# Mortality at Advanced Ages in The United Kingdom

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**Abstract**

In the United Kingdom, the Government Actuary's Department (GAD) produces official life tables and population projections for the population as a whole and for each constituent country of the United Kingdom. The Continuous Mortality Investigation Bureau (CMIB) of the Faculty and Institute of Actuaries collects data on the mortality experience of U.K. life offices and produces mortality tables for various classes of insurance products based on these data, which include projections of future mortality.

As in other developed countries, mortality rates at advanced ages have fallen quite dramatically over the twentieth century in the United Kingdom. The paper gives an overview of the data available to GAD and CMIB and discusses the problems encountered in estimating mortality rates at old ages in the United Kingdom for both the general population and those taking out insurance. It describes the current and past methodologies used to construct mortality rates at advanced ages for official population life tables and the CMIB mortality tables of insured lives. Possible methods for projecting mortality rates at advanced ages are also discussed.

The paper includes an update of some of the results for the U.K. population presented at the 2002 Society of Actuaries conference following the results of the 2001 U.K. census and discusses the work involved in preparing the latest CMIB mortality tables, which are planned to be published during 2005.

## 1. Introduction

Although data on numbers of deaths by sex and age at death are available for the United Kingdom, data on the population numbers by year of age for ages 90 and over from which mortality rates at those ages can be calculated are not published (other than for censuses, and even here there are issues of data quality; see Thatcher 1994).

Estimates of mortality rates at these old ages are required for a variety of purposes, including the compilation of population life tables and for analyzing past trends in mortality to inform the assumptions for projecting mortality at these ages for the U.K. national population projections. These projections are prepared by the Government Actuary's Department (GAD) at the request of the Registrars General of England and Wales, Scotland and Northern Ireland. GAD also produces official life tables for England and Wales (English life tables) and Scotland (Scottish life tables) based on population and deaths data for a three-year period around each census date. Ungraduated life tables known as interim life tables are also produced annually for the United Kingdom and each constituent country; these are calculated using data for consecutive three-year periods.

Until relatively recently, the derivation and projection of mortality rates for the very old were not analyzed in great detail because the numbers involved were very small. However, these age groups are continuing to grow, and there is substantial interest within government and elsewhere in their projected numbers—particularly with regard to the impact on pensions and the provision of long-term care.

One complexity when considering data at the U.K. level is that responsibility for producing demographic statistics such as the population estimates and numbers of deaths for each country of the United Kingdom falls to the respective Registrars General of England and Wales, Scotland and Northern Ireland. Statistics relating to the United Kingdom as a whole are then obtained by aggregation. This has led to some difficulties in analyzing data where the individual countries collect or publish data in different ways or where historical data are available for differing periods within each country.

The Continuous Mortality Investigation Bureau (CMIB) of the Faculty and Institute of Actuaries collects deaths and in-force data by age and gender from participating U.K. life offices for various population subgroups who have taken out insurance contracts, including annuities, and publishes graduated mortality rates derived from these data, together with projections of future mortality rates by year and age.

This paper updates work presented at the 2002 Society of Actuaries symposium on mortality at the highest ages and discusses work being carried out by the CMIB. Section 2 gives an overview of the available population data and discusses some of the problems encountered in estimating mortality rates at old ages in the United Kingdom. Section 3 describes some of the methodologies used to construct mortality rates at advanced ages for official life tables. Section 4 discusses some of the methodologies proposed for estimating mortality rates at ages 85 and over. Section 5 discusses the results of an investigation into suitable methods for estimating mortality rates at old ages in the United Kingdom, carried out as part of a review carried out in 2000–2001 of the methodology for projecting mortality rates in the U.K. national population projections. Sections 6 and 7 describe recent work undertaken by GAD to construct a database of historical mortality rates for the United Kingdom and constituent countries going back to 1961 and the mortality rates derived from this database. Possible methods for projecting mortality rates at advanced ages are discussed in Section 8. Section 9 discusses the current direction of the CMIB's investigations into mortality and, in particular, projection of mortality at older ages. Section 10 discusses some of the findings of the paper. Section 11 provides a short conclusion.

## 2. Data Sources

The principal available sources of data to estimate age-specific population mortality rates in the United Kingdom and its constituent countries are the midyear population estimates and the numbers of deaths, by age and sex, provided by the Registrars General for England and Wales, Scotland and Northern Ireland. Data for the United Kingdom are obtained by aggregation.

### 2.1 Population Data

The decennial population censuses provide the base figures from which official national population estimates for each country of the United Kingdom are derived. Censuses have been carried out every 10 years since 1801, excepting 1941. The latest census was carried out in 2001. The population estimates are obtained by rolling forward the estimates by age and sex produced after a census, using data on subsequent births, deaths and net migration. These rolled-forward estimates are generally subject to increasing error as they move farther away from the last census year and are eventually revised following the next census. It might be expected that the estimates derived for a census year should be the most reliable, since it is a legal requirement in the United Kingdom to complete and return census forms. However, data from recent censuses have had to be adjusted to allow for undercounts of certain population subgroups. The initial results of the 2001 census produced a mid-2001 population estimate for the United Kingdom that was around 1.2 million lower than predicted by rolling forward the mid-1991 estimates, which, in turn, were produced following the results of the 1991 census. Part of this shortfall was attributed to an overestimation of the mid-1991 population estimates and part to underestimates in international migration between the censuses. Also, as a result of evidence from demographic analysis and longitudinal surveys and investigations in local authority areas where the discrepancies were especially great, the initial mid-2001 estimates have been revised upwards by 275,000. However, even after these adjustments a residual discrepancy of around 200,000 remains unexplained. Final revised estimates for 2001 and 2002 were published on September 9, 2004, and revised historical population estimates for 1992–2000 on October 7, 2004.

A further problem affecting the reliability of census data for the very old is age misreporting (Humphrey 1970; Thatcher 1992). Errors were found in the data in the 1971 and 1981 censuses, where verification checks were carried out on a sample of those at old ages. For example, of the 3,727 supposed centenarians in the 1981 census data, only 1,644 could be successfully traced through the National Health Service Central Register, which allows the date of birth to be checked against previous declarations. There were many cases where the age differed by a very round number such as 10 or 20 years. Other processing errors were also found. As a result, population estimates for the United Kingdom and the constituent countries are not published by single year of age and sex for ages over 90 because of their dependence on the census figures at high ages. As far as the authors are aware, no checks were carried out on the centenarian ages in the 1991 or 2001 census, although the number of centenarians produced in the 1991 census appeared high compared to other sources and estimates of the total number of centenarians in 1991 (Thatcher 1994, 1999a).

As well as misreporting of ages recorded at advanced ages in censuses, misreporting of age at date of death can introduce further errors into the population estimates.

Although some administrative data sources (e.g., birth registrations, school rolls and pensions data) could be used to check information about various age groups, there are generally no other sources of population estimates for the United Kingdom or its constituent countries against which comparisons can be made.

Population estimates are usually published showing numbers as at the midpoint of each calendar year, by sex and age last birthday at that time. The available data for each country of the United Kingdom are described in Appendix A.

**2.2 Deaths Data**

Numbers of deaths by age and sex up to the oldest age of death on a calendar-year basis are available from 1910 onwards for England and Wales combined. For Scotland deaths are available by single year of age up to 99, with aggregate totals for ages over 100 for the period 1911–1973 and up to the oldest age of death from 1974 onwards. For Northern Ireland deaths to the oldest age are available from 1968. Before that the numbers are available up to age 99, with aggregate totals for ages 100 and over.

Since registration of death before a body may be buried or cremated has been a legal requirement in each country of the United Kingdom for many years, registration of deaths is believed to be virtually complete. Registration and certification of deaths usually follows a standard procedure throughout the United Kingdom. Except in circumstances that involve referral to a coroner, a doctor must produce a medical certificate. This is taken to the registrar who completes the registration details using information provided by the informant (a person legally allowed to register the death). There are various checks on the entry and validation of the data supplied, although not all the information provided may be internally verifiable other than for certain aspects of its consistency.[1]

All deaths registered in England and Wales or in Scotland, whether of residents or nonresidents of that country, are included in the data for that country. Deaths of nonresidents in Northern Ireland have been included in the data only since 1991. Deaths of people normally resident in the United Kingdom that occur and are registered abroad are excluded from the figures. No estimates are made of these numbers of deaths. For the purposes of calculating mortality rates, it is assumed that the numbers of deaths abroad, by age and gender, of residents of each constituent country of the United Kingdom are equal to the numbers of deaths in those countries of nonresidents.

Because of the legal requirement to register deaths and the various procedures outlined in the previous paragraph, these data are thought to be reliable, even for the very old, by Thatcher and others. However, an exercise carried out by the Northern Ireland Statistics and Research Agency has suggested that there is some incorrect notification of the date of birth, and hence the age at death, in Northern Ireland, particularly for the very old, with an estimated 5 percent of death certificates showing age at death in excess of 90 years old being in error by at least two years (Office for National Statistics2000). There are no other sources of data on deaths for each country as a whole.

**2.3. Other Sources of Data**

Although there are no other sources for making population estimates and deaths by age and sex covering each country of the United Kingdom, the Department for Work and Pensions (DWP) maintains a 5 percent sample (based on National Insurance numbers) of pensioner data, which is regularly updated, with notified deaths being removed and the relevant new pensioners added. The accuracy of these data is discussed further in Section 5.

The DWP also is involved in the identification and verification surrounding the sending of messages of congratulations by the Queen to people reaching their 100th birthday and to those reaching their 105th and each successive birthday. Data have been available in the past on the number of such messages issued each year, but data for recent years have not been analyzed as yet. In practice, lists of potential recipients are compiled every few weeks prior to the relevant birthdays. Verification checks are carried out, and a record is kept of cases verified and whether a message is requested or not. Since messages are not issued to everyone and there may be problems covering those in communal establishments, these data do not provide

[1] A description of some of these checks is given in the Office for National Statistics reference volumes *Mortality Statistics by Cause*, Series DH2. For further information on the registration of deaths in the United Kingdom, see Devis (2000).

accurate numbers of the very elderly. However, they can provide a lower bound on numbers at the oldest ages.

There are organizations other than the national statistical offices of the Registrars General, such as the CMIB, that collect mortality data from subgroups or samples of the population. Such investigations often apply only to subsections of the population that are not, usually, representative samples of the whole population. Thus the resulting mortality rates by age and sex are not necessarily indicative of those for the whole population.

Other sources that could provide useful data to inform analyses of past and future trends are longitudinal surveys, such as the English Longitudinal Study of Ageing. The surveys take a sample of people and follow progress throughout their lives. Such studies allow for an analysis of mortality that links various characteristics of the sample's members, something that is not possible using death registrations, which contain only a small amount of extra information. However, it is difficult to find the characteristics of a relatively small population, which the very elderly comprise, from such sample surveys. Also, most longitudinal surveys in the United Kingdom have not yet been in progress for very long periods.

Further difficulties in analyzing mortality rates for the very elderly in the United Kingdom can arise since a relatively large proportion of the very elderly is in communal establishments rather than households; this can cause problems for data collection for longitudinal studies, and such institutions are often outside the sampling frame of other surveys such as household surveys.

### 3. National Population Life Tables

A series of life tables, based on data for England and Wales combined (but somewhat confusingly called the English life tables),[2] have been produced at fairly regular intervals since 1843. The reports on the life tables have traditionally contained a record of mortality rates at each age, usually calculated from death registration data and population numbers for a three-year period centered on the census year, together with a graduated set of those rates. The latter, together with the resulting life tables, have been used in the past as a standard for measuring changes in mortality rates over time and for various purposes such as the assessment of monetary lump sums awarded as damages in court cases involving fatal accidents and personal injuries.

These national life tables are produced by applying a method of graduating the derived crude death rates over an age range starting with the young and usually ending in the eighties or nineties. Mortality rates at the very youngest ages are

[2] In the nineteenth century Wales was regarded as a region of England.

usually compiled directly from the records of births and deaths at these ages. Often the main graduation method used did not work well at the oldest ages beyond 90, and a separate method was used at these ages, usually some form of extrapolation from the rates graduated over the main body of the tables. A brief description of the methods used for graduating mortality rates and for extending the graduations to the oldest age for English Life Tables No. 7 (1901–1910) onwards are given in Appendix B. Further details are included in the appropriate English Life Table (ELT) publications.

A similar set of Scottish life tables generally have been produced at the same time as English life tables, beginning with the census of 1871. These have usually adopted the same graduation method as was used for the corresponding ELT, except for the 1950–1952 tables when King's method (together with the Gompertz extrapolation for mortality rates at the oldest ages) was used for Scotland rather than fitting a mathematical curve. Life tables for Northern Ireland were published after the census of 1926; these were calculated using King's method but based on using data in 10-year age groups rather than five-year age groups. A second set of tables were produced covering the period 1950–1952. The graduations for these tables were carried out by fitting mathematical curves to the data. For females, a logistic curve was found to suffice; for males, a good fit was found in increasing the resulting rates for women by 10 percent and adding a second term equivalent to a normal curve.

Figures 1 and 2 show the resulting published graduated values of $q_x$ for ages 80–104 for males and for ages 80–106 for females for a selection of English life tables over the twentieth century. As can be seen, the shape of the mortality curve varies between the different ELTs. The figures illustrate the wide disparities that arise from the differing methods adopted to graduate mortality rates at the oldest ages in the life tables. Mortality rates at these ages were assumed to increase most rapidly for ages 100 and over in ELT No. 8. Recent ELTs show a more gentle increase in mortality at the oldest ages. Different patterns emerge for males and females. For example, while mortality rates in the ELTs for 1970–1972 and 1990–1992 converge for males, they diverge for females. Interestingly, the curves for ELT No. 11, which were graduated using an amalgam of a logistic and a normal curve, display mortality rates that increase at a declining rate at the oldest ages for both males and females. These may be more in line with recent thinking on the pattern for mortality rates at the oldest ages, which several commentators have suggested can best be modeled by a logistic function (Thatcher et al. 1998).

