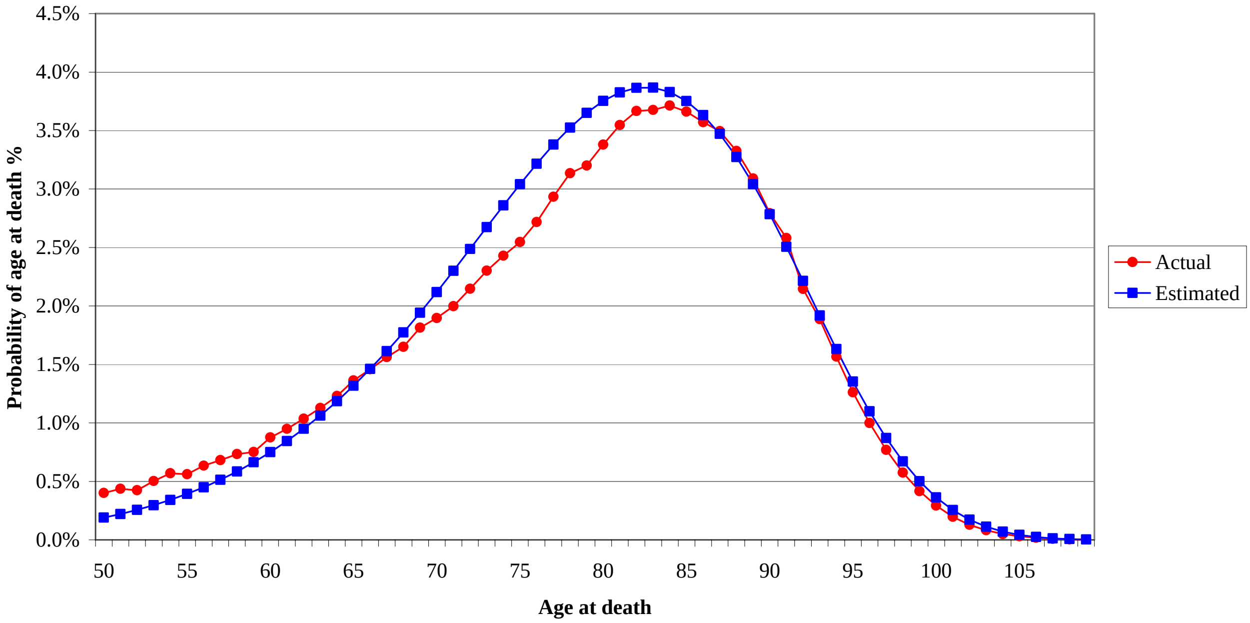


Table 31 provides results of the goodness-of-fit analyses, together with the values of the parameters that were required by the Weibull distribution for each of the selected years.

TABLE 31
Values of Parameters for Best-Fit Models and Measure of Goodness of Fit (Japanese Males)

	1960	1970	1980	1990	1998
Parameter α	7.04	7.57	8.40	8.84	8.75
Parameter β	74.46	76.76	80.14	82.46	83.72
R^2 value	0.990	0.997	0.996	0.993	0.993
Skewness	0.175	0.253	0.370	0.417	0.386

We derived the following time-dependent formulas for each of the parameters required by the Weibull distribution and provide details of the analysis of goodness of fit as follows:

Parameter α = 0.0491 ⊚ (Calendar Year – 1959) + 7.2353; R^2 value: 0.938,
Parameter β = 0.2534 ⊚ (Calendar Year – 1959) + 74.802; R^2 value: 0.981.

From this we projected the age at death for selected percentiles for selected future calendar years. The results are set out in Table 32.

TABLE 32
Age at Death for Specified Percentile in Selected Calendar Years (Japanese Males)

Population Percentile	Calendar Year								
	1998	1999	2000	2005	2010	2015	2020	2025	2030
50th percentile		82.1	82.4	83.7	85.0	86.3	87.6	88.9	90.1
90th percentile		93.5	93.7	94.9	96.1	97.2	98.4	99.6	100.7
95th percentile		96.2	96.4	97.6	98.7	99.8	101.0	102.1	103.2
99th percentile		100.8	101.0	102.0	103.1	104.2	105.2	106.3	107.2

Alternate Results: United States and Japan

In the previous sections we have presented two models of age at death and two models of mortality rates. Further analysis of the models of mortality rates is useful, as the structure of the models is conducive to considering alternative assumptions as to rates of mortality improvement. The logistic model of mortality rates by individual year provided a closer fit than that of the logistic model of cumulative mortality rates, and we therefore consider some alternative assumptions in respect of that model.

