GI ADV Model Solutions
Spring 2023

1. Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5e) Calculate the price for a proportional treaty.

(5g) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the expected losses in the layer using an exposure rating approach.

Commentary on Question:

This part required understanding how an underlying limit affects the pricing for an excess layer. ILF = increased limits factor.

For each of the five rows in the table:

- Step 1: The coverage starts at the underlying limit and ends at the underlying limit plus the policy limit.
- Step 2: The reinsured layer starts at 1 million plus the underlying limit and ends at the minimum of 4 million plus the underlying limit and the policy limit plus the underlying limit.
- Step 3: The reinsurance exposure factor is [ILF at reinsurance end point – ILF at reinsurance start point] divided by [ILF at policy end point – ILF at policy start point].
- Step 4: Expected loss in layer is the expected loss ratio times the subject premium times the reinsurance exposure factor.

1. Continued

The sum of the expected loss in layer over the five rows is the total expected loss in layer.

(b) Calculate the expected technical ratio (loss ratio plus commission ratio) for the treaty.

The commission at each loss ratio (LR) is determined by the sliding scale. The commission "slides" from 30% at a 50% LR to 10% at a 90% LR in which the commission reduces by 0.5% for every 1% incremental increase in LR. The expected technical ratio is the sum over all possibilities of the probability of a LR times the technical ratio (LR plus commission ratio) at that LR.

(c) Assess whether the sliding scale commission is balanced.

Commentary on Question:
This may be done by comparing the commission at the expected LR (ELR) and the expected commission. In this scenario, these results are reasonably close so one may conclude that the scale is balanced. Because it is not exactly balanced, an answer stating that it is not balanced was also acceptable.

ELR = 57.2% [= 40% × 15% + 50% × 35% + … + 90% × 2%]
Commission at ELR = 26.4% [using the scale for a 57.2% LR]
Expected commission = 25.7% [= 30% × 15% + 30% × 35% + … + 10% × 2%]

Yes, the scale is balanced because the expected commission is close to the commission at the ELR.

2. Learning Objectives:

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6c) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of the individual risk rating methods of schedule rating and experience rating.

Solution:

(a) Describe how an insured's risk control activities affect each of the following individual risk rating plans:

(i) Schedule rating

(ii) Prospective experience rating

(iii) Retrospective experience rating

(i) The insured receives a credit for the presence of a risk control measure at the time the policy is written.
(ii) Risk control measures must have actually lowered claims from the expected amounts in the historical experience period used for rating to be reflected in the rate.
(iii) Risk control measures must actually lower claims from the expected amounts during the policy period to be reflected in the rate.

(b) Explain why insurers use schedule rating.

To incorporate judgment about specific risk characteristics of the insured that are either not considered at all or are not adequately reflected in the manual rating process.

(c) Describe how the NCCI formula differs from the basic formula.

In the numerator, actual claims are split into primary and excess claims. Actual primary claims are given full credibility while actual excess claims are credibility weighted with expected excess claims. Also, a ballast amount is added to both the numerator and the denominator to limit year-to-year variability.

2. **Continued**

(d) Identify two other characteristics of insureds that would make retrospective experience rating inappropriate.

Commentary on Question:
There are more than two other characteristics. The model solution is an example of a full credit solution.

- Significant fluctuations in premium volume from year to year
- Poor claims experience

3. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (d) to (f). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (d) to (f) is for explanatory purposes only.

Solution:

(a) State two advantages of using the overdispersed Poisson distribution as opposed to the Poisson distribution.

- Can match two moments instead of one.
- Maximum likelihood estimation matches the usual Cape Cod estimate.

(b) Describe a situation where incremental losses may <u>not</u> be independent.

Commentary on Question:

There are two situations provided in the source reading. Only one was required for full credit. The model solution is an example of a full credit solution.

All periods are equally affected by a change in loss inflation.

(c) Describe a situation where incremental losses may <u>not</u> be identically distributed.

There are different risks and mix of business in each period.

(d) Demonstrate that the MLE of *ELR* is 0.5251.

Commentary on Question:

Refer to the Excel solutions spreadsheet.