Similar patterns can also be seen in Figures 3 and 4, which show the mortality rates from the most recent ELTs from 1950–1952 onwards. While the rates for a given age decrease for successive life tables for ages up to the midnineties, crossovers occur at higher ages, and the rates of increase in mortality rates with age vary considerably. It should be emphasized that, because of the different methodologies used to graduate mortality rates at ages 90 and over for each ELT, it is difficult to disentangle the actual underlying changes in mortality at these ages from the effects of the changes in the methodology used to close the life table.

**Figure 1 - Probabilities of dying by single year of age - Males qx**
**(Source: English Life Tables)**

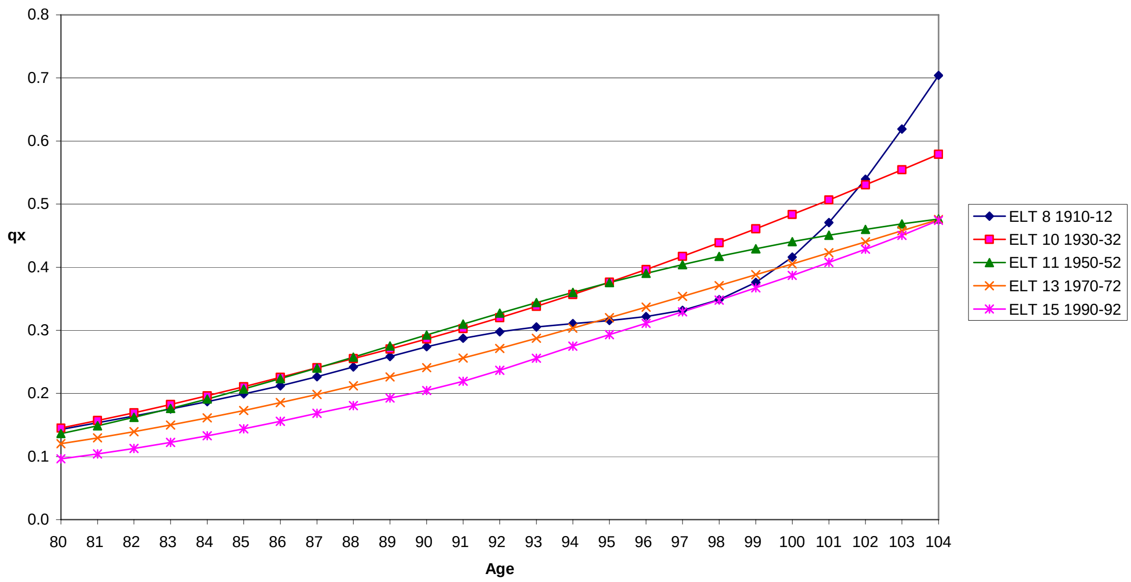



**Figure 2 - Probabilities of dying by single year of age - Female qx**
**(Source: English Life Tables)**

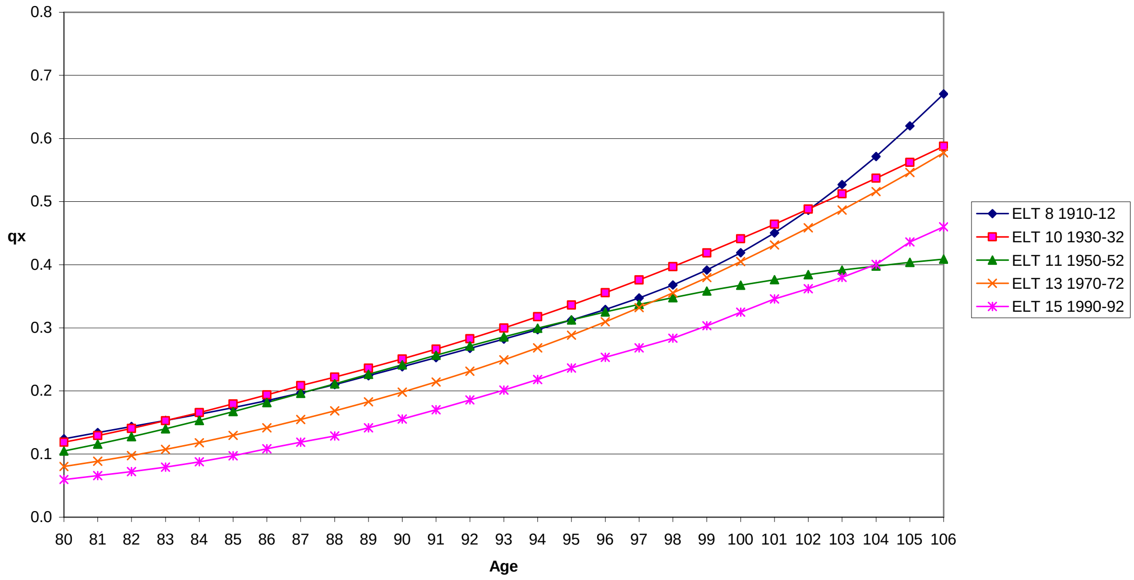


**Figure 3 - Probabilities of dying by single year of age - Males qx**
**(English Life Tables 1951-1991)**

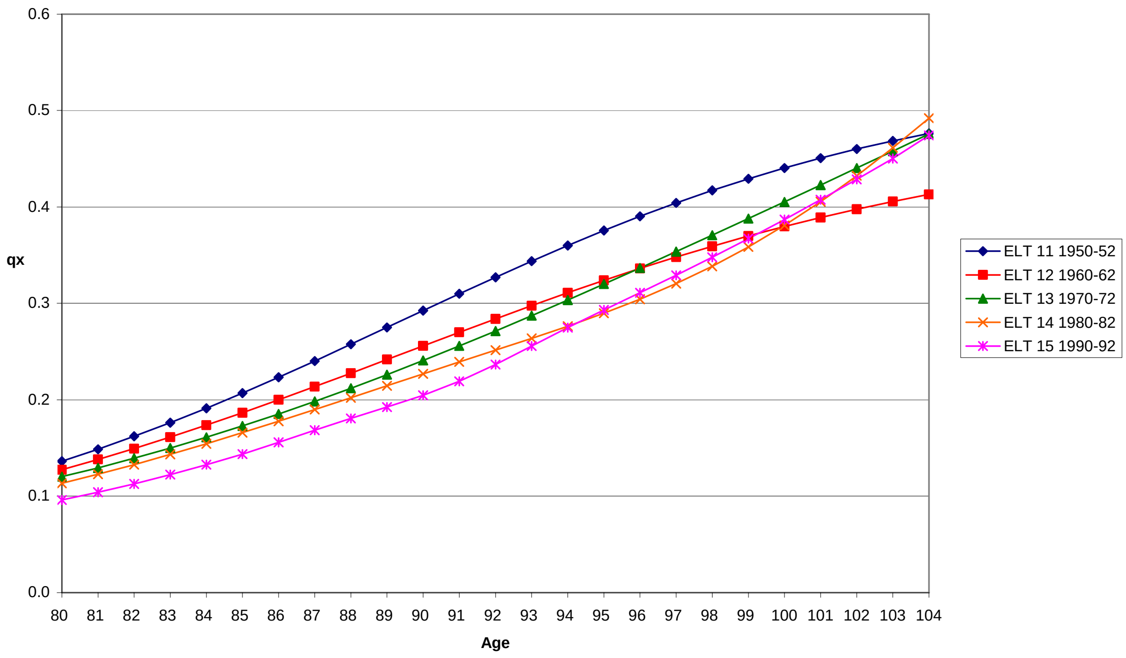
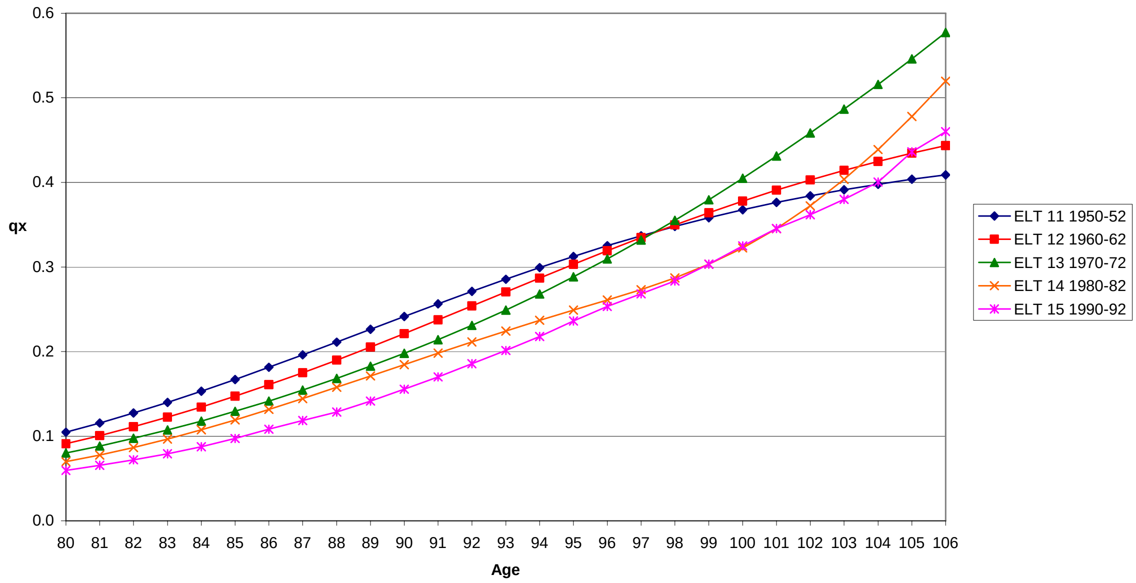



**Figure 4 - Probabilities of dying by single year of age - Females qx**
**(English Life Tables 1951-1991)**




## 3.1 Interim Life Tables

As well as the detailed investigations that take place following a census to prepare the decennial national life tables, interim life tables are produced annually. These are calculated from ungraduated mortality rates derived from data on deaths and population estimates for three consecutive calendar years (e.g., 2001–2003). These life tables are produced for each country of the United Kingdom as well as for

Great Britain and the United Kingdom as a whole. The derived crude values of $m_x$ are converted to values of $q_x$ using the formula $q_x = 2m_x/(2 + m_x)$. Values of $m_x$ and $q_x$ are calculated up to age 110; however, because of the small numbers of deaths involved at these ages and since the resulting crude death rates are not graduated, these values are published only up to age 100. Where there are no deaths recorded at a given old age, $m_x$ is assumed to equal 0.5.

The population estimates by year of age for those aged 90 and over have been derived using a variant of the extinct generation method described in Section 4 of the paper. In each case the resulting estimates were adjusted to equal the official total estimates for the population aged 90 and over by a simple scaling factor. In all cases, in calculating the death rates, deaths and population estimates for ages 110 and above are amalgamated.

**4. Methods for Estimating Mortality Rates for People Aged 85 and Over**

Various methodologies have been proposed for estimating population numbers and mortality rates at these oldest ages. Most of these generate population numbers from death registrations. For the purposes of estimating the number of very old people, these data are considered to be more reliable than estimates obtained from censuses. These methods include the method of extinct generations (Vincent 1951) and the survivor ratio method (Thatcher et al. 2002).

The method of extinct generations is a way of generating population numbers from death registrations. It is based on the assumption that when all the members of a given cohort (in this case, people born in a given calendar year) have died, the numbers that were alive earlier can be reconstructed if the dates of death of everyone in that cohort are known. This assumes that international migration can be ignored, which will usually be the case if the method is confined to ages that are high enough and if death registration data are accurate. Ideally, the method requires data on deaths by year of birth and age at death.

The survivor ratio method is a modified version of the extinct generations method that does not involve waiting until all the members of a cohort are dead. At old ages only a small proportion of the original members of a cohort will still be alive. It is assumed that the ratio of the number of survivors to the number in the cohort who died over a given period can be estimated from the experience of previous cohorts. This "survivor ratio" then can be applied to the number of deaths for a cohort over the same number of years to obtain estimates for the number of survivors.

This gives an estimate of the number of survivors in cohorts that are not yet extinct. The population of these cohorts in previous years can then be constructed using the number of deaths in previous years, as in the extinct generations method.

The method requires qualitative judgments on a number of issues: the number of years' deaths to include in the denominator of the survivor ratio; the number of cohorts across which to average survivor ratios; and the nature of this averaging (e.g., weighted or unweighted). If mortality rates are falling over the period chosen, a further correction factor could also be introduced. If reliable estimates for the total population are available, say, for the over-nineties, but these estimates are not reliable for individual ages, then a correction factor can be applied so that the estimates agree with this total.

Another data-driven method that might be used for ascertaining death rates at older ages would be to extrapolate from the rates for ages up to the age for which rates can be determined from data available. This approach is discussed by Boleslawski and Tabeau (2001), who consider the performance of 14 different models for extrapolating the age pattern of mortality among the over-eighty-fives, using data for individual years of age for France, the Netherlands and Norway. Their findings are that a polynomial of degree five or six, fitted using the weighted least-squares method, provided the best extrapolation for cases where mortality data for the over-eighty-fives are not reliable and satisfactory data exist for ages between 60 and 84.

A database containing the available official statistics on deaths at ages 80 and over in 30 countries since 1960 (and earlier, in many cases) is held at Berkeley University and has been analyzed extensively. Thatcher et al. (1998) have tested six possible models for mortality above ages 80 fitted to data sets comprising 13 industrialized countries from the database for overlapping periods 1960–1970, 1970–1980, 1980–1990 and for the cohorts born in 1871–1880, for males and females separately. This research suggested that the data were closer to the logistic model than to the Gompertz, Weibull and Heligman and Pollard models.

Thatcher (1999b) has fitted logistic functions of the form $\mu_x = \kappa z/(1+z) + \gamma$ to data from the English life tables and other historic life tables from the medieval period onward, where

$\mu_x$ is the force of mortality
$\kappa$ was taken to equal 1
$z = \alpha \exp(\beta x) = \exp[\beta(x-\phi)]$, with $\phi = -\ln(\alpha)/\beta$.

The results show that over the period 1841–1981, $\alpha$ has decreased quite dramatically, $\beta$ has risen very slowly from around 0.10 to 0.11, and thus $\phi$ has increased, while $\gamma$ had become negligible. In terms of interpretation the force of mortality is dominated by $\gamma$ at ages in the thirties and by $\phi$ at the oldest ages (90 and over). Here $\alpha$ is the dominant parameter in the middle range of ages, and $\beta$ is a measure of the relative rate at which the force of mortality increases with age.