We have calculated annuities as previously on two scenarios:

Age-specific mortality improvements after 1998 are assumed to be 50 percent of those associated with the principal results ("Scenario 1")
Age-specific mortality improvements after 1998 are assumed to be 200 percent of those associated with the principal results ("Scenario 2").

These scenarios are intended purely as an illustration of the impact of such assumptions, rather than representing a probable or likely pattern of future mortality improvements that might be associated with particular events or changes in behavior. The results are set out in Tables 33–36.

TABLE 33
Annuities Payable in Arrears on Different Mortality Improvement Assumptions (U.S. Females)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) Scenario 1	13.95	12.30	10.54	8.75	7.01	5.38	3.96	2.78	1.86
(2) Scenario 2	15.05	13.32	11.46	9.54	7.64	5.86	4.30	3.01	2.01
(3) Principal	14.31	12.63	10.84	9.01	7.21	5.54	4.07	2.86	1.91
((1) / (3)) − 1	−2.5%	−2.7%	−2.8%	−2.8%	−2.8%	−2.8%	−2.7%	−2.6%	−2.5%
((2) / (3)) − 1	5.1%	5.5%	5.7%	5.9%	5.9%	5.8%	5.7%	5.5%	5.2%

TABLE 34
Annuities Payable in Arrears on Different Mortality Improvement Assumptions (U.S. Males)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) Scenario 1	12.34	10.65	8.94	7.26	5.70	4.31	3.13	2.19	1.46
(2) Scenario 2	13.45	11.62	9.73	7.87	6.14	4.60	3.31	2.28	1.51
(3) Principal	12.71	10.97	9.20	7.46	5.84	4.40	3.19	2.22	1.48
((1) / (3)) − 1	−2.9%	−2.9%	−2.8%	−2.7%	−2.5%	−2.2%	−1.8%	−1.5%	−1.0%
((2) / (3)) − 1	5.8%	5.9%	5.8%	5.5%	5.0%	4.4%	3.7%	2.9%	2.1%

TABLE 35
Annuities Payable in Arrears on Different Mortality Improvement Assumptions (Japanese Females)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) Scenario 1	15.53	13.85	11.98	9.98	7.95	6.00	4.28	2.85	1.78
(2) Scenario 2	17.52	15.85	13.87	11.66	9.33	7.04	4.99	3.30	2.03
(3) Principal	16.23	14.53	12.61	10.53	8.39	6.34	4.50	3.00	1.86
((1) / (3)) − 1	−4.3%	−4.7%	−5.0%	−5.2%	−5.3%	−5.2%	−5.0%	−4.7%	−4.4%
((2) / (3)) − 1	7.9%	9.0%	10.0%	10.8%	11.2%	11.2%	10.8%	10.0%	9.2%

TABLE 36
Annuities Payable in Arrears on Different Mortality Improvement Assumptions (Japanese Males)

	Age								
	60	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
(1) Scenario 1	13.47	11.73	9.91	8.07	6.32	4.74	3.39	2.31	1.50
(2) Scenario 2	15.22	13.3	11.33	9.25	7.23	5.40	3.84	2.59	1.66
(3) Principal	14.05	12.26	10.37	8.45	6.62	4.95	3.54	2.40	1.55
((1) / (3)) − 1	−4.2%	−4.3%	−4.5%	−4.5%	−4.4%	−4.2%	−4.0%	−3.7%	−3.5%
((2) / (3)) − 1	8.3%	8.9%	9.3%	9.5%	9.4%	9.0%	8.5%	7.9%	7.2%

Discussion

Implied Improvements in Life Expectancy

All four models that we have described and used in this paper are based on the assumption that there is a time dependency to the values of the parameters underlying those models. The results given in the previous sections would suggest that this assumption has some validity over the period of the investigation 1960–1998. However, it is not necessarily probable that such historic relationships will be repeated in the future. Indeed, such historic relationships are likely to be the result of interaction between different factors whose relative contribution may have changed over the period of the investigation: for example, changes in the prevalence of smoking or the introduction of new treatments for cardiovascular disease.