3. Continued

1. For each row of data provided, calculate *G*(at the end of the interval) and *G*(at the beginning of the interval).
   - The end of the interval is "To" months minus 6.
   - The beginning of the interval is "From" months minus 6 if "From" months >=12 and 0 if "From" months is 0.
2. The MLE of *ELR* is the total of losses divided by the total of subject premium times [*G*(at the end of the interval) minus *G*(at the beginning of the interval)].

(e) Estimate the scale factor, σ^2.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

1. For each row of data provided, calculate the expected increment as subject premium times [*G*(at the end of the interval) minus *G*(at the beginning of the interval)] times the MLE of *ELR*.
2. For each row of data provided, calculate the square of [the increment minus the expected increment] divided by the expected increment.
3. Then, the scale factor equals the sum of the amounts in 2 divided by [the number of data points minus the number of parameters estimated] which is equal to 294.0381.

(f) Estimate the process standard deviation of the loss reserve for all accident years combined.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

The ultimate loss estimate for all accident years (AYs) combined is the total subject premium times the MLE of *ELR*. The loss reserve estimate for all AYs combined is the ultimate loss estimate for all AYs combined minus the sum of incremental payments. The estimate of the process standard deviation of the loss reserve for all AYs combined is the square root of [loss reserve estimate for all AYs combined times the estimate of the scale factor] which is equal to 1,569.93.

4. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question required the candidate to respond in Excel for parts (c) to (h). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (c) to (h) is for explanatory purposes only.

Solution:

(a) State whether or not this implies that the errors in reserve estimates for different accident periods are independent. Justify your answer.

No. This is because the estimators for the different periods are all influenced by the same development factors, so the errors are positively correlated.

(b) State the other two statistical assumptions underlying the chain ladder model.

- The conditional expected accumulated total claim amount at a given development period is the accumulated total claim amount at the previous development period times a development factor that does not vary by accident period.
- The conditional variance of the accumulated total claim amount at a given development period is the accumulated total claim amount at the previous development period times a proportionality constant that does not vary by accident period.

4. Continued

(c) Demonstrate that the standard error for the first half of 2021 has been correctly calculated.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

For the following, let j = accident period (1st Half 2019 is 1, 2nd Half 2019 is 2, 1st Half 2020 is 3, …) and k = development period. Then to square the development triangle, the incremental payments are calculated as $c_{j,k} = c_{j,k-1} \times f_{k-1}$.

$$SE_{2021.1} = \sqrt{c_{5,8}^2\left(\frac{\alpha_4^2}{f_4^2}\left(\frac{1}{c_{5,4}}+\frac{1}{c_{1,4}+c_{2,4}+c_{3,4}+c_{4,4}}\right)+\frac{\alpha_5^2}{f_5^2}\left(\frac{1}{c_{5,5}}+\frac{1}{c_{1,5}+c_{2,5}+c_{3,5}}\right)+\frac{\alpha_6^2}{f_6^2}\left(\frac{1}{c_{5,6}}+\frac{1}{c_{1,6}+c_{2,6}}\right)+\frac{\alpha_7^2}{f_7^2}\left(\frac{1}{c_{5,7}}+\frac{1}{c_{1,7}}\right)\right)} = 4{,}862$$

(d) Calculate the standard error for the full year of 2021.

Commentary on Question:
Refer to the Excel solutions spreadsheet.

$$SE_{2021} = \sqrt{SE_{2021.1}^2 + SE_{2021.2}^2 + 2c_{5,8}c_{6,8}\left(\frac{\left(\alpha_4^2/f_4^2\right)}{\left(c_{1,4}+c_{2,4}+c_{3,4}+c_{4,4}\right)}+\frac{\left(\alpha_5^2/f_5^2\right)}{\left(c_{1,5}+c_{2,5}+c_{3,5}\right)}+\frac{\left(\alpha_6^2/f_6^2\right)}{\left(c_{1,6}+c_{2,6}\right)}+\frac{\left(\alpha_7^2/f_7^2\right)}{\left(c_{1,7}\right)}\right)} = 8{,}773$$

(e) Describe Mack's nonparametric test for correlations between development factors.

Rank the development factors in each column. Calculate Spearman's rank correlation coefficient for each pair of adjacent columns. The test statistic is a weighted average of those coefficients, which is compared to the distribution of the test statistic under the null hypothesis of no correlation.

(f) Describe the adjustment that Venter suggests to correct for correlation between adjacent development factors.

Add the covariance to the product of adjacent development factors.

4. **Continued**

(g) Describe Mack’s nonparametric test for calendar year effects.

Identify within each column of development factors those with rank larger than the mean rank and those with rank smaller than the mean rank. For each diagonal, record the number of ranks larger than the mean rank or the number of ranks smaller than the mean rank, whichever is smaller. The test statistic is the sum of these values over all diagonals, which is compared to the distribution of the test statistic under the null hypothesis of no calendar year effects.

(h) Describe a model that Venter suggests could account for calendar year effects.

Use a multiplicative model with factors for accident period, development period and diagonal.

5. Learning Objectives:

7. The candidate will understand the application of game theory to the allocation of risk loads.

Learning Outcomes:

(7a) Allocate a risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Note that the scenarios, U to Z, are not independent.
L = Loss, p = Probability, Var = Variance, Cov = Covariance,
MV = Marginal Var, RL = Risk Load

Solution:

(a) Calculate the renewal risk load for each treaty using the Marginal Variance method.

For each treaty (P, Q, and R), and the total of all combined, calculate:

- $E(L)$ = sum over scenarios: $p \times L$
- $E(L^2)$ = sum over scenarios: $p \times L^2$
- $Var(L) = E(L^2) - E(L)^2$

For each treaty combination, (P+Q, P+R, Q+R), calculate:

- $E(L)$, $E(L^2)$ and $Var(L)$
- Covariance = (Var – Var for the two treaties)/2

For each treaty (P, Q, and R):

- MV = Var for the total – Var for the other two treaties combined
- Renewal RL = $\lambda \times$ MV

(b) Calculate the renewal risk load for each treaty using the Shapley method.

For each treaty combination, (P+Q, P+R, Q+R), calculate:

- $E(L)$, $E(L^2)$ and $Var(L)$, Covariance

For treaty P, Shapley Value = Var for P + Cov for P+Q + Cov for P+R
For treaty Q, Shapley Value = Var for Q + Cov for P+Q + Cov for Q+R
For treaty R, Shapley Value = Var for R + Cov for P+R + Cov for Q+R
For each treaty (P, Q, and R): Renewal RL = $\lambda \times$ Shapley Value

5. Continued

(c) Explain how the risk loads calculated using the Covariance Share method would differ from those using the Shapley method.

The Shapley method allocates the covariance equally between the accounts while the Covariance Share method allocates the covariance in proportion to the loss size.

6. **Learning Objectives:**

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4b) Calculate the expected value premium for increased limits coverage and excess of loss coverage.

(4c) Explain and calculate the effect of economic and social inflationary trends on first dollar and excess of loss coverages.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

CDF = cumulative distribution function

Solution:

(a) Calculate the expected payment per loss for a policy with a limit of 50,000 using both the size method and the layer method.

First, calculate the CDF for the loss distribution.

Size method: sum of loss times probability for loss amounts from 1,000 to 50,000 plus 50,000 times (1 minus the CDF at 50,000) = 20,031

Layer method: 50,000 minus 1,000 times (sum of the CDF from 1,000 to 49,000) = 20,031

(b) Calculate the expected payment per loss for the layer from 50,000 to 100,000 using both the size method and the layer method.

Size method: sum of loss times probability for loss amounts from 51,000 to 100,000 plus 100,000 times (1 minus the CDF at 100,000) minus 50,000 times (1 minus the CDF at 50,000) = 7,517

Layer method: 50,000 minus 1,000 times (sum of the CDF from 50,000 to 99,000) = 7,517

6. **Continued**

(c) Calculate the new expected payment per loss for a policy with a limit of 50,000 using <u>either</u> the size method or the layer method.

Commentary on Question:
Either method could have been used. The model solution provided uses the size method.

Note: 50,000 / 1.25 = 40,000

Size method:
1.25 times (sum of loss times probability for loss amounts from 1,000 to 40,000) plus 50,000 times (1 minus the CDF at 40,000) = 22,314

(d) Calculate the new expected payment per loss for the layer from 50,000 to 100,000 using <u>either</u> the size method or the layer method.

Commentary on Question:
Either method could have been used. The model solution provided uses the size method.

Note: 50,000 / 1.25 = 40,000 and 100,000 / 1.25 = 80,000

Size method:
1.25 times (sum of loss times probability for loss amounts from 41,000 to 80,000) plus 100,000 times (1 minus the CDF at 80,000) minus 50,000 times (1 minus the CDF at 40,000) = 8,949

(e) Calculate the trend factor for the policy with a limit of 50,000 <u>and</u> the trend factor for the layer from 50,000 to 100,000.

Limit 50,000: (answer from (c) divided by answer from (a)) = 1.1140
Layer: (answer from (d) divided by answer from (b)) = 1.1905

(f) Calculate the increased limit factor for 100,000 with a basic limit of 50,000, both before <u>and</u> after trend. Explain any difference.

Before: (answer from (a) plus answer from (b)) / answer from (a) = 1.3753
After: (answer from (c) plus answer from (d)) / answer from (c) = 1.4011

The increase in the increased limit factor is due to the leveraged effect of inflation.

7. Learning Objectives:

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4g) Estimate the premium asset for retrospectively rated policies for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Discussion of Estimating the Premium Asset on Retrospectively Rated Policies, Feldblum

Commentary on Question:

This question required the candidate to respond in Excel for part (d). An example of a full credit solution for this part is in the Excel solutions spreadsheet. The solution in this file for part (d) is for explanatory purposes only.

Solution:

(a) Describe what each of *A*, *B*, and *x* represent.

A is (Premium with no incurred losses / Standard premium) minus one.
B is the slope factor relating premium changes with loss changes.
x is the standard loss ratio.

(b) Describe how the PDLD method differs from Fitzgibbon's method with respect to the function relating retrospective premium to losses incurred.

The PDLD method assumes the function is a set of line segments of decreasing slope for increasing incurred losses.

(c) Describe the two methods for calculating PDLD ratios.

- Use the retrospective rating parameters to derive them.
- Use historical booked premium and reported loss development to estimate them.

(d) Calculate the premium asset on retrospectively rated policies as of December 31, 2022 arising from policy years 2020 and 2021 using the PDLD method.

Commentary on Question:

CPDLD = Cumulative PDLD, PLE = Percentage of Loss Emerged Since Prior Evaluation, ELE = Expected Loss Emergence after Last Completed Retrospective Adjustment, PY = Policy Year

7. Continued

For each PY calculate:

- Cumulative PDLD (CPDLD) ratios as follows: sum (PDLD ratio times PLE) from the next adjustment for the PY to the last adjustment divided by sum of PLE from the next adjustment for the PY to the last
  - For example, the PY 2020 CPDLD ratio = sum (PDLD ratio times PLE) from 2nd to 4th adjustment divided by sum of PLE from 2nd to 4th adjustment.
- Expected future premium = CPDLD ratio times ELE
- Estimated total premium = Premium booked from prior retrospective adjustment plus expected future premium

Premium asset = sum over PYs estimated total premium minus sum over PYs premium booked as of year-end 2022.

8. **Learning Objectives:**

3. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(3a) Describe a risk margin analysis framework.

(3b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(3c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel for part (c). An example of a full credit solution for this part is in the Excel solutions spreadsheet. The solution in this file for part (c) is for explanatory purposes only.

Solution:

(a) Describe two of these components, other than the three listed above.

Commentary on Question:

There are many components. Only two were required for full credit. The model solution is an example of a full credit solution.

- Portfolio preparation: Determine valuation portfolio, claim groups and techniques to deploy for each claim group.
- Analysis of correlation effects: Select correlation coefficients between valuation classes and between outstanding claim and premium liabilities for internal systemic risk and for each external systemic risk category.

(b) Identify four subjective decisions that are required in this approach.

- risk indicators
- measurement and scoring criteria
- importance afforded to each risk indicator
- CoVs that map to each score from the balanced scorecard

8. **Continued**

(c) Calculate the following:

(i) Total independent risk CoV for both valuation classes combined (***X***)

(ii) Correlation between the valuation classes for outstanding claims for internal systemic risk

(iii) Internal systemic risk CoV for premium liabilities for both valuation classes combined (***Y***)

(iv) Total external systemic risk CoV for both valuation classes combined (***Z***)

Commentary on Question:
OC = Outstanding Claims, PL = Premium Liabilities, VC = Valuation Class,
AUT = Auto VC, LIA= Liability VC
IND = Independent Risk, ISR = Internal Systemic Risk,
ESR = External Systemic Risk

(i) ***X*** = square root of the sum of the squares (over VC and type of liability) of the % of total liabilities × independent risk CoV

(ii) Correlation between the VCs for OC for ISR = [(square of total ISR CoV for OC × % of OC in total liabilities) − (sum over VC of the squares of OC ISR CoV × % of OC in total liabilities)] ÷ [2 × (OC ISR CoV for AUT × % AUT OC in total liabilities) × (OC ISR CoV for LIA × % LIA OC in total liabilities)]

(iii) ***Y*** = square root of [sum over VC of the squares of (ISR CoV for PL × % PL in total liabilities) + 2 × (ISR CoV for AUT PL × % AUT PL in total liabilities) × (ISR CoV for LIA PL × % LIA PL in total liabilities) × amount from part (ii)] ÷ % PL in total liabilities

(iv) ***Z*** = square root of [square of (ESR CoV for AUT × % AUT in total liabilities) + square of (ESR CoV for LIA × % LIA in total liabilities) + 2 × ESR correlation × (AUT ESR CoV × % AUT in total liabilities) × (LIA ESR CoV times % LIA in total liabilities)]

9. Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5a) Understand the types of reinsurance and key reinsurance terms.

(5k) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) and (c). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (b) and (c) is for explanatory purposes only.

Solution:

(a) Explain why the risk transfer in this reinsurance contract would not be categorized as "reasonably self-evident" to permit reinsurance accounting.

Commentary on Question:

The model solution is an example of a full credit solution.

Aggregate excess of loss reinsurance is not one of the types of insurance usually categorized as reasonably self-evident. Furthermore, this contract has characteristics that limit risk transfer such as dictating the timing of payments at a specific date in a lump sum.

(b) Determine whether or not this reinsurance contract transfers sufficient risk to permit reinsurance accounting using the Expected Reinsurer Deficit (ERD) test with a threshold of 1%.

Commentary on Question:

M = Million

For each claim amount:

- Reinsurance before loss participation = minimum of [100M and maximum of (0 and (claim amount – 150M))]
- Cedant's loss participation = 65% × maximum of (0 and (reinsurance before loss participation – 48M))

9. **Continued**

- Reinsurance after loss participation = reinsurance before loss participation − cedant's loss participation
- Reinsurer's net economic loss = maximum of [0 and −1× (48M − reinsurance after loss participation × ($1.04^{-3.5}$))]

p = probability of reinsurer having a net economic loss = 6%
T = average reinsurer net economic loss when one occurs = sum of probability × reinsurer's net economic loss ÷ p = 7.83M
P = 48M

ERD = p × T ÷ P = 6% × 7.83M ÷ 48M = 0.98%
The ERD is less than the threshold of 1% so it fails the risk transfer test. It does not transfer sufficient risk to permit reinsurance accounting.

(c) Reinsurance accounting may be applicable even if the risk transfer in this reinsurance contract is not categorized as "reasonably self-evident" and the contract does not meet the conditions for risk transfer from a quantitative test.

Describe when this may apply.

Commentary on Question:
The model solution is an example of a full credit solution.

This can apply when the reinsurer has assumed substantially all risk from the cedant.

10. Learning Objectives:

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the total IBNR for claims excess of 400,000 as of December 31, 2022 using each of the following approaches:

(i) Development factors calculated using a simple average

(ii) Theoretically-derived development factors based on Siewert's formula

(i) Development factors calculated using a simple average

- Create development triangle of reported claims excess of 400,000 by subtracting the development triangle of reported claims at 400,000 limit from the development triangle of reported claims at total limits.
- Create a triangle of age-to age development factors with the created triangle.
- Average age-to-age development factors and then calculate cumulative development factors (CDFs) with them.
- Calculate IBNR with the CDFs minus one times the reported excess claims on the diagonal of the triangle.

(ii) Theoretically-derived development factors based on Siewert's formula

- Create a triangle of age-to age development factors with the development triangle of reported claims at total limits.
- Select age-to-age development factors and then calculate CDFs with them.
- Calculate the CDFs for claims excess of 400,000 using the formula $CDF_t \times (1 - R_{72}) \div (1 - R_t)$ for t = 12 to 72.
- Calculate IBNR with the CDFs minus one times the reported excess claims