**5. Estimates of Mortality Rates for Those Aged 85 and over in the United Kingdom**

As discussed earlier, mortality rates by single year of age for the over-nineties cannot be calculated directly by dividing numbers of deaths at a given age by the midyear population at that age, because the annual population estimates are not available by single year of age for these ages. For the decennial life tables, these rates usually have been estimated by extrapolation from graduated rates for younger ages.

The growth in numbers at these oldest ages is giving rise to substantial interest in their projected numbers—for example, with regard to the impact on pensions and the provision of long-term care. As a result, greater attempts have been made from the early 1990s to allow for more detailed analysis of the actual past mortality trends at those ages when setting assumptions for use in the U.K. national population projections.

Various methodologies proposed for estimating and projecting mortality rates at ages over 85 were examined as part of a recent review of the methodology for projecting mortality rates in the U.K. national projections (see Government Actuary's Department 2001). These included deriving estimates using extinct generations and survival ratio methods, fitting curves to mortality rates over the entire age range and methods involving extrapolation of past trends such as the Lee-Carter methodology (Lee 2000). Alternative sources of data were also investigated.

Kannisto and Thatcher have constructed a database, covering the years 1911–1998, that gives population estimates for England and Wales by sex for each individual age from age 80 to the highest attained age. This is based on the methods of extinct generations and survivor ratios described earlier (labeled the KT method for ease of reference in this paper). These results agreed with work done by GAD.

The Kannisto-Thatcher method produces population estimates as at January 1 and, to do so, requires only the numbers of deaths by age as at that date. As deaths data for U.K. countries are only published as "age attained at death," each figure for calendar-year deaths is split between the two cohorts affected, with half allocated to each cohort. Kannisto (1988) has suggested that at ages over 100 the split is nearer to 55:45 than 50:50, but that below age 100 the split is nearer 50:50. Thatcher (1992) has carried out calculations using both assumptions, which suggest that assuming a

50:50 split has little effect on the values of $q_x$ compared to those derived assuming a 55:45 split.

Andreev (1999) has analyzed the existence of age-heaping (the tendency to record ages ending in certain digits) in death registrations in the Kannisto-Thatcher database. For England and Wales there was some evidence of age-heaping in the years 1911–1980 at ages 81 and 91 for males and females as well as age 84 for females, with age-heaping being more evident for females than males. No evidence was found in the database for Scotland. There was no evidence of age exaggeration for the data for England and Wales or for Scotland.

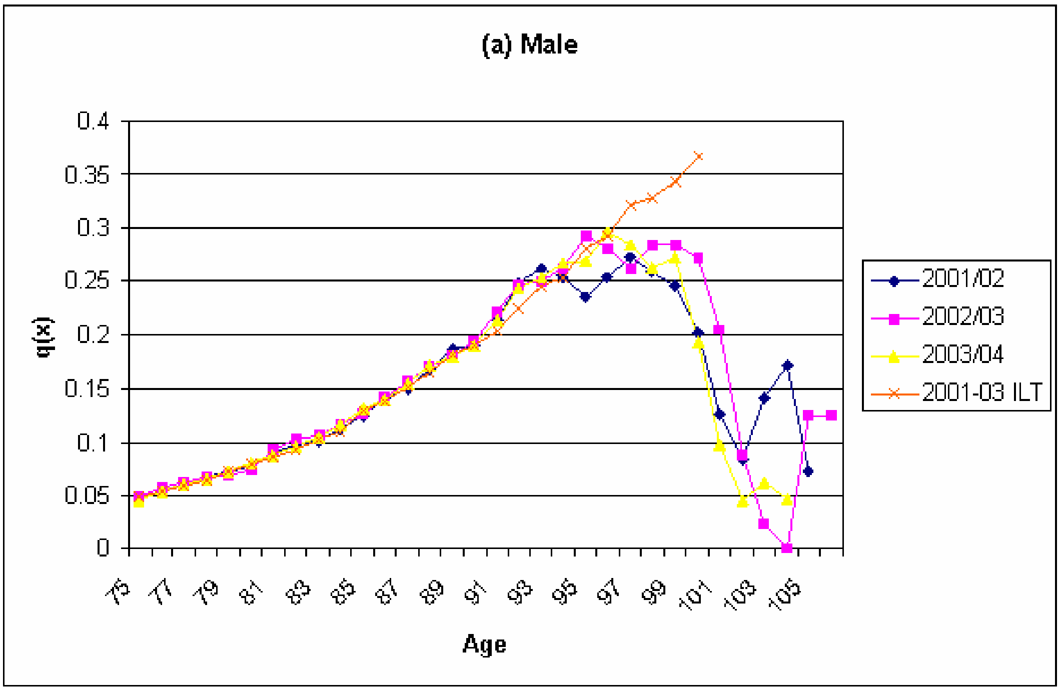
The survivor ratio method has been tested against the method of Das Gupta (Das Gupta 1990) and a method called the decline of mortality method, proposed by Andreev as part of a wider study of the database (Andreev 1999) using data for 13 countries. The results suggested that the survivor ratio method, linked to the official population estimates for the total at ages 90 and over, performed better than the other methods. The survivor ratio method has been chosen to be applied to countries that do not have reliable population registers when updating the database. Full details of the tests and criteria are available from the Max Planck Institute.

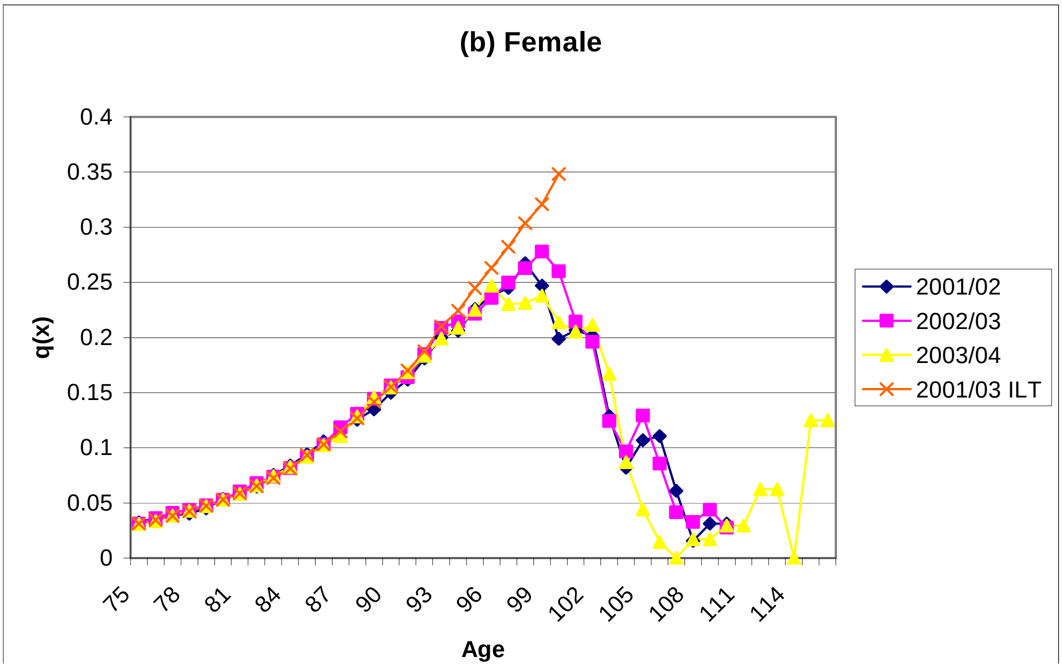
The 5 percent sample (based on the National Insurance numbers of people in Great Britain covered by the U.K. social security system) of the database of those in receipt of state pensions held by the DWP has been investigated as an alternative source of data on mortality rates at old ages. This database, which should contain everyone in the population in receipt of state pension, is regularly updated, with notified deaths being removed and the relevant new pensioners added. Mortality rates can be determined by a comparison of the numbers aged $x$ in one year and $x$ +1 in the next year. The review found that, for data for 1997–1999, there was a good match between mortality rates derived from these data and from the corresponding interim life tables up to around age 97, after which the rates diverged. The latest data continue to suggest that there is a discrepancy between the DWP pensioner data and numbers at old ages derived from the Kannisto-Thatcher method. Figure 5 compares the probability of someone aged $x$ dying before reaching age $x$ +1 ($q_x$) derived from DWP data covering 2001–2003 (inclusive) with those from GAD's interim life tables (ILTs) for Great Britain, which were produced using population data for the same period. For ages above 90 the age distribution of the population used in the ILT have been broadly derived from the Kannisto-Thatcher age distributions. Up to about age 96, the shape of the rates derived from the DWP data broadly matches the ILT curve. Beyond this age, however, the rates diverge.

There are various reasons why the figures rated up from the DWP sample would not necessarily equal the official population estimates, related to factors such as foreign residents in Great Britain not entitled to receive state benefits, former residents living overseas who have their state pensions paid through U.K. bank accounts, late deletion of deaths and so on. However, the DWP data imply that in Great Britain the number of centenarians is around 475 percent greater in 2003 than in 1991. The number of deaths of centenarians over the period has only risen by around 90 percent. If both these were true, they would imply implausibly high reductions in mortality rates for ages over 100. Also, the dramatic falls in mortality rates from age 100 onwards, for both males and females, calculated from the DWP sample, seem unlikely, although the figures may be affected to some extent by people living overseas returning to the United Kingdom as they grow older.

Data on the number of messages sent by the Queen to people aged 100 and 105 and over also suggest there has been a much smaller increase in this population group over the period 1991–1996 than the number of pensioners recorded in the DWP sample. Given these various concerns, the use of the DWP database as an alternative source of data for deriving mortality rates for the very old has been discounted until such time as further investigations are made into the reasons for these discrepancies and the reliability of the data being recorded.

**Figure 5 Comparison of 2001 to 2003 Interim Life Table mortality rates with rates calculated from financial year DWP pensioner data**




The fitting of mathematical curves to mortality rates derived for all ages was also investigated as part of the review. Practical difficulties were found in attempting to fit one curve to the entire age range of U.K. data, and it was felt that, even where a curve could be fitted, the projection of the parameters of the curves could well be problematic, and this option was not pursued further. However, fitting of curves to the oldest ages only may be a useful way of graduating the crude mortality rates at these ages calculated using deaths data and population estimates derived from a survivor ratio method. There has been growing interest in methods that involve fitting various types of spline functions to historical mortality data to produce mortality curves or surfaces.

The review concluded that other sources of U.K. data on numbers of very old people discussed in this section appeared to be less reliable or credible in terms of the shape of the mortality rates produced as compared with those given by the Kannisto-Thatcher survivor ratio method, which appeared to give reliable population figures that can be used to produce mortality rates at older ages for the United Kingdom and its constituent countries. Since the time of the review no more effective method has been encountered.

## 6. Construction of a U.K. Database of Historic Mortality Rates for Those Aged 85 and Over

### 6.1 Methodology and Calculation

This section discusses the work currently being carried out within GAD to construct a database of historical population estimates, and hence mortality rates, for the United Kingdom and its constituent countries using the Kannisto-Thatcher survivor ratio method (see Thatcher et al. 2002). The estimates produced for the final year of the database are controlled so that the totals of the estimates for those aged 90 and over by sex are equal to the official estimates of the population aged 90 and over by sex for that year. A choice has to be made as to the numbers of cohorts and previous years to be used for calculating the survivorship ratios. The survival of the five preceding cohorts over a five-year period has been used, in line with the parameters used in the database originally kept by the Max Planck Institute for Demographic Research. Work on checking the database constructed is still ongoing following the recent revisions to the population estimates for England and Wales from 1982 onwards, and hence the results given in this paper are provisional only.

There are some problems with using such a model for U.K. purposes. First, the assumed split of deaths in a calendar year between cohorts is somewhat arbitrary. Second, the method produces population estimates as at January 1 rather than the midyear estimates required. The latter are obtained by interpolation. The method can be adapted to provide midyear population estimates directly if deaths figures are available on the basis of midyear to midyear numbers by age defined at either the beginning or the end of the period. Deaths data in this format have been provided for Scotland.

The method also depends on the data on ages at death being correct. Although the data for the United Kingdom is believed to be good because of the requirement to register deaths, age misreporting may still occur, as suggested by the study carried out in Northern Ireland, mentioned in Section 2. Andreev (1999) found some evidence of age misreporting in data for England and Wales but none for Scotland.

Another problem can occur at the ages where the estimates derived from the KT method are joined to the official estimates. For example, the official estimates may be thought reliable at a given age such as 89, or, say, for the total for ages 90 and over. It is desirable that a relatively smooth run into the official ones can be obtained. This can be achieved to some extent by adjusting the correction factor in the method to provide the requisite figures for recent years, although this may not work consistently for estimates for earlier years. An alternative approach where the total for an age group is used as a control is to apply the age distribution produced by the KT method for a particular year to the control total for that age group from the official estimates to obtain estimates for each year of age that total the official estimate.

While the KT method may be used for countries of any population size, problems of consistency may arise where estimates are also required for smaller regions. In the United Kingdom estimates are required for the United Kingdom as a whole and for each individual country. This leads to considerations of whether the KT method should be applied to produce U.K. estimates from an aggregate of U.K. deaths, with the results disaggregated in some way to provide estimates for each constituent country, or whether it might be applied to each country individually with U.K. figures obtained by aggregation. Given that the population estimates are provided separately for each country by the relevant national statistical offices and data for the United Kingdom are obtained by aggregation, it was decided that databases would be constructed for England and Wales combined, Scotland and Northern Ireland. The database for the United Kingdom is then obtained by aggregation. Further work is required to ascertain whether this gives figures for the United Kingdom that vary significantly from those obtained by applying the KT method directly to U.K. data.

The assumption of nil migration at the oldest ages may be less robust when considering databases for each country of the United Kingdom rather than the United Kingdom as a whole, since internal migration between countries may be more prevalent than international migration at these ages. In theory, this would not matter for producing figures at the U.K. level since the overall number of deaths and hence the population estimates in the United Kingdom would not be affected, just redistributed between countries within the United Kingdom.

The databases discussed in this paper have not been constrained to provide population estimates that total the official estimates for ages 90 and over other than in the final year of the database. However, for purposes where figures derived from official sources are to be used, such as official life tables, estimates that have been constrained to equal the totals are currently used, being derived by applying a scaling factor to the resulting estimates so that the total over the relevant age range equals the official estimate.

The use of the KT methodology for estimating the age-specific population estimates at ages 90 and over does mean that the historic estimates need to be recalculated each year when the population estimates and deaths data for the following year are released, at least for those cohorts believed not to be extinct.

There have been slight differences in the method of calculation of the elderly populations of each individual country because of problems with data availability. The calculation methods for England and Wales combined, Scotland and Northern Ireland are described below.

### 6.2 England and Wales Combined

An updated version of the Max Planck database calculated to January 1, 2003, has been produced. The method operates to the start of year rather than midyear as it uses calendar-year deaths data. This is the latest year for which the method can be operated, the control total for the population aged 90 and over being derived from the average of the mid-2002 and mid-2003 population estimates. The resulting population as at January 1, 2003, was then aged on to January 1, 2004, using data on deaths in the calendar year 2003, and the two populations averaged to produce estimates for mid-2003. Following the revisions to the historical population estimates made after the 2001 census, the inconsistencies between the published estimates at age 89 and those produced for ages 89, age 90 and age 90+ by the KT method reported in the previous paper appear to have diminished, at least for females.

### 6.3 Scotland

Midyear to midyear deaths at ages 80 and above were provided by the General Register Office for Scotland for the period from 1974 onwards in the age definition used for aging on population estimates. The KT method was used to produce a mid-2003 age distribution. The resulting KT figures for age 89 for females appear relatively consistent with the official estimates; however, the figures for males are more variable.

### 6.4 Northern Ireland

Calendar-year deaths have been used to calculate a database with populations at January 1, 2003. As with the England and Wales database, the January 1, 2003, population has been aged on to January 1, 2004, and the two populations averaged to produce a mid-2003 age distribution. For most years the KT estimates for ages 89 are lower than the official estimates, and the differences are quite variable from year to year; however, the numbers involved are much smaller than for England and Wales and smaller than for Scotland and hence may be expected to exhibit greater variability.

**6.5 Results**

Table 1 shows the population estimates derived for specimen years for the United Kingdom, aggregated from estimates for England and Wales, Scotland and Northern Ireland derived using the KT method. The marked increases in the numbers of both males and females in their nineties and over illustrate the importance of estimating populations at these ages. The table also shows a large increase in the number of females aged over 100 as well as over age 105 since 1961. The increases for males for these age groups are lower than for females; this is partly due to factors such as war deaths.

The figures in the table also suggest that mortality rates have been improving at ages in the nineties for both males and females over the last 40 years at least. For example, the number of males aged 90 has increased by 125 percent over the period 1961–1991 while the number of males aged 100 (who are more or less the survivors from those aged 90 in 1961 and 1991) has increased by 325 percent from 1971 to 2001. For females the corresponding increases are 215 percent at age 90 and 405 percent at age 100.

Tables 2 and 3 show the derived estimates for England and Wales combined, Scotland, Northern Ireland and the United Kingdom for specimen years, together with the official population estimates for the same year.

**TABLE 1**

**Population at Individual Ages 90 and Over in the United Kingdom, Estimated by Survival Ratio Method Using Death Registrations for 1961–2003 and Population Estimates for Mid-2003**

| Age | 1961 | 1971 | 1981 | 1991 | 2001 |
|---|---|---|---|---|---|
| | | | Males | | |
| 90 | 6,225 | 8,533 | 9,437 | 14,084 | 23,778 |
| 91 | 5,074 | 6,253 | 7,047 | 10,129 | 18,112 |
| 92 | 3,523 | 4,459 | 5,209 | 6,972 | 13,513 |
| 93 | 2,378 | 3,112 | 3,674 | 4,807 | 9,683 |
| 94 | 1,546 | 2,137 | 2,508 | 3,338 | 6,686 |
| 95 | 842 | 1,402 | 1,637 | 2,261 | 4,451 |
| 96 | 495 | 872 | 1,099 | 1,484 | 2,889 |
| 97 | 272 | 544 | 698 | 985 | 1,843 |
| 98 | 156 | 319 | 459 | 619 | 1,107 |

| Age | 1961 | 1971 | 1981 | 1991 | 2001 |
|---|---|---|---|---|---|
| | | | **Males** | | |
| 99 | 96 | 173 | 259 | 371 | 643 |
| 100 | 53 | 89 | 145 | 215 | 380 |
| 101 | 31 | 63 | 73 | 128 | 197 |
| 102 | 15 | 36 | 40 | 77 | 94 |
| 103 | 7 | 19 | 23 | 42 | 48 |
| 104 | 3 | 7 | 11 | 19 | 28 |
| 105 | 2 | 1 | 8 | 11 | 13 |
| 106 | 0 | 1 | 3 | 7 | 5 |
| 107 | 0 | 1 | 0 | 3 | 2 |
| 108 | 0 | 0 | 0 | 1 | 1 |
| 109 | 0 | 0 | 0 | 0 | 0 |
| 110+ | 0 | 0 | 0 | 0 | 0 |
| Total | 20,718 | 28,021 | 32,330 | 45,553 | 83,473 |
| | | | **Females** | | |
| 90 | 16,203 | 26,732 | 34,554 | 51,286 | 66,852 |
| 91 | 12,586 | 20,580 | 27,108 | 40,445 | 55,510 |
| 92 | 8,932 | 14,987 | 21,014 | 31,019 | 45,230 |
| 93 | 6,216 | 10,906 | 15,644 | 23,466 | 35,557 |
| 94 | 4,540 | 7,828 | 11,308 | 17,348 | 26,802 |
| 95 | 3,038 | 5,471 | 8,085 | 12,549 | 19,739 |
| 96 | 2,002 | 3,603 | 5,604 | 8,844 | 14,108 |
| 97 | 1,280 | 2,339 | 3,823 | 6,089 | 9,916 |
| 98 | 744 | 1,496 | 2,518 | 4,127 | 6,771 |
| 99 | 454 | 905 | 1,636 | 2,706 | 4,453 |
| 100 | 281 | 544 | 991 | 1,667 | 2,755 |
| 101 | 151 | 312 | 571 | 1,010 | 1,700 |
| 102 | 78 | 160 | 299 | 615 | 980 |
| 103 | 43 | 87 | 162 | 351 | 541 |
| 104 | 25 | 42 | 90 | 194 | 288 |
| 105 | 11 | 17 | 49 | 104 | 148 |
| 106 | 4 | 11 | 21 | 51 | 75 |
| 107 | 1 | 7 | 12 | 27 | 36 |
| 108 | 0 | 3 | 6 | 12 | 17 |
| 109 | 0 | 1 | 2 | 5 | 8 |
| 110+ | 0 | 1 | 3 | 6 | 10 |
| Total | 56,589 | 96,032 | 133,500 | 201,921 | 291,496 |

## TABLE 2
## Estimates Derived Using Survivor Ratio Method and Official Population Estimates at Ages 85 to 89 and Ages 85+ and 90+ for England and Wales, Scotland and Northern Ireland

| Age | 1981 | | 1991 | | 2001 | |
|---|---|---|---|---|---|---|
| | KT Estimate | Official Estimate | KT Estimate | Official Estimate | KT Estimate | Official Estimate |
| England and Wales | | | | | | |
| Males: | | | | | | |
| 85 | 27,899 | 27,792 | 44,321 | 43,924 | 54,460 | 54,061 |
| 86 | 22,651 | 22,893 | 36,058 | 36,623 | 48,627 | 49,285 |
| 87 | 18,262 | 17,895 | 28,839 | 28,860 | 41,575 | 41,456 |
| 88 | 14,567 | 14,595 | 22,455 | 22,834 | 34,479 | 34,012 |
| 89 | 11,230 | 11,195 | 17,076 | 17,308 | 27,778 | 27,030 |
| 90+ | 29,067 | 31,589 | 41,269 | 41,791 | 75,828 | 77,108 |
| 85+ | 123,676 | 125,959 | 190,018 | 191,340 | 282,747 | 282,952 |
| Females | | | | | | |
| 85 | 80,705 | 80,197 | 108,737 | 107,989 | 111,354 | 110,872 |
| 86 | 68,843 | 67,997 | 94,873 | 95,535 | 106,202 | 108,219 |
| 87 | 57,959 | 55,698 | 81,452 | 81,394 | 96,518 | 97,023 |
| 88 | 48,390 | 46,498 | 68,749 | 69,274 | 84,198 | 84,353 |
| 89 | 39,332 | 38,398 | 56,869 | 57,287 | 71,791 | 71,007 |
| 90+ | 120,770 | 125,796 | 181,898 | 181,033 | 262,984 | 263,069 |
| 85+ | 415,999 | 414,584 | 592,578 | 592,512 | 733,047 | 734,543 |
| Scotland | | | | | | |
| Males: | | | | | | |
| 85 | 2,451 | 2,471 | 3,722 | 3,776 | 4,333 | 4,334 |
| 86 | 2,012 | 2,042 | 2,994 | 3,046 | 4,082 | 4,077 |
| 87 | 1,637 | 1,639 | 2,439 | 2,489 | 3,374 | 3,381 |
| 88 | 1,225 | 1,250 | 1,798 | 1,853 | 2,956 | 2,778 |
| 89 | 968 | 968 | 1,418 | 1,490 | 2,215 | 2,111 |
| 90+ | 2,433 | 2,712 | 3,243 | 3,329 | 6,095 | 6,367 |
| 85+ | 10,726 | 11,082 | 15,614 | 15,983 | 23,055 | 23,048 |
| Females | | | | | | |
| 85 | 7,713 | 7,686 | 10,212 | 10,222 | 10,263 | 10,262 |
| 86 | 6,326 | 6,187 | 8,759 | 8,756 | 9,878 | 9,843 |
| 87 | 5,298 | 5,156 | 7,574 | 7,623 | 8,990 | 8,960 |
| 88 | 4,477 | 4,329 | 6,331 | 6,376 | 7,529 | 7,411 |
| 89 | 3,428 | 3,371 | 5,096 | 5,178 | 6,335 | 6,252 |
| 90+ | 10,153 | 10,859 | 15,987 | 16,130 | 22,838 | 23,018 |
| 85+ | 37,395 | 37,588 | 53,959 | 54,285 | 65,833 | 65,746 |

| Age | 1981 | | 1991 | | 2001 | |
| --- | --- | --- | --- | --- | --- | --- |
| | KT Estimate | Official Estimate | KT Estimate | Official Estimate | KT Estimate | Official Estimate |
| Northern Ireland | | | | | | |
| Males: | | | | | | |
| 85 | 788 | 906 | 986 | 1,076 | 1,290 | 1,278 |
| 86 | 634 | 729 | 807 | 866 | 1,130 | 1,127 |
| 87 | 485 | 533 | 647 | 707 | 954 | 958 |
| 88 | 382 | 417 | 491 | 533 | 762 | 801 |
| 89 | 310 | 334 | 368 | 411 | 595 | 583 |
| 90+ | 830 | 966 | 1,041 | 1,142 | 1,550 | 1,617 |
| 85+ | 3,429 | 3,887 | 4,340 | 4,735 | 6,281 | 6,364 |
| Females: | | | | | | |
| 85 | 1,892 | 2,099 | 2,520 | 2,689 | 2,838 | 2,786 |
| 86 | 1,590 | 1,734 | 2,196 | 2,394 | 2,613 | 2,628 |
| 87 | 1,325 | 1,398 | 1,851 | 1,946 | 2,318 | 2,315 |
| 88 | 1,063 | 1,163 | 1,542 | 1,673 | 1,946 | 2,006 |
| 89 | 840 | 927 | 1,272 | 1,385 | 1,589 | 1,674 |
| 90+ | 2,577 | 2,934 | 4,036 | 4,360 | 5,674 | 5,651 |
| 85+ | 9,287 | 10,254 | 13,417 | 14,447 | 16,978 | 17,060 |

## TABLE 3

### Estimates Derived Using Survivor Ratio Method and Official Population Estimates at Ages 85 to 89 and Ages 85+ and 90+ for the United Kingdom

| Age | 1981 | | 1991 | | 2001 | |
| --- | --- | --- | --- | --- | --- | --- |
| | KT Estimate | Official Estimate | KT Estimate | Official Estimate | KT Estimate | Official Estimate |
| United Kingdom | | | | | | |
| Males: | | | | | | |
| 85 | 31,138 | 31,169 | 49,029 | 48,776 | 60,083 | 59,673 |
| 86 | 25,297 | 25,664 | 39,859 | 40,535 | 53,839 | 54,489 |
| 87 | 20,384 | 20,067 | 31,925 | 32,056 | 45,903 | 45,795 |
| 88 | 16,174 | 16,262 | 24,744 | 25,220 | 38,197 | 37,591 |
| 89 | 12,508 | 12,497 | 18,862 | 19,209 | 30,588 | 29,724 |
| 90+ | 32,330 | 35,267 | 45,553 | 46,262 | 83,473 | 85,092 |
| 85+ | 137,831 | 140,928 | 209,972 | 212,058 | 312,083 | 312,364 |
| Females: | | | | | | |
| 85 | 90,310 | 89,982 | 121,469 | 120,900 | 124,455 | 123,920 |
| 86 | 76,759 | 75,918 | 105,828 | 106,685 | 118,693 | 120,690 |
| 87 | 64,582 | 62,252 | 90,877 | 90,963 | 107,826 | 108,298 |
| 88 | 53,930 | 51,990 | 76,622 | 77,323 | 93,673 | 93,770 |
| 89 | 43,600 | 42,696 | 63,237 | 63,850 | 79,715 | 78,933 |
| 90+ | 133,500 | 139,589 | 201,921 | 201,523 | 291,496 | 291,738 |
| 85+ | 462,681 | 462,426 | 659,954 | 661,244 | 815,858 | 817,349 |

Table 4 shows the ratio of the KT estimates to the official estimates for the ages and years given in Tables 2 and 3. In general, for England and Wales the KT estimates for age 90 and over are lower than the official estimates from 1971 onwards, with males showing the greater differences. The fit for those aged 90 and over in Scotland is not very good for males over the period 1981–2002. For Scottish females, the KT estimates for the 90 and over group are lower at the beginning of the 1980s but then rise above the estimates by the late 1980s before fluctuating around 100 percent thereafter. The figures for age 90 and over in Northern Ireland show the greatest divergence between the KT estimates and the official population estimates. The KT estimates for 85 and over for each country are generally lower than or similar to the official estimates for most years, although for Scottish females the KT estimate is very close to the official estimate for all years 1981–2002. Again, Northern Ireland shows the greatest divergence between the two estimates for most years, for both male and females.

The fit for the age 85 and over group is generally better than that for the age 90 and over group in all years. This is not surprising. If the age distributions at death are correct and the assumptions made about splitting these deaths between birth cohorts are also correct, then the only difference for a given birth cohort in the actual figures for that cohort and the KT estimates is the absolute difference between these two figures in 2003. As the cohort is projected back, this absolute difference becomes smaller in percentage terms, and hence the fit, in percentage terms, should improve.

Generally it appears that the larger the population, the lower the average absolute percentage differential between the derived KT estimate and the official estimate for the population aged 90 and over for the period 1991–2001. The one exception is that the differential for females in Scotland is lower than for males in England and Wales, although the latter comprise a larger population.

**TABLE 4**

**Ratio of KT Estimates to Official Estimates**

| | **Males** | | | **Females** | | |
|---|---|---|---|---|---|---|
| **Age** | **1981** | **1991** | **2001** | **1981** | **1991** | **2001** |
| England & Wales: | | | | | | |
| 85 | 1.004 | 1.009 | 1.007 | 1.006 | 1.007 | 1.004 |
| 86 | 0.989 | 0.985 | 0.987 | 1.012 | 0.993 | 0.981 |
| 87 | 1.021 | 0.999 | 1.003 | 1.041 | 1.001 | 0.995 |
| 88 | 0.998 | 0.983 | 1.014 | 1.041 | 0.992 | 0.998 |
| 89 | 1.003 | 0.987 | 1.028 | 1.024 | 0.993 | 1.011 |
| 90+ | 0.920 | 0.988 | 0.983 | 0.960 | 1.005 | 1.000 |
| 85+ | 0.982 | 0.993 | 0.999 | 1.003 | 1.000 | 0.998 |
| Scotland: | | | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 85 | 0.992 | 0.986 | 1.000 | 1.004 | 0.999 | 1.000 |
| 86 | 0.985 | 0.983 | 1.001 | 1.022 | 1.000 | 1.004 |
| 87 | 0.999 | 0.980 | 0.998 | 1.028 | 0.994 | 1.003 |
| 88 | 0.980 | 0.970 | 1.064 | 1.034 | 0.993 | 1.016 |
| 89 | 1.000 | 0.952 | 1.049 | 1.017 | 0.984 | 1.013 |
| 90+ | 0.897 | 0.974 | 0.957 | 0.935 | 0.991 | 0.992 |
| 85+ | 0.968 | 0.977 | 1.000 | 0.995 | 0.994 | 1.001 |
| Northern Ireland: | | | | | | |
| 85 | 0.869 | 0.916 | 1.009 | 0.902 | 0.937 | 1.019 |
| 86 | 0.869 | 0.932 | 1.003 | 0.917 | 0.917 | 0.994 |
| 87 | 0.909 | 0.915 | 0.996 | 0.948 | 0.951 | 1.001 |
| 88 | 0.916 | 0.921 | 0.951 | 0.914 | 0.922 | 0.970 |
| 89 | 0.927 | 0.895 | 1.021 | 0.906 | 0.918 | 0.949 |
| 90+ | 0.859 | 0.912 | 0.959 | 0.878 | 0.926 | 1.004 |
| 85+ | 0.882 | 0.917 | 0.987 | 0.906 | 0.929 | 0.995 |
| UK: | | | | | | |
| 85 | 0.999 | 1.005 | 1.007 | 1.004 | 1.005 | 1.004 |
| 86 | 0.986 | 0.983 | 0.988 | 1.011 | 0.992 | 0.983 |
| 87 | 1.016 | 0.996 | 1.002 | 1.037 | 0.999 | 0.996 |
| 88 | 0.995 | 0.981 | 1.016 | 1.037 | 0.991 | 0.999 |
| 89 | 1.001 | 0.982 | 1.029 | 1.021 | 0.990 | 1.010 |
| 90+ | 0.917 | 0.985 | 0.981 | 0.956 | 1.002 | 0.999 |
| 85+ | 0.978 | 0.990 | 0.999 | 1.001 | 0.998 | 0.998 |

## 7. Mortality Rates in the United Kingdom at Old Ages

Figures 6 and 7 show the values of the death rates, $m_x$, by single year of age over selected five-year periods for the United Kingdom derived from the KT database constructed for the United Kingdom for 1969–2003. The death rates shown are calculated by dividing deaths at the given age over a five-year period by the sum of the midyear populations over the same period. Figure 6 shows that mortality for males since 1969 rises gradually by age until age 99, but that fluctuations occur at ages 100 and higher. Up to age 98 the mortality rates for a given age are lower for successive periods. However, the results for the period 1969–1973 are rather more erratic, with mortality rates remaining relatively constant at some ages in the nineties before rising again, but with rates at the highest ages at levels sometimes lower than in succeeding periods. One reason for this may be greater variability due to the relatively small numbers of deaths in this period (the rates have not been smoothed in any way, other than by being calculated on aggregate data). A greater prevalence of age misreporting also may have occurred. Similar patterns occur for female mortality, but with mortality rates in the late 1960s appearing less erratic than for males. Mortality rates for given ages are generally lower for successively later periods up to age 98.

**Figure 6: Values of mx by age for five year periods - Males, UK, 1969-2003**

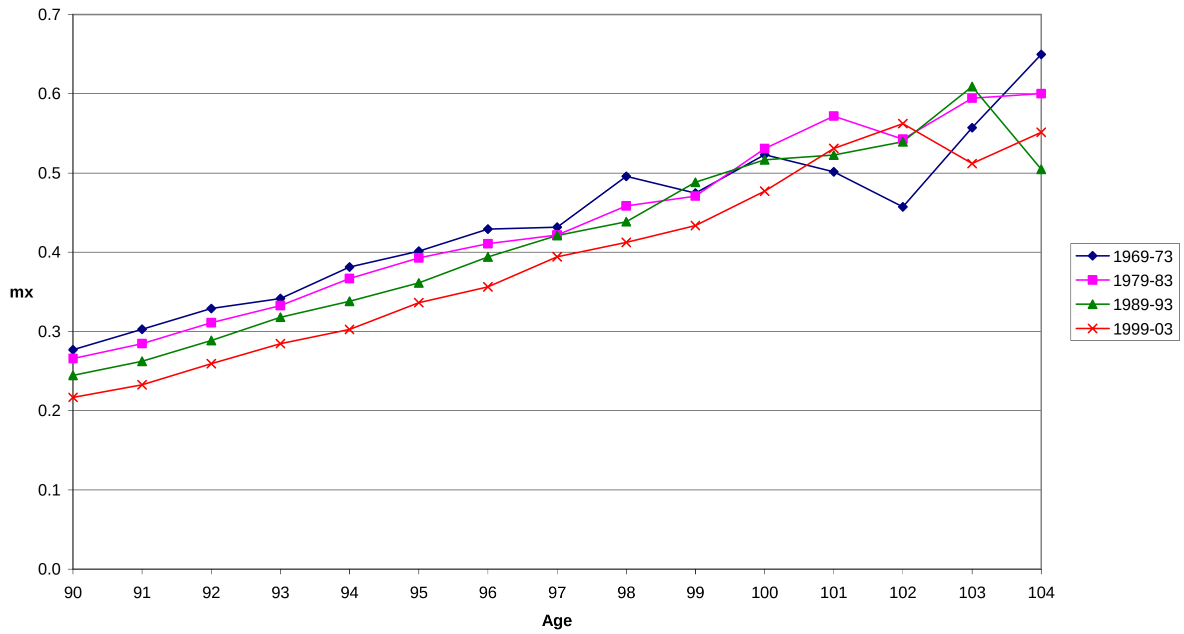



**Figure 7: Values of mx by age for five year periods - Females, UK, 1969-2003**

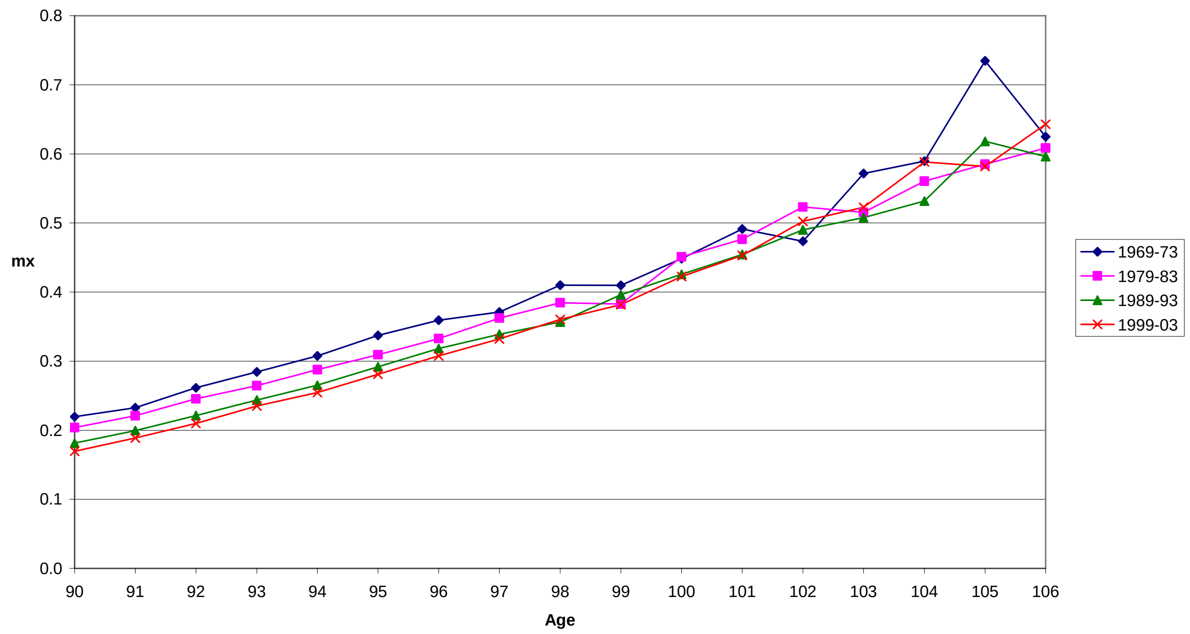


**Figure 8: Values of mx by age for five year periods - Males, UK, 1964-2003**

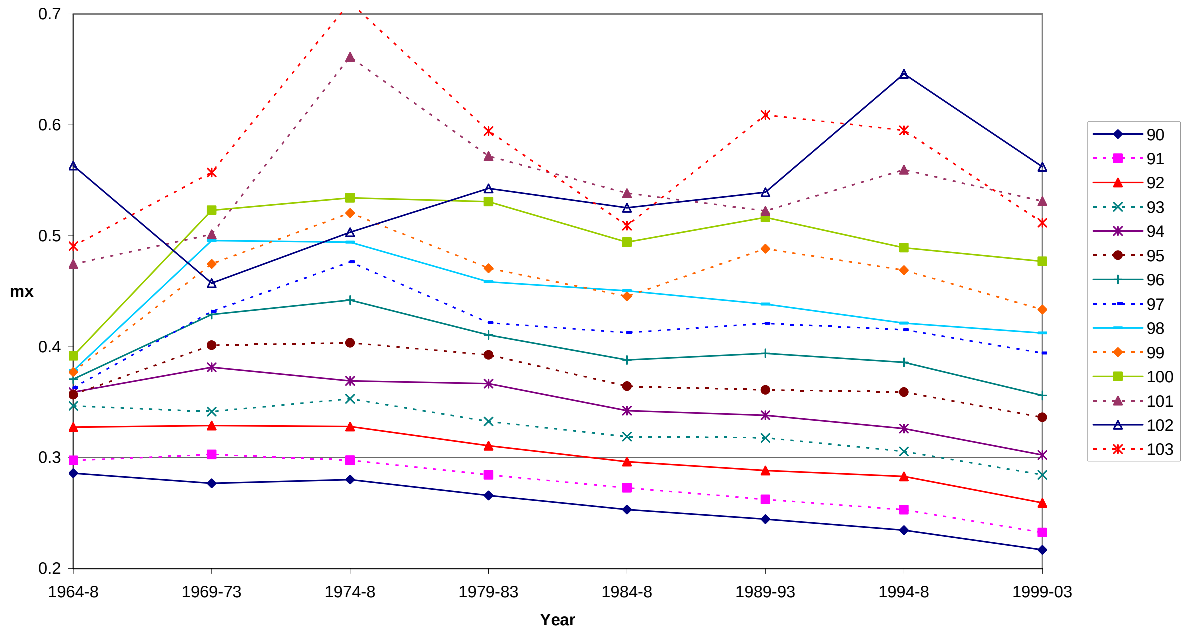
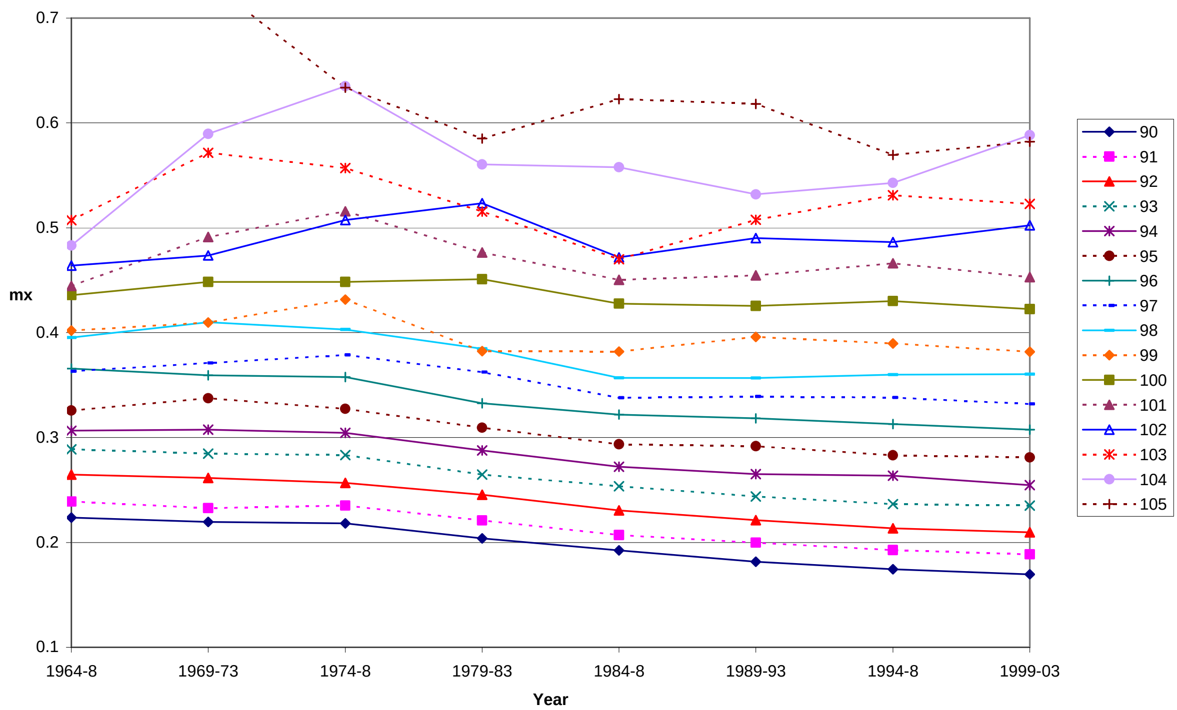



**Figure 9: Values of mx by age for five year periods - Females, UK, 1964-2003**




Figures 8 and 9 present the same data in a different format. It can be seen that for females the mortality rates generally increase with age up to 100 for a given period. Similar patterns exist at ages above 100, but there are more erratic movements. In general, the slopes of the lines are downwards for ages up to 100, indicating continually falling death rates at all ages over the last 35 years. Beyond that age, there appears to have been relatively little improvement in mortality rates over the period.

For males, the patterns are rather more difficult to discern, mainly because of the crossovers at various ages seen in the data for 1964–1968 and 1969–1973. Death rates appear to remain static, or to increase, between 1964–1968 and 1974–1978. From the early 1980s, death rates at most ages up to 98 then decline over the following 20 years. Death rates for higher ages are much more erratic, and there is little evidence of a general decline in death rates at ages over 100.

An analysis of the ratio of the male-to-female values of $q_x$ shows that, in general, for a given year, the values of $q_x$ for males converge toward the female values with increasing age; that is, there is a decreasing differential with age. However, the data also suggest that for ages up to 100 the absolute differentials between the male and female values for a given age were relatively constant until the mid-1990s, after which the trend appears to be decreasing differentials. It is possible that these are cohort effects with some cohorts generally exhibiting wider mortality differentials between males and females than others.

Tables 5 and 6 show the values of $q_x$ for quinquennial periods derived from the values of $m_x$, and Tables 7 and 8 show the ratio of the value for a given quinquennium to that for the preceding quinquennium at the same age (i.e., the proportionate decrease or increase in mortality rates for a given age over a five-year period).

Overall, mortality rates ($q_x$) in the United Kingdom for 1999–2003 at age 90 have fallen by around 22 percent of their value in 1964–1968 for both males and females. Over this period female rates have fallen at all ages up to 100, and for males, up to age 96. However, these figures are dependent on the rates at either end of the period under consideration; for instance, over the period 1974–1978 to 1999–2003 mortality rates at age 90 fell by around 20 percent for both males and female and at age 99 by 14 percent and 10 percent, respectively.

**TABLE 5**

**Values of $q_x$ for Quinquennial Periods, Males, United Kingdom, 1964–2003**

| Age | 1964–1968 | 1969–1973 | 1974–1978 | 1979–1983 | 1984–1988 | 1989–1993 | 1994–1998 | 1999–2003 |
|---|---|---|---|---|---|---|---|---|
| 85 | 0.17728 | 0.17461 | 0.17202 | 0.16557 | 0.15641 | 0.14874 | 0.13954 | 0.13109 |
| 86 | 0.19549 | 0.18618 | 0.18562 | 0.18088 | 0.16872 | 0.16073 | 0.15134 | 0.14102 |
| 87 | 0.20911 | 0.20031 | 0.19530 | 0.19406 | 0.17814 | 0.17365 | 0.16434 | 0.15338 |
| 88 | 0.22119 | 0.21366 | 0.21086 | 0.20476 | 0.19104 | 0.18520 | 0.17734 | 0.16826 |
| 89 | 0.24041 | 0.22946 | 0.22493 | 0.21287 | 0.20413 | 0.19969 | 0.19238 | 0.18336 |
| 90 | 0.25018 | 0.24319 | 0.24578 | 0.23464 | 0.22478 | 0.21794 | 0.20996 | 0.19559 |
| 91 | 0.25892 | 0.26299 | 0.25910 | 0.24908 | 0.24005 | 0.23185 | 0.22465 | 0.20835 |
| 92 | 0.28140 | 0.28244 | 0.28179 | 0.26904 | 0.25813 | 0.25214 | 0.24796 | 0.22958 |
| 93 | 0.29547 | 0.29170 | 0.30012 | 0.28518 | 0.27515 | 0.27430 | 0.26513 | 0.24913 |
| 94 | 0.30446 | 0.32034 | 0.31173 | 0.30988 | 0.29235 | 0.28923 | 0.28055 | 0.26273 |
| 95 | 0.30279 | 0.33424 | 0.33578 | 0.32824 | 0.30820 | 0.30586 | 0.30445 | 0.28796 |
| 96 | 0.31285 | 0.35323 | 0.36199 | 0.34062 | 0.32500 | 0.32916 | 0.32344 | 0.30227 |
| 97 | 0.30725 | 0.35503 | 0.38480 | 0.34817 | 0.34210 | 0.34778 | 0.34398 | 0.32935 |
| 98 | 0.31813 | 0.39716 | 0.39625 | 0.37297 | 0.36763 | 0.35959 | 0.34800 | 0.34182 |
| 99 | 0.31729 | 0.38357 | 0.41304 | 0.38113 | 0.36430 | 0.39246 | 0.37992 | 0.35628 |
| 100 | 0.32753 | 0.41461 | 0.42161 | 0.41949 | 0.39623 | 0.41057 | 0.39305 | 0.38515 |
| 101 | 0.38351 | 0.40087 | 0.49700 | 0.44467 | 0.42424 | 0.41426 | 0.43720 | 0.41964 |
| 102 | 0.43945 | 0.37215 | 0.40204 | 0.42688 | 0.41600 | 0.42476 | 0.48828 | 0.43888 |
| 103 | 0.39404 | 0.43574 | 0.52427 | 0.45818 | 0.40587 | 0.46679 | 0.45857 | 0.40747 |
| 104 | 0.41629 | 0.49026 | 0.53333 | 0.46154 | 0.43243 | 0.40293 | 0.54412 | 0.43228 |
| 105 | 0.22955 | 0.57594 | 0.70588 | 0.49351 | 0.42991 | 0.45333 | 0.51163 | 0.56842 |

**TABLE 6**

**Values of $q_x$ for Quinquennial Periods, Females, United Kingdom, 1964–2003**

| Age | 1964–1968 | 1969–1973 | 1974–1978 | 1979–1983 | 1984–1988 | 1989–1993 | 1994–1998 | 1999–2003 |
|---|---|---|---|---|---|---|---|---|
| 85 | 0.13375 | 0.12915 | 0.12827 | 0.11991 | 0.10829 | 0.10178 | 0.09688 | 0.09455 |
| 86 | 0.14857 | 0.14430 | 0.14036 | 0.13300 | 0.12033 | 0.11327 | 0.10852 | 0.10380 |
| 87 | 0.16039 | 0.15396 | 0.15030 | 0.14357 | 0.13254 | 0.12496 | 0.11890 | 0.11578 |
| 88 | 0.17059 | 0.16792 | 0.16286 | 0.15685 | 0.14409 | 0.13553 | 0.13226 | 0.12777 |
| 89 | 0.18922 | 0.18093 | 0.17801 | 0.17416 | 0.15865 | 0.15051 | 0.14549 | 0.14241 |
| 90 | 0.20115 | 0.19776 | 0.19665 | 0.18497 | 0.17561 | 0.16636 | 0.16039 | 0.15640 |
| 91 | 0.21351 | 0.20842 | 0.21047 | 0.19896 | 0.18752 | 0.18156 | 0.17581 | 0.17239 |
| 92 | 0.23373 | 0.23119 | 0.22759 | 0.21856 | 0.20660 | 0.19919 | 0.19287 | 0.18982 |
| 93 | 0.25237 | 0.24916 | 0.24813 | 0.23377 | 0.22502 | 0.21735 | 0.21147 | 0.21046 |
| 94 | 0.26583 | 0.26659 | 0.26422 | 0.25151 | 0.23957 | 0.23403 | 0.23297 | 0.22576 |
| 95 | 0.28019 | 0.28869 | 0.28135 | 0.26790 | 0.25609 | 0.25469 | 0.24800 | 0.24648 |
| 96 | 0.30929 | 0.30460 | 0.30340 | 0.28517 | 0.27728 | 0.27460 | 0.27048 | 0.26660 |
| 97 | 0.30738 | 0.31304 | 0.31847 | 0.30675 | 0.28906 | 0.28992 | 0.28914 | 0.28482 |
| 98 | 0.33009 | 0.34022 | 0.33537 | 0.32260 | 0.30281 | 0.30272 | 0.30502 | 0.30541 |
| 99 | 0.33469 | 0.34000 | 0.35501 | 0.32107 | 0.32080 | 0.33057 | 0.32621 | 0.32056 |
| 100 | 0.35789 | 0.36628 | 0.36629 | 0.36803 | 0.35240 | 0.35090 | 0.35402 | 0.34884 |
| 101 | 0.36367 | 0.39436 | 0.40995 | 0.38482 | 0.36765 | 0.37038 | 0.37809 | 0.36937 |
| 102 | 0.37657 | 0.38289 | 0.40479 | 0.41477 | 0.38178 | 0.39371 | 0.39120 | 0.40151 |

| Age | 1964–1968 | 1969–1973 | 1974–1978 | 1979–1983 | 1984–1988 | 1989–1993 | 1994–1998 | 1999–2003 |
|---|---|---|---|---|---|---|---|---|
| 103 | 0.40464 | 0.44448 | 0.43570 | 0.40990 | 0.38050 | 0.40488 | 0.41963 | 0.41447 |
| 104 | 0.38917 | 0.45530 | 0.48201 | 0.43777 | 0.43611 | 0.42013 | 0.42695 | 0.45462 |
| 105 | 0.52466 | 0.53735 | 0.48124 | 0.45265 | 0.47485 | 0.47214 | 0.44332 | 0.45082 |

**TABLE 7**

**Ratio of $q_x$ for Quinquennial Periods to Preceding Quinquennial Period, Males, United Kingdom, 1969–2003**

| Age | 1969–1973 | 1974–1978 | 1979–1983 | 1984–1988 | 1989–1993 | 1994–1998 | 1999–2003 |
|---|---|---|---|---|---|---|---|
| 85 | 0.985 | 0.985 | 0.962 | 0.945 | 0.951 | 0.938 | 0.939 |
| 86 | 0.952 | 0.997 | 0.974 | 0.933 | 0.953 | 0.942 | 0.932 |
| 87 | 0.958 | 0.975 | 0.994 | 0.918 | 0.975 | 0.946 | 0.933 |
| 88 | 0.966 | 0.987 | 0.971 | 0.933 | 0.969 | 0.958 | 0.949 |
| 89 | 0.954 | 0.980 | 0.946 | 0.959 | 0.978 | 0.963 | 0.953 |
| 90 | 0.972 | 1.011 | 0.955 | 0.958 | 0.970 | 0.963 | 0.932 |
| 91 | 1.016 | 0.985 | 0.961 | 0.964 | 0.966 | 0.969 | 0.927 |
| 92 | 1.004 | 0.998 | 0.955 | 0.959 | 0.977 | 0.983 | 0.926 |
| 93 | 0.987 | 1.029 | 0.950 | 0.965 | 0.997 | 0.967 | 0.940 |
| 94 | 1.052 | 0.973 | 0.994 | 0.943 | 0.989 | 0.970 | 0.936 |
| 95 | 1.104 | 1.005 | 0.978 | 0.939 | 0.992 | 0.995 | 0.946 |
| 96 | 1.129 | 1.025 | 0.941 | 0.954 | 1.013 | 0.983 | 0.935 |
| 97 | 1.156 | 1.084 | 0.905 | 0.983 | 1.017 | 0.989 | 0.957 |
| 98 | 1.248 | 0.998 | 0.941 | 0.986 | 0.978 | 0.968 | 0.982 |
| 99 | 1.209 | 1.077 | 0.923 | 0.956 | 1.077 | 0.968 | 0.938 |
| 100 | 1.266 | 1.017 | 0.995 | 0.945 | 1.036 | 0.957 | 0.980 |
| 101 | 1.045 | 1.240 | 0.895 | 0.954 | 0.976 | 1.055 | 0.960 |
| 102 | 0.847 | 1.080 | 1.062 | 0.975 | 1.021 | 1.150 | 0.899 |
| 103 | 1.106 | 1.203 | 0.874 | 0.886 | 1.150 | 0.982 | 0.889 |
| 104 | 1.178 | 1.088 | 0.865 | 0.937 | 0.932 | 1.350 | 0.794 |
| 105 | 2.509 | 1.226 | 0.699 | 0.871 | 1.054 | 1.129 | 1.111 |

**TABLE 8**

**Ratio of $q_x$ for Quinquennial Periods to Preceding Quinquennial Period, Females, United Kingdom, 1969–2003**

| Age | 1969–1973 | 1974–1978 | 1979–1983 | 1984–1988 | 1989–1993 | 1994–1998 | 1999–2003 |
|---|---|---|---|---|---|---|---|
| 85 | 0.966 | 0.993 | 0.935 | 0.903 | 0.940 | 0.952 | 0.976 |
| 86 | 0.971 | 0.973 | 0.948 | 0.905 | 0.941 | 0.958 | 0.957 |
| 87 | 0.960 | 0.976 | 0.955 | 0.923 | 0.943 | 0.952 | 0.974 |
| 88 | 0.984 | 0.970 | 0.963 | 0.919 | 0.941 | 0.976 | 0.966 |
| 89 | 0.956 | 0.984 | 0.978 | 0.911 | 0.949 | 0.967 | 0.979 |
| 90 | 0.983 | 0.994 | 0.941 | 0.949 | 0.947 | 0.964 | 0.975 |
| 91 | 0.976 | 1.010 | 0.945 | 0.943 | 0.968 | 0.968 | 0.981 |
| 92 | 0.989 | 0.984 | 0.960 | 0.945 | 0.964 | 0.968 | 0.984 |
| 93 | 0.987 | 0.996 | 0.942 | 0.963 | 0.966 | 0.973 | 0.995 |
| 94 | 1.003 | 0.991 | 0.952 | 0.953 | 0.977 | 0.995 | 0.969 |
| 95 | 1.030 | 0.975 | 0.952 | 0.956 | 0.995 | 0.974 | 0.994 |
| 96 | 0.985 | 0.996 | 0.940 | 0.972 | 0.990 | 0.985 | 0.986 |

| Age | 1969–1973 | 1974–1978 | 1979–1983 | 1984–1988 | 1989–1993 | 1994–1998 | 1999–2003 |
|---|---|---|---|---|---|---|---|
| 97 | 1.018 | 1.017 | 0.963 | 0.942 | 1.003 | 0.997 | 0.985 |
| 98 | 1.031 | 0.986 | 0.962 | 0.939 | 1.000 | 1.008 | 1.001 |
| 99 | 1.016 | 1.044 | 0.904 | 0.999 | 1.030 | 0.987 | 0.983 |
| 100 | 1.023 | 1.000 | 1.005 | 0.958 | 0.996 | 1.009 | 0.985 |
| 101 | 1.084 | 1.040 | 0.939 | 0.955 | 1.007 | 1.021 | 0.977 |
| 102 | 1.017 | 1.057 | 1.025 | 0.920 | 1.031 | 0.994 | 1.026 |
| 103 | 1.098 | 0.980 | 0.941 | 0.928 | 1.064 | 1.036 | 0.988 |
| 104 | 1.170 | 1.059 | 0.908 | 0.996 | 0.963 | 1.016 | 1.065 |
| 105 | 1.024 | 0.896 | 0.941 | 1.049 | 0.994 | 0.939 | 1.017 |

## 8. Projecting Mortality Rates at the Oldest Ages

Currently projections of mortality rates for the oldest ages in the U.K. national population projections are carried out using the same methodology as used for projecting rates at younger ages. This method involves elements of extrapolation and target setting.[3]

At the time that GAD agrees on the assumptions for the projections with the Registrars General, crude mortality rates by age and sex are generally known only for the years up to and including the year preceding the base year of the projection. Mortality rates for the base year are estimated by analyzing trends in mortality rates by age and sex over the previous 40 years and extrapolating forward for one year to the base year. This extrapolation is also used to determine a set of improvement factors by age and sex for mortality rates in the base year of the projections. Target improvement rates for the twenty-fifth year of the projection period are also set (25 years being considered a reasonable period for making assumptions of future rates of mortality improvement). For the 2002-based projections, it is assumed that the rate of mortality improvement will be 1.0 percent a year in 2027 at all ages for both males and females (1.0 percent pa is the approximate annual rate of improvement in mortality rates for the United Kingdom over the twentieth century standardized using the 2001 population estimates). Mortality improvement factors for each sex by age and calendar year for years between the base year of the projections and the target year are obtained by interpolation between the improvement factors assumed for the base year and those assumed for the target year, assuming that the rate of decline (or increase) from the improvement factors in the base year to those assumed for the target year will be greater in the earlier years of the projection.

The historic data for cohorts born before 1947 show evidence of differential rates of improvement in mortality between generations. For example, mortality rates for generations born around 1931 are still improving at faster rates than earlier and later generations. There is also evidence for similar cohort effects for those born in years before 1931. It is not known whether these outcomes result from something

[3] For more details of methodology adopted see Government Actuary's Department (2002).

inherent within the generations concerned (e.g., differences in childhood environment, growing up during the war years), or whether this is merely the result of various period effects that happen to have produced greater improvements for these generations. Thus, for cohorts born before 1947, the interpolation to determine the annual improvement factors by age and calendar year is carried out along a year of birth cohort, rather than along the year of age.

At one level the assumptions made for projecting rates at very old ages will have little effect on the overall population numbers, and hence it could be argued that a similar method should be used here as for other ages. However, there is always particular interest in the projected numbers of centenarians when projections are published. A methodology that produced rapidly varying estimates between projections may cast some doubt on the whole projection process even though such estimates are, by nature, likely to be more variable than estimates covering younger ages.

Several countries produce population projections that involve the adoption of target assumptions in target years for projecting mortality, and this would seem a reasonable methodology for use at the oldest ages. Other possible methods include extrapolation of current rates of improvement into the future, for example, using a Lee-Carter methodology (Lee 2000) or attempting to fit mathematical curves to the data and extrapolating the parameters of the curve. The latter can be difficult, however, and has not proved amenable to projecting mortality rates in the United Kingdom. There are problems in trying to fit curves that do not involve a large number of parameters to mortality rates over the whole range of ages. Even where this can be done, there is no guarantee that it will prove easy to project the values of these parameters to produce reasonable future mortality rates. For these reasons, such a method has been rejected for use in projecting mortality rates at all ages for the United Kingdom. The difficulties may be eased if curves are fitted to a subgroup comprising the oldest ages only, since fewer parameters may be needed. There is a growing body of evidence as to which curves may be most suitable for this purpose (e.g., the logistic curve). However, if such a method is used only for the oldest ages, problems will inevitably arise in future years of the projections at ages where the method is joined to that chosen for younger ages. (Such problems also exist in constructing life tables, but the problems arising in attempting to maintain reasonable continuity at the ages where the methods join arise only for a very small number of years, rather than the several decades covered in a set of population projections.) Problems may also arise if an extrapolation method such as the Lee-Carter method is used for this particular age group. The straightforward application of an extrapolation method may produce mortality rates that reduce with increasing age, or that increase indefinitely, which may be considered unlikely by most demographers.

The review of the methodology for projecting mortality rates in the U.K. national projections recommended that the "targeting" methodology in use be retained, but that ways of more clearly relating the targeting of the rates of mortality improvement to available data and other relevant evidence should be considered. For projecting mortality at old ages, consideration needs to be given to whether the same target improvement factors should be assumed as for younger ages. (An assumed rate of improvement of 1 percent pa would imply a ratio over a five-year period comparable to those shown in Tables 7 and 8 of 0.951; comparison with the figures in Tables 7 and 8 suggests that an assumed rate of improvement at these oldest ages of 1 percent pa in 2027 is not unreasonable in light of current rates of improvement at these ages.)

The database of historical mortality rates discussed earlier in this paper is intended to provide the basis for analysis of past mortality trends at the oldest ages. The methods discussed above for projecting mortality rates could all play a role in setting the assumptions. Suitable target rates of improvement might be derived from consideration of the results of extrapolating recent levels of improvement, fitting curves to derived mortality rates at these ages and extrapolating the parameters, consulting demographic and medical experts and considering the results of recent research and practical developments.

Given the relatively small improvements that have occurred in mortality rates at the oldest ages in comparison with the improvements at younger ages, one of the fundamental issues is whether this general feature will continue in the future or whether we can reasonably project more dramatic changes at older ages.

## 9. The Continuous Mortality Investigation Bureau

The Continuous Mortality Investigation Bureau (CMIB) is a body funded by the U.K. life insurance industry, and run by the Faculty of Actuaries and Institute of Actuaries, with responsibility for collecting and analyzing mortality and other data from participating companies. Mortality data have been collected on a continuous basis since 1924, and in more recent times its activities have been extended to income protection insurance and critical illness insurance. In practical terms, the investigation of each line of business is delegated to a subcommittee, supported by a permanent secretariat.

Data are collected from participating companies in the form of schedules of numbers of claims (or annuities ceasing payment by death) during the preceding calendar year, and numbers of policies in force at the end of each calendar year; in the case of some classes of pension and annuity business, total amounts of annuity are collected as well. Thus, the investigation is not of deaths, but claims, and various crude adjustments are needed to allow for persons with duplicate policies. Data are always subdivided by individual age and sex, and then by other categories determined by the evolving nature of the business; for example, data in respect of assurances are now split by smoker/nonsmoker status whenever possible.

Classes of business for which the very oldest ages are particularly relevant are pensioners and annuitants. In U.K. practice, "pensioners" are members of pension schemes of various kinds, whose pensions in payment are secured by an annuity with a life insurer, while "annuitants" are persons who have bought an individual annuity contract, either with savings from an individual retirement plan or with other funds. During several decades of shifting pensions legislation in the United Kingdom, different classes of annuitant have been created, each arguably subjected to different incentives to save for retirement, so the CMIB has set up new investigations as has seemed necessary. Experience has shown that these often have appeared to show different mortality.

An important duty of the CMIB, perhaps the most important, is to produce the standard tables for use by actuaries in life insurance companies. Since these have a defined role in insurance company regulation, particular trouble is taken over their production. New standard tables have been produced every 10 years or so, the last being the "80" Series tables based on the experience of 1979–1982, the "92" Series tables based on the experience of 1991–1994, and, at the time of writing, new "00" Series tables are being prepared based on the experience of 1999–2002. Tables for pensioners and annuitants have, and continue to be, produced in two stages. First, base tables are prepared, which are straightforward graduations of the data from the quadrennium. Second, the base tables are projected forward to allow for future improvements in longevity.

The methodology used for graduating the base tables for some time has been based on maximum likelihood fitting of a Gompertz-Makeham family of functions (of the general form "polynomial + exp(polynomial)") to the force of mortality. It is described in detail in Forfar et al. (1988). In some experiences the data are either very scarce or suspected to be unreliable at very high ages, and beyond a certain age are not used for the graduation. In these cases mortality rates at the highest ages are just extrapolations of the parametric model and sometimes require ad hoc adjustments, either of the rates themselves or of the parameters, to appear reasonable. In guiding such decisions, the CMIB, like GAD, has regard to the special investigations of oldest-old mortality discussed in previous sections.

Currently the projection methodology is attracting more interest, because improving longevity is recognized as a significant factor in capitalizing annuity business. Allowance for improvements has been made since the a(55) annuitants' tables were produced based on the immediate postwar experience. Originally the improvements were introduced indirectly via the monetary annuity values, but latterly explicit projection formulas were used, which, in the form of reduction factors applied to the base table, resulted in a two-dimensional table indexed by age and calendar year. The "92" Series reduction factors were based on rates of mortality declining exponentially to asymptotic values, the latter chosen to produce greater improvements at younger ages than at older ages.

The experience of the 1990s suggested that even these most recent projections underestimated actual improvements in mortality, and further investigation focused on birth cohorts. GAD had found evidence that the cohorts born in the years around 1931 had had notably higher rates of improvements, while the CMIB Assured Lives data for males showed a similar feature for the cohorts born in the years around 1926. No conclusive explanation has been advanced for this "cohort effect," though some hypotheses have been offered, including changing smoking habits, healthy diet because of rationing during the Second World War, the introduction of the National Health Service in 1948 and increasing affluence from the 1950s and 1960s.

The CMIB issued new interim projections in 2002 (CMIB Working Paper No.1) in which the existing reduction factors, based on exponential decline, were adjusted in an ad hoc manner to allow for the main cohort effect. A new feature, in what was presented to the profession, was that three possible scenarios were put forward, called short, medium and long cohorts, depending on the period during which the excess rate of improvement enjoyed by the main cohort were supposed to wear off. Previous CMIB projections had presented a single scenario. At the time of writing, a working party of the CMIB is evaluating recent advances in projection methodologies, with a view to proposing a method suitable for use with the "00" Series tables, and adequate to meet the needs of life insurers under the new regulatory regime being introduced in the United Kingdom. In doing so, the CMIB is to some extent covering ground already covered by GAD and, indeed, has found GAD's researches very helpful. However, the exercise has somewhat different objectives.

The Financial Services Authority (the U.K. insurance regulator) is introducing new rules for capital adequacy based on stochastic evaluations of risk. While the initial impetus may have come from asset risk, the same philosophy applies to all forms of risk that reasonably may be modeled, including mortality risk. Thus, the CMIB working party is examining projection methodologies capable of producing quantitative measures of risk as well as sample or central projections. The three scenarios for the interim projection of cohort mortality issued in 2002 are recognized as the first, rather crude, step toward projection methodologies that explicitly include risk. This is potentially a large step forward from previous approaches, and in March 2004 the CMIB working party issued CMIB Working Paper No. 3, a discussion document on the issues raised. It discussed, among other things, contrasting methodologies such as time-series approaches (typified by the Lee-

Carter model in much of the literature) and extrapolation of smoothed regression models; model, parameter and stochastic risk, and the problems of quantifying them; and the extent to which measures of uncertainty based on large populations could be applied to different, smaller populations such as an insurer's annuitant portfolio. The last point, in particular, distinguishes the CMIB's task from that of GAD; the CMIB has very few large and mature experiences available to it, while the pensioners' and annuitants' tables that must be projected are sometimes quite small and sometimes were set up comparatively recently.

**10. Discussion**

Population estimates and mortality rates are required at the oldest ages for the United Kingdom and constituent countries for constructing life tables and for carrying out national population projections. However, while data on deaths are available at all ages, population numbers are not available by year of age for ages 90 and above. Various methods have been used over the last decade to estimate these figures. Recently this work has focused on the methodologies proposed by Kannisto and Thatcher. Their methods have been used to derive a database of population estimates at ages over 90 (or 85 for earlier years) for the United Kingdom and constituent countries.

This database appears to provide good population estimates for the age group 85 and over, compared to the official population estimates, as well as for the age group 90 and over, especially for more recent years. For work carried out using official demographic data, it has been decided to use the officially published figures as far as possible. This means that the official estimates will be used for individual ages up to 89, with estimates derived using the KT methodology for older ages. These would be used for official life tables and for deriving the starting population numbers for the base year of a set of national population projections. The mortality rates resulting from the U.K. database constructed back to 1961 will be analyzed when setting assumptions for future population projections.

The previous paper reported that, in constructing the database, problems had occurred at the ages where the official estimates and the KT estimates join. The recent revisions to the population estimates from 1982 onwards following the 2001 census have meant that these problems appear to have diminished compared to those reported previously, but further investigations need to be carried out to verify whether there is still a problem.

While the KT method is obviously an approximation, it has been shown to be robust by the work that has been carried out at the Max Planck Institute for Demographic Research. There are parameters to be chosen in carrying out the method, chiefly over how many cohorts and how many years back the survival ratios are operated. However, experiments varying these parameters do not appear to provide any great improvement to the discontinuity problem above.

The operation of the KT method controlled to the overall population of 90 and over given by the estimates is dependent on the assumption that the errors in the individual age data underlying the total (which are the reason why they themselves are not used) average out and the overall 90 and over estimate gives a good approximation. The KT figures shown in the paper were produced using the latest available population estimates for ages 90 and over (for 2003 at the time of writing the paper) as the control total. It may be better to control on the estimates for those aged 90 and over produced for the latest census, or the estimates for the date nearest the census, on the assumption that those will be the most correct, given the tendency for errors in the estimates to accumulate as they move farther away from a census year.

In producing projections, there is a move toward providing measures of uncertainty. For the national population projections, this has been done in the past by producing variant projections that use different assumptions on future mortality improvements, fertility, migration, etc. The latest CMIB projections also provide for three different scenarios of future mortality improvement, and there has been much discussion as to how measures of uncertainty might be provided in the forthcoming projections.

**11. Conclusions**

Mortality rates in the United Kingdom have been falling over the twentieth century. However, there is a lack of data to derive mortality rates by single year of age at ages 85 and over (or 90 and over in more recent years of the last century) to analyze the rates of reduction at these ages. Data are available from the historic English and Scottish life tables. However, mortality rates derived at the oldest ages in these life tables suffer from unreliability in the estimated numbers at these ages as reported in the census on which the national life tables before the 1980–1982 series are based. The differing methodologies adopted for graduating mortality rates at the oldest ages also make comparisons at these ages between life tables difficult.

The unreliability of census data for ages 90 and over has also meant that official population estimates are not published by individual year of age above age 89. However, the growing numbers aged 90 and over in the U.K. population have meant that reliable estimates by single year of age are required for this age group. The paper describes recent work carried out by the U.K. Government Actuary's Department to construct a database of population estimates and historical mortality rates for the United Kingdom by each year of age from 1961 onwards. Further work is required to check the results and to investigate discontinuities between the estimates derived using the KT methodology and the official population estimates.

Section 7 of the paper shows that mortality rates derived from the population estimates in this database and the registered numbers of deaths by age suggest that mortality is declining at old ages up to age 100 for males and females. The picture beyond these ages is not clear, partly because the numbers of deaths at these ages are still very small; what evidence there is suggests that mortality rates beyond age 100 are not declining by any significant amount. Data from the insured populations covered by the CMIB investigations also display rather erratic movements in mortality rates at old ages (of course, the CMIB data are rather lower in number than for the population as a whole and suffer from problems such as late reporting of deaths).

The recent and continuing large increases in the numbers of the elderly and very elderly in the United Kingdom mean that there is much more interest in the projections of their numbers. The interest of the U.K. insurance regulator in the stochastic evaluation of risk, including mortality risk, and recent discussions of the effects of increasing longevity on the future and types of pension schemes provided in the United Kingdom also emphasize the importance of being able to measure and project mortality rates at the oldest ages. Given the difficulties in calculating current and historical rates of mortality at the oldest ages, the projection of these mortality rates is particularly problematical, and measures of the uncertainty of the projections should be provided, where possible, both in the national population projections and in the CMIB projections of mortality rates for insured populations.

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## Appendix A

## Published Population Estimates for the Constituent Countries of the United Kingdom

### England and Wales combined; Scotland

| Period | Published data on population estimates |
|---|---|
| Pre-1959 | Prior to 1959, population estimates usually published in five-year age groups, except for the data provided by a census |
| 1959–1970 | Population estimates available by sex and single year of age up to 84, together with a totals figure for age 85 and over |
| 1971 onwards | Population estimates available by sex and single year of age up to 89, together with a totals figure for age 90 and over |

### Northern Ireland

| Period | Published data on population estimates |
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| Pre-1959 | Prior to 1959, population estimates usually published in five-year age groups, except for the data provided by a census |
| 1959–1970 | Unpublished records are available with individual age data, often by single year of age, for people up to 95 and then by aggregate total for the 95-plus. Data for 1966 are available only in five-year age bands. |
| 1971–1980 | Population estimates available by sex and single year of age up to 89, together with a totals figure for age 90 and over |
| 1981–1989 | Population estimates available by sex and single year of age up to 84, together with a totals figure for age 85 and over[4] |
| 1990 onwards | Population estimates available by sex and single year of age up to 89, together with a totals figure for age 90 and over |

[4] Previously data by single year of age for the period 1981–1990 were available up to the age of 89, but when revisions were made to the Northern Ireland population estimates for these years, revised figures were produced by age only for people under 85.

**Appendix B**

**Methodologies Adopted for Graduating English Tables during the Twentieth Century and for Extending the Graduations to the Oldest Ages**

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| ELT No. 7 (1901–1910) | The first English life tables produced in the twentieth century, ELT No. 7 were based on data for 1901–1910. The main criteria adopted in deciding on a method for graduating the data for ELT No. 7 were that the method should be simple in theory, easy in application, and produce curves of smooth graduation and curves that adhere closely to the original data, with the last being the most important desideratum. The report suggests that the previous ELT Nos. 1–6 showed mortality rates that were felt to be too low at the oldest ages. The report discusses the existence of exaggeration and misreporting of age, both in the censuses and at death. It was noted that if the same misstatements appeared in the same proportions in both the census and in registering deaths, and if on both occasions the same misstatements occurred, the result is to understate the rate of mortality. However, the extent of age misstatements was not known, and no correction was attempted.<br><br>ELT No. 7 used census data for 1901 and 1911 to derive the exposed-to-risk by age and sex over the period 1901–1910. Data from the censuses were provided in quinquennial age groups up to 99 and a further group of 100 and over. The mean populations were then derived in five-year age groups up to 100, with a final group of 100 and over. This latter group was then divided into 100–104 and 105 in the same proportions as existed among centenarians enumerated in the 1911 census. Deaths data for the period were derived by apportioning the deaths in various age groupings according to the distribution of the deaths in the three years 1910–1912, to obtain numbers of deaths in the same age groupings as the population figures. Graduated quinquennial pivotal values for the population and the deaths respectively were then derived for each age group 5–9 to 100–104. Values of the central death rate, $m_x$, were obtained by dividing |

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| | the pivotal value of the number of deaths by the pivotal value of the population for ages 12, 17, to 97. These values were then converted to rates of mortality using the formula $q_x = 2m_x / (2 + m_x)$, where $q_x$ is the probability that someone aged $x$ will die before reaching age $x$+1.<br><br>The intervening values of the rates of mortality for ages 17–92 were then obtained by osculatory interpolation between transforms of the quinquennial values of $q_x$. This method of graduation became known as King's method, named after the actuary George King, who proposed the methodology and who carried out the preparation of the life tables. The method was used to prepare several subsequent English life tables.<br><br>Values of log $p_x$ (where $p_x = 1-q_x$) for ages 89, 90, 91, 92 and 97 used to obtain fourth differences. The life table was completed from age 93 upwards by summation of these differences. |
| ELT No. 8 (1910–1912) | Similar method to that used for ELT No. 7 |
| ELT No. 9 (1920–1922) | At the oldest ages the previous method of extrapolation was found to produce decreasing values of $q_x$ above age 100. A Gompertz graduation was used to obtain values of $q_{85}$ upwards. Examination of the data suggested that $\log_{10} p_{89}/\log_{10} p_{84}$ was approximately equal to the value of $\log_{10} p_{94}/\log_{10} p_{89}$. The tenth root of $\log_{10} p_{94}/\log_{10} p_{84}$ was applied to $\log_{10} p_{84}$ to give values for $q_{85}$ upwards. The values obtained by this method were adopted for all ages above 84. |
| ELT No. 10 (1930–1932) | Gompertz extrapolation was used to produce values of $q_{87}$ and older ages, based on an examination of the values of colog $p_{x+5}$/colog $p_x$, which gave a good agreement of the actual and expected numbers of deaths. The resulting mortality rates appeared to be heavy for ages 95–99 compared to the actual number of deaths recorded. However, the numbers of deaths recorded at these ages appeared small compared to numbers at ages 90–94, especially for males. |

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| ELT No. 11 (1950–1952) | The report on the English life tables commented that King's method automatically produces, over each five-year age group, a very close agreement between actual and expected deaths. This is obtained by retaining features that a more powerful method of graduation would obliterate as of no real significance. Also, the method does not graduate crude death rates since ungraduated rates are not calculated. Since population numbers and deaths are graduated separately, the method is also susceptible to features that affect population numbers but not the mortality rates, such as fluctuations in numbers of births. Pivotal values at oldest ages are likely to be unreliable, and, in previous ELTs, the graduated values for oldest ages was carried out by ad hoc methods that bore no relation to the main method used for graduating the mortality rates.<br><br>Other graduation methods were considered, and it was found that for males from age 21 and for females from age 27 national mortality rates could be represented very closely by a mathematical formula of similar form for each sex. These curves were consisted of a combination of a logistic curve and a normal curve, of the form<br><br>$m_x = a + b/(1 + e^{-\alpha(x-k))}) + c \cdot e^{-\beta(x-n)^2}$.<br><br>The logistic part of the curve contributed by far the greater amount to the values of $m_x$ at the oldest ages.<br><br>The graduated mortality rates gave expected deaths for men aged 95 and over that were 10 percent higher than the actual number. The crude rates at these ages seemed to be low compared to those in the 90–94 age group, a feature that was apparent in the 1930–1932 data, and it was felt that this was due primarily to misstatements of age. |
| ELT No. 12 (1960–1962) | ELT No. 12 was also graduated by fitting a mathematical curve to the data. |
| ELT No. 13 (1970–1972) | The fitting of a mathematical curve to the ungraduated data proved problematic. Graduation by cubic splines |

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| | for ages 2–95 was adopted for each sex. The graduations at older ages were completed by extrapolation, assuming a limiting age of 110, although no details as to methodology are given. |
| ELT No. 14 (1980–1982) | ELT No. 14 were the first English life tables to use an exposed-to-risk calculated from the midyear population estimates rather than the enumerated census population adjusted by data on registered deaths. Graduation by cubic splines was again adopted. At ages over 95 the underlying data were felt to be suspect, and the crude death rates formed an erratic set of values. For males over age 92, the central death rates were extrapolated using a cubic polynomial determined by requiring it to have the same value and first and second derivatives at age 92 as the quadratic defined by the graduated values of $m_{90}$, $m_{91}$ and $m_{92}$ and a somewhat arbitrary value of $m_x$ of 0.75 at age 105 taken from data collected for an investigation of the mortality of centenarians covering the years 1950–1979.[5] For females, the extrapolation was carried out in a similar manner, but over the age range 93 to 95 with a value for $m_{105}$ of 0.65. The value of $q_{112}$ was taken to equal 1 for both men and women. |
| ELT No. 15 (1990–1992) | The exposed to risk for ELT No. 15 was determined from the midyear population estimates for 1990, 1991 and 1992 for ages up to 89. However, these estimates were not provided at individual ages over 90. The age distribution in the 1991 census for ages over 90 was also felt to be unreliable. A method based on survival rates was used to estimate age distributions for those aged 90 and over for each year 1990, 1991 and 1992. These age distributions were then applied to the official population estimates totals of those aged 90 and over for the relevant year to obtain revised numbers at each age. It was found that the method used gave population estimates at ages just under 90 that were close to the official estimates.<br><br>The method of reverse survival rates used involved |

5 See Thatcher (1981) and Editorial (1984).

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| | first calculating, for each age $x$ for each calendar year up to and including 1993, the ratio of the numbers of deaths at ages ($x$−2) to ($x$+2) in the calendar year to the number of deaths aged ($x$−3) to ($x$+1) in the previous year. These ratios were then extrapolated into future years. The estimated numbers of deaths at each age $x$ in the years from 1994 onwards were then obtained by applying the appropriate projected ratio of deaths at age $x$ in the year to the number of deaths at age ($x$−1) in the preceding calendar year. The population numbers at each age 90 and over in 1990, 1991 and 1992 were then obtained by summing the deaths backwards along a cohort from the future year in which the deaths for the cohort were projected to be negligible.<br><br>The resulting crude death rates were graduated using natural cubic splines, with knots at each of the data points. The method could be extended to the highest ages as the values at these ages have little effect on the graduation at younger ages. However, a cut-off point was taken at those ages beyond which the crude death rates ceased an upward progression, otherwise the graduating spline may turn downwards. As a result, the maximum age was chosen to be 102 for males and 104 for females.<br><br>The method used in ELT No. 14 for extrapolating mortality rates for the oldest was not adopted since it was found that the resulting extrapolated values depended critically on the arbitrarily chosen fixed values and the initial conditions. Experiments using straight-line extrapolations of $m_x$ (or implied $q_x$), and transforms of these rates suggested that the most satisfactory results, judged on the basis of age-to-age progression of the mortality rates and the relationship between the extrapolated male and female rates, were obtained by using a regression line fitted to the values of the function $\ln(q_x/(1-q_x))$ from ages 85–103 for males (104 for females) and then extrapolating beyond. The extrapolated values of $\ln(q_x/(1-q_x))$ were converted back to give values of $q_x$ directly. An arbitrary limiting age of 121 was chosen for both males and females for the resulting life tables. |