With this proviso stated, all four models project continuing reductions in mortality rates at all ages and for both sexes. We would note, however, that unless there were conflicting elements within the model, it would not be possible for an extrapolative model to produce a trend in future mortality rates that was fundamentally different from that over the investigation period.

Mortality improvements for Japan are projected to be greater than for the United States, reflecting the higher pattern of mortality improvements over the period of investigation. A reasonable argument might be that developed nations would expect to see converging life expectancy, reflecting an increased difficulty for new medical advances to extend life expectancy, a greater availability of previous advances to both the population as a whole and to other countries, and increasing convergence in behavior between different populations. As a consequence, since Japan has the highest life expectancy among developed nations, future mortality improvements for Japan might be expected to be less than for other developed nations. However, as was noted by Oeppen, Vaupel. and others, Japanese females in particular have had both the highest life expectancy and the highest mortality improvements among developed countries in recent years.

Implications for Population Age Structure

The two models of age at death clearly illustrate the potential aging of the population. The model of trends in percentiles projects that life expectancy at the 99th percentile according to a life table in year 2005 will be age 101 for U.S. females and age 97 for U.S. males, whereas the equivalent figures are ages 103 and 99 for Japanese females and males, respectively.

However, the population age structure is influenced by not only changes in mortality rates at different ages, but also the birth rate and net effect of immigration. It would therefore be expected that changes in mortality rates will have a lesser impact on the United States' population age structure than that of Japan, as the United States has a higher birth rate and higher rates of immigration.

Commentary on Expert Option as to the Existence of Limits to Improvements in Life Expectancy

Vaupel and Oeppen in their paper "Broken Limits to Life Expectancy" (2002) noted that improvements in life expectancy had been greater in the second half of the twentieth century than had been expected by the World Health Organisation and government actuarial departments. Earlier assumptions as to an upper limit to life expectancy had now been exceeded in some developed countries. Instead they observed that if you consider the country with the highest life expectancy in any given calendar year and these highest life expectancies are plotted over a period of 160 years, a linear relationship is evident such that highest life expectancy increases by 0.25 years with each calendar year.

There is no clear reason why this relationship should exist, and changes in the highest life expectancy would be expected to be dependent on the interplay between different countries, medical advances, changes in behavior, and variations in the hazards faced. Further, the fact that this relationship has existed for such a long period of time is neither a guarantee nor an indication that this relationship will continue into the future. However, the continuation of this relationship is a scenario that, in our view, should be considered.

Further examination of trends in age at death for different percentiles in the population has led Christensen and Vaupel (1996) to observe that there does not appear to be any evidence of convergence between the different percentiles that might be evidence of the presence of an imminent limit to life expectancy.

The concept of limits to life expectancy was first suggested by August Weissman in the nineteenth century in terms of an absolute limit that would free a population from the burden of supporting infirm, elderly members. This concept has been widely discredited in part through observation of species in captivity. However, such limits do exist at the cellular level, as observed by Leonard Hayflick as to the number of times that fibroblasts involved in wound healing are able to divide.

Each division of the genetic material is marked by a reduction in the length of a location at the end of each chromosome called the telomere. It is thought that this change in the telomere length affects gene expression and ultimately leads to cell senescence, when further divisions are impossible. The intriguing observation is that further divisions are linked to increasing loss of cellular function.

The phenomenon of aging at the level of the organism is well known, but the mechanisms are much less clear. Various evolutionary theories of aging have been put forward that highlight the importance of ensuring survival to sexual maturity, with either mutations in later life not being selected against or genes having beneficial and deleterious effects at different points in the lifespan.

An alternative concept as propounded by Hayflick and others is that genes play no direct part in the process of aging. Instead, in order to ensure survival to sexual maturity, natural selection favors genes that provide a functional overcapacity, and aging represents the result of random damage within and outside cells that gradually reduces this capacity.

