ILA LAM Model Solutions
Spring 2021

1. Learning Objectives:

1. The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

Learning Outcomes:

(1a) With respect to stochastic models:

- Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
- Describe and apply the theory and uses of real world versus risk neutral assumptions.
- Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
- Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
- Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
- Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.

Sources:

Common Misunderstandings of Risk-Neutral Valuation, Stroman, Financial Reporter, SoA, June 2019

Stochastic Modeling is on the Rise, Product Matters, Nov 2016

Stochastic Analysis of Long-Term Multiple-Decrement Contracts, 2008 (excluding Attachments)

LAM-135-19: Stochastic Modeling, Theory and Reality from and Actuarial Perspective, sections I.A, I.B-I.B.3.a, I.B.4 & I.D-I.D.3

LAM -139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019; Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a GMAB)

1. Continued

Commentary on Question:
Candidates generally performed well in this question.

Stronger candidates were able to provide thorough critique and analysis to demonstrate their understanding on the topics. Some candidates failed to comment on whether the statement (part a) or the parameters (part c) is right or wrong. Credit is deducted in these cases.

Note – Part (a) statements A & D and Part (b) were determined to focus on information (Stat/GAAP standards) that were not a core part of the syllabus, so they were excluded from grading. Points for these sections were reallocated to the other components of question 1.

Solution:
(a) Critique the following proposals or statements. Justify your answer.

A. *VM-20 AAA stochastic log volatility scenarios are good for setting the outer loop of the nested stochastic framework.*

B. *The outer loop's scenarios are retrospective and should be strictly calibrated to historical data to produce realistic paths for economic variables.*

C. *The expected 1-year risk-free rates listed below suggest the nested scenarios are properly calibrated*:

Projection Year	**1**	**2**	**3**	**4**	**5**
Outer Loop	3%	3%	4%	3%	3%
Inner Loop	2%	1%	2%	2%	2%

D. *The new Stat and GAAP standards create increased needs in hedging, especially for GMIB riders*.

E. *If a GMAB rider is added to this product, the nested stochastic framework needs to be redesigned*.

F. *CTE based measurements are far superior to VaR based measurements for stochastic modeling.*

1. Continued

Commentary on Question:
Most candidates performed well in this part.

For sub-part E, some candidates were not able to demonstrate their understanding on GMAB rider.
For sub-part F, some candidates were able to distinguish the difference between VaR and CTE but were unable to critique that one is not far superior to the other. Partial credit is given in these cases.

A. This subsection could not be answered with information provided so it was excluded from grading.

B. Partially True.
The goal of the outer loop (real world scenarios) is to provide basis for future expectations/outcomes.
The past is often viewed as a good basis for future expectations. However, it's more reasonable to calibrate the model strategically based on history, as well as incorporating some judgement-based considerations about future market movements, reasonability of certain scenarios and relationship between assumed parameters and results.

C. False. The scenarios are not properly calibrated.
The outer loop utilizes real-world scenarios and the inter loop utilizes risk-neutral scenarios.
The expected future path of the short-term risk-free rate is lower in real-world (outer loop) scenarios than in risk-neutral (inner loop) scenarios due to term premiums in the risk-neutral scenarios.

D. This subsection could not be answered with information provided so it was excluded from grading.

E. False.
GMAB provides a floor on the AV for a certain period and can be modeled as a European put option. The simplest way for the insurer to hedge its exposure is to purchase a similar option from an investment bank.
It does not demand the need for dynamic replication hedging and stochastic-on-stochastic projections so the nested framework does not need to be updated for this additional rider.

1. Continued

F. False.
VaR and CTE measures have their own pros and cons, and one is not superior to the other.
CTE measures may be more sensitive to severe low-frequency loss scenarios than VaR. However, CTE measures for extreme loss scenarios may be tempered by the remaining tail.
VaR is a point estimate and does not provide additional information beyond this. However, VaR is widely used and can be easily communicated and understood by management.

(b) The stochastic simulation was performed twice, once without hedging and once with hedging. Resulting worst Greatest Present Value of Accumulated Deficiencies (GPVAD) are provided below:

GPVAD	Without Hedging	With Hedging
CTE 70	480	350
CTE 98	920	650

(i) Calculate the capital required, ignoring the deterministic reserve floor. Show all work.

(ii) Calculate the total asset requirement (TAR) under the statutory view assuming a hedge effectiveness ratio of 80%. Show all work.

Commentary on Question:
Part (b) could not be answered with information provided so it was excluded from grading.

1. Continued

(c) Your company is considering acquiring a block of Term Life business and wishes to assess the mortality risk through a stochastic valuation. You are given:

	Present value of future cash flow deviations from the best estimate scenario		
Percentile	**Underwriting**	**Volatility**	**Catastrophe**
95%	-280	-59	-431
90%	-250	-43	-320
75%	-120	-20	-152
25%	186	32	101
10%	210	38	135
5%	267	46	240

Analyze the reasonableness of the initial results presented above, from a stochastic modeling perspective. No calculations are required for this question.

Commentary on Question:
Most candidates were able to identify that parameters for underwriting and volatility are indeed reasonable. Better candidates were able to take note on the fact that catastrophe is a one-sided risk/event.

Candidates who only provided explanation on underwriting and volatility parameters being reasonable would only get partial credits.

Parameters for Underwriting and Volatility appear reasonable while the ones for Catastrophe appear incorrectly set up and need further investigations.

The intended catastrophes modeled reflects sharp increase in mortality for a short period of time and are all one-sided events. When these events occur, they should only have adverse impact on the mortality results and hence, it should only show negative impacts in the worst/high percentile scenarios. There should not be any impacts in the low percentile scenarios contrary to what the data shows.

Volatility impact is the smallest among the three, which is reasonable. Volatility risk oscillates around the best estimate on an annual basis and one would anticipate modest variation around the best estimate. Underwriting impact is approximately 4-5 times of that of volatility using comparable parameters, which is also reasonable.

Other reasonable analysis/responses are acceptable.

2. Learning Objectives:

3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

Learning Outcomes:

(3a) With respect to Asset-Liability Models:

- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  - Major product risks for which ALM can be a useful tool for their management.
  - Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
- Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
- Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
- Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
- Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
- Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

(3b) With respect to asset adequacy analysis and cash flow testing, describe and evaluate actuarial practice with respect to:

- Modeling and selecting assets and related assumptions (incl. modeling assets with contingent cash flow risks).
- Handling liability cash flow contingencies and risks.
- Setting up projection model parameters and assumptions.
- Describe how Interest Rate Forwards and Futures and Swaps can be used in ALM, and apply the mathematics in given situations.

Sources:

LAM-131-19: Life Insurance Accounting, Asset/Liability Management Ch 22

LAM-117-14: Key Rate Durations: Measures of Interest Rate Risk

2. Continued

LAM-118-14: Revisiting the Role of Insurance Company ALM w/in a RM Framework

LAM-140-19: Asset Adequacy Analysis Practice Note, 2004 , questions: 3, 5, 10-16, 18-20, 27, 29-31, 39, 42-60, 65-68, 71-82, 85 & 89

Commentary on Question:
This question tests the candidate's understanding of ALM, specifically the use of effective duration and key rate duration. It also tests the candidate's knowledge of asset adequacy testing (as part of ALM and appropriate methods/assumptions to use).

Solution:

(a) Describe the four main approaches by which a company can address its Asset Liability Management (ALM) needs.

Commentary on Question:
The candidate must give a reasonable description for each item in order to receive full credit; only partial credit is awarded if they only simply list the four approaches.

1. Investment Strategy
The most direct approach to ALM. This approach takes the inforce liability structure as given and aims to manage investment risk by means of a complementary investment strategy.

2. Product Design
Build ALM considerations into product design and pricing by striking a strategic balance between costs, risks and contributions to the overall product design objective. The ALM considerations involve the determination of what degree of investment risk is acceptable in exchange for the benefits of selling a competitive product.

3. Reinsurance
The simplest approach to mitigating investment risk can be through reinsurance. Reinsurance of traditional life insurance products generally encompasses investment risk, insofar as a reinsurer shares in the overall experience on a block of business. There is also a market for reinsurance specific to investment risks.

4. Holism
Holistic technique focuses on risk at the enterprise level, rather than at the product or line-of-business level as has been typical in past ALM practice. Holism qualifies as a separate ALM approach because it seeks to identify and exploit existing or potential synergies in a company's diverse business activities.

2. Continued

(b)

(i) Explain why MRK might want to minimize surplus volatility instead of minimizing asset-only volatility in their ALM practice.

(ii) You are given the following information on MRK Life's balance sheet:

	Market Value	Effective Duration
Assets	1,000	10
Liability	800	15

Assume a -0.5% parallel shift in the interest rate curve. Calculate the change to MRK's surplus. Show all work.

(iii) You are given the following information on two types of interest rate hedging instruments available in the market:

Hedging Instrument	Notional	Effective Duration
Swap 1	100	15
Swap 2	100	-10

Recommend a suitable hedging portfolio using the above swaps to minimize the surplus volatility solved for in part (ii). Show all work.

Commentary on Question:
For full credit, candidate should relate the answer to the given company information (aka MRK has interest sensitive products and thus would choose to minimize liability int rate exposure)

b(i):
Given that MRK Life's insurance liabilities are interest sensitive, minimizing surplus volatility is more advantageous as it aims to manage both assets and liability interest rate exposure. Managing surplus volatility aims to address the duration mismatch from both asset and liability.

b(ii)
(See "ILA LAM Solutions Spring 2021_Q2 Cal.xlsx" - "Part B calc" tab)

MV change = - MV*Duration*Shock
Change to Assets = 1,000*10*(-0.5%) = +50
Change to Liabilities = 800*15*(-0.5%) = +60
Change to Surplus = 50 - 60 = -10

2. Continued

b(iii)
(See "ILA LAM Solutions Spring 2021_Q2 Cal.xlsx" - "Part B calc" tab)

MRK would need a MV of +10 to offset its surplus decrease in part ii.

Change to swap = - Notional*Duration*Shock
Change to swap 1 = 100*15*(-0.5%) = +7.5
Change to swap 2 = 100*15*(-0.5%) = -5
MRK should execute 133.33 notional of swap 1 to offset its surplus decrease
(10/7.5*100 = 133.33)

(c) MRK is considering the following two bond portfolios:

	Market Value of Zero-Coupon Bonds			Effective Duration
	10-year	**20-year**	**30-year**	
Portfolio 1	100	100	400	25
Portfolio 2	0	300	300	25

Interest rates experience the following change:

Rate	Change
10-year	+0.05
20-year	+0.1
30-year	-0.05

Calculate the change in return for each portfolio. Show all work.

Commentary on Question:
Candidate could calculate either the rate of return or the amount of return.

see "ILA LAM Solutions Spring 2021_Q2 Cal.xls" - "Part C calc" tab

Since the bonds are zero coupon, their durations are 10, 20, and 30 (which can also be proven from the given effective duration).
The key rate duration (KRD) for each portfolio = the proportion of MV * duration of bond
The scenario return at each duration = - KRD * change in spot rate

The change in return for each portfolio is the sum of changes of each KRD

2. Continued

Rate of Return:
Portfolio 1 = +0.583
Portfolio 2 = -0.25

Amount of Return:
Portfolio 1 = 350
Portfolio 2 = -150

(d) Critique each of the following statements related to MRK's annual asset adequacy analysis:

A. Asset adequacy analysis is purely a required regulatory exercise and provides no other value to MRK.

B. Either cash flow testing or gross premium valuation would be appropriate to use for MRK's asset adequacy analysis.

C. A projection period of 20 years is adequate for MRK's analysis.

D. The current interest rate environment should not influence MRK's opinion on asset adequacy.

Critique each statement.

Commentary on Question:
For full credit, the candidate should give an explanation; no credit is given for simply stating True or False. Candidate should also relate their answer to the given company's (MRK) situation in order to receive full credit.

A: Asset adequacy analysis results can be valuable to MRK, especially since the business is interest sensitive, as it can inform management of possible problems that could arise due to the underlying characteristics or current management of the business.

B: GPV is appropriate when the liabilities are not interest sensitive, whereas CFT may be more appropriate where cash flows vary significantly under different economic or interest rate scenarios. Given that MRK's liabilities are interest sensitive, MRK should only use CFT and not GPV for their analysis.

C: MRK's business is long duration and therefore a 20-year projection period is likely too short. The analysis should extend long enough to where a longer period would not materially affect the analysis. It is highly likely that a longer projection period would give different results; MRK should consider using a longer projection period.

2. Continued

D: Asset adequacy analysis specifies that reserves must be adequate under "moderately adverse conditions". An actuary's opinion on "moderately adverse" may be different in a low interest rate environment vs a high interest rate environment. In a low-rate environment, certain scenarios such as "pop down" could be viewed as going beyond moderately adverse and perhaps would not bear as much weight in the opinion. This would be especially true for long term business like MRK's, where it might be argued that such a low interest rate scenario projected beyond 20 years is much more than "moderately adverse".

3. Learning Objectives:

2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

Learning Outcomes:

(2h) Describe and evaluate the guidance in the Actuarial Standards of Practice

(2j) Describe and evaluate considerations around the governance of expert judgment in actuarial modelling

Sources:

The Effect of Deflation or High Inflation on the Insurance Industry, 2012 (excluding pp. 11-14)

Modeling – 4th Draft, Actuarial Standards Board, December 2018, Sections 3 & 4

Commentary on Question:

The goal of the question is to assess the candidate's knowledge of the impact of inflation on life insurance products, and their ability to assess how to design appropriate assumptions and methods for given products. The candidate must demonstrate knowledge of the direct and indirect impacts of inflation on insurance products. The candidate must also be able to apply the elements of the ASB draft modeling standard (Actuarial Standard of Practice No. 56) to the review of a given model.

Grading points are allocated equally between parts (a) and (b).

Solution:

(a) Critique the following statements pertaining to the given inflation assumptions:

A. *A regime switching model is used for inflation where the rate of inflation at any point is an autoregressive process, but the dynamics are dictated by the prevailing regime*.

B. *It is reasonable to assume that inflation is the only parameter that changes for each scenario modeled for both products*.

C. *The number of scenarios for the Payout Annuities is reasonable*.

D. *The relationship between inflation and investment returns is reasonable*.

Commentary on Question:

The question presents the modelling processes for a company that sells 2 products: participating whole life insurance and a single premium annuity. The candidate is asked to critique the statements above as they pertain to inflation

3. Continued

A

The core concept of regime switching is that at any point in time, economic variables may be modeled by the dynamics within that regime. However, changes in the economy may build such that the assumed process for financial variables is no longer appropriate. In these cases, the economy is said to switch regimes.

- A regime switching model for inflation rates is one where the expected inflation rate and its volatility have set values in one regime
- The model assigns a probability to switch between regimes, but once inside one of the regimes, the model calculates an inflation level based on the underlying assumptions (e.g., volatility) of that regime.
- An auto-regressive model is a different type of model where the current inflation rate is influenced by the previous level of inflation. Changes are based on transition probabilities
- These are two separate modeling approaches.

B

Inflation is not the only parameter that changes, but it is not enough to just answer yes or no.

- Depending on the prevailing inflation regime, the lapse rates may vary
- Under the deflation regime, we would expect to see lower lapses since the guaranteed interest rate on these products would be stronger when compared to inflation
- In the normal regime we would expect to see normal levels of lapses
- Under the hyper-inflation regime, we would expect to see higher lapses as the higher inflation will erode the investment gains and there are likely to be more attractive products in the market that are willing to offer higher interest rates
- Interest rates can also vary

C

- The single premium annuity model has far fewer scenarios than the whole life model
- Given that the single premium annuity has inflation-indexed payments (75% of annuities), it should at least be the same number of scenarios as the whole life model, or more

D

- It is not reasonable to assume investment return moves closely with inflation rate
- The assumed relationship between inflation and investment returns should be consistent between the two models

3. Continued

- For annuities, the annual inflation rate is set equal to the average CPI rate over the past 2 years.
- This means that the investment return is expected to move in the same direction or magnitude as the inflation rate.
- The relationship may not be reasonable because assets are usually invested relatively long-term to match the long-term liabilities feature

(b) Recommend changes to the modeling processes for both products in order to better manage the interest rate risk.

Commentary on Question:
There are a number of recommendations that could be made to the modelling process, and appropriate credit was awarded for each recommendation, up to the maximum for the question. Therefore it was not necessary to correctly state each recommendation below

- The Company should adopt a single inflation/investment return model. Both products are subject to the same economic environment, so a single model makes sense.
- The Company could consider simply using an autoregression model for inflation. The products sold are fairly simple, so the use of a regime-switching model seems to be overly complex
- Studies have shown that the relationship between inflation and investment returns varies by asset

Par whole life

- For WL, the Company should consider a dynamic lapse formula tied to inflation (lapses are higher when inflation is high)
- The stress testing can be conducted more frequently, say quarterly, to assess the sensitivity.
- The current model was developed two years ago by an external consultant. It is important for the company to have enough knowledge about the model and ensure that the intended purpose of the model is consistent with the company's intent

Annuity

- For the single premium annuity product, it is subject to a larger risk of inflation risk since a majority of policies (75%) is sold with an inflation-indexed payment amount
- Average CPI rate is used but it is often criticized for various biases
- More scenarios should be modeled
- More stress testing should be conducted to measure the impact of extreme scenarios

3. Continued

- The model was developed by in house staff 5 years ago. Periodic model validation and peer review should be done to ensure the model is up to date.
- May also ask for comments from an external consultant, with more knowledge expertise

4. Learning Objectives:

4. The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:

(4a) With respect to Economic Scenario Generators:

- Describe the need for ESGs and explain the structure of ESG models and components.
- Describe and apply basic default free interest rate models, including one-factor continuous time models.
- Assess the propriety of a particular ESG model and related assumptions for particular applications.

(4b) With respect to Equity-Linked models:

- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

Sources:

Investment Guarantees Ch 7 (pg 115-125), Hardy, 2003

Investment Guarantees Ch 8 (pg 133-143), Hardy, 2003

LAM-139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019; Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a GMAB)

Commentary on Question:

Candidates generally did well on part a) of the question but struggled with part b). Candidates who scored better on part b) generally understood that they had to calculate the put option cost at time 1 which was dependent on which scenario happened in the first period and in addition they demonstrated an understanding that the cumulative P&L impact was the mean of the impacts of the possible scenarios

Solution:

(a) Calculate the cost of a replicating portfolio that immunizes market risk over a 1-year period. Assume a no arbitrage market.

Commentary on Question:

Candidates generally did well on this part of the question. Candidates received full credit if they calculated the correct option price either from first principles using the principle of no arbitrage and a risk neutral calculation or by directly recalling the formula for the price of a replicating portfolio at time=0

4. Continued

$$P = \{C_u(1-p^*) + C_d p^*\}e^{-rt} \quad where\ p^* = \frac{S_u - S_0 e^{rt}}{S_u - S_d}$$

See Excel worksheet attached for model solution

(b) Calculate the expected cumulative Profit or Loss if the policyholder surrenders their policy at the end of year 2.

Commentary on Question:
This question tested the candidate's ability to calculate an option price and the ability to analyze the possible outcomes of a set of scenarios on the surrender value of a simple GMSB product. The candidates needed to demonstrate knowledge of the possible payout of the hedging strategy provided by the put options, and the impact of these on the company P&L. To receive full credit, candidates would need calculate the correct put option costs as well as possible hedge payouts and the expected impact of these on the P&L.
Most candidates struggled with this part:

- *Few candidates were able to calculate a correct put option cost at time 1*
- *Many candidates did not recognize that in a negative return scenario, even without a surrender – the put option pays out at time 1 and this payout goes to the P&L*
- *Some candidates failed to differentiate between cashflows that only impacted the Account value and/or payout vs cashflows that impacted the company P&L*
- *Many candidates in the calculation of the put option – misused the volatility in the Black-Scholes formula leading to them being unable to select the correct values from the provided normal distribution*
- *Some candidates wrongly identified some scenarios as being more likely than others*

Candidates were given partial credit for:

- *Recognizing they needed to use the Black-Scholes formula to evaluate the option price*
- *Calculating the correct components that go into the P&L*
- *Showing that the expected loss was the average of the possible scenario outcomes*

$$P_0 = Ge^{-rT}\Phi(-d_2) - S_0(1-m)^T\Phi(-d_1)$$

$$Where\ d_1 = \frac{\log(S_0(1-m)/G) + \left(r + {\sigma^2}/{2}\right)T}{(\sigma\sqrt{T})}$$

See attached Excel sheet for worked solution