This leads easily to the concept put forward by Olshanky, Carnes, and Cassel (1990) of a "biological warranty" or a practical upper limit to life expectancy. This is, in effect, the result of the combination of a given level of functional overcapacity and a given rate of aging. There is debate as to the extent that diseases such as atherosclerosis and Alzheimer's are part of or independent of the aging process. However, proponents of the "biological warranty" concept note that the practical upper limit may be close and that extrapolation of past trends in life expectancy is not supported by the underlying biological processes.

However, all demographers and biogerontologists are in agreement that the absence of current treatments for the effects of aging does not mean that such treatments could not be developed in the future. Such treatments might include the results of stem cell research or rejuvenation of cell function as suggested by initial in vitro experiments with the enzyme telomerase. The effect of such treatments would depend on the penetration in the population and possible latent adverse side effects.

It is also possible that, regardless of such treatments, changes in behavior will have a significant and possible negative impact on future life expectancy. As the significant reductions in the prevalence of smoking in the second half of the twentieth century have had a significant effect on lung cancer and cardiovascular disease, changes in diet and reduced levels of exercise are leading to large increases in the prevalence of obesity and type 2 diabetes.

Conclusions

We would note the following points from our investigations as presented in this paper:

The Weibull distribution provides a valuable model of age at death

The logistic function provides a valuable model of mortality rates for ages 50 and over

There appears to be a clear time dependency to the values of parameters that we have used in respect of the Weibull distribution for age at death and in respect of the logistic functions for both mortality rates and for cumulative probability of death

All four models project that mortality rates will continue to reduce, and therefore it would be unreasonable not to allow for the possibility of further mortality improvements in considering annuitant mortality

Life expectancy for Japan is currently greater than that for the United States, and particularly so for females. None of the projections from the models indicate that these differences will decline.

References

Beard, R. 1971. Some aspects of theories of mortality, causes of death analysis, forecasting and stochastic processes. Biological Aspects of Demography (ed. W. Brass). Taylor and Francis, London.

Carnes, B., S. Olshansky, and D. Grahn. 2003. "Biological Evidence for Limits to the Duration of Life." *Biogerontolgy* 4: 31–45.

Christensen, K., and J. Vaupel. 1996. "Determinants of Longevity: Genetic, Environmental and Medical Factors." *Journal of Internal Medicine* 240: 333–341.

Gavrilov, L., and N. Gavrilova. 2002. "Evolutionary Theories of Aging and Longevity." *The Scientific World Journal* 2: 339–356.

Gompertz, B. 1825. On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies. *Philos. Trans. Royal Society London.* 115: 513-585.

Hayflick, L. 1965. The Limited in Vitro Lifetime of Human Diploid Cell Strains. *Experimental. Cellular. Research.* 37: 614-636.

Hossack, I. B., J. H. Pollard, and B. Zehnwirth. 1999. *Introductory Statistics with Applications in General Insurance*. 2nd ed. Cambridge: Cambridge University Press.

International Longevity Center USA 2000. *Biomarkers of Aging: From Primitive Organisms to Man.* City: International Longevity Center-USA, Tucson, Arizona.

Kannisto, V. 1994. Development of Oldest-Old mortality: 1950-1990. Odense University Press, Odense, Denmark.

Klugman, S. A., H. H. Panjer, and G. E. Willmot. 1998. *Loss Models: From Data to Decision.* New York: John Wiley & Sons.

Miller, I., and M. Miller. 1999. *John E. Freund's Mathematical Statistics.* 6th ed. Englewood Cliffs, N.J.: Prentice Hall.

Oeppen, J., and J. Vaupel. 2000. "Enhanced: Broken Limits in Life Expectancy." *Science* 296: 1029–1031.

Olshansky, S., B. Carnes, and C. Cassel. 1990. "In Search of Methuselah: Estimating the Upper Limits to Human Longevity." *Science* 250: 634–640.

Olshansky, S., L. Hayflick, and B. Carnes. 2002. "Position Statement on Human Aging." *Journal of Gerontology A: Biological Sciences and Medical Sciences* 57: B292–297.

Perks, W. 1932. On some experiments in the graduation of mortality statistics. *Journal of Institute of Actuaries* 63: 12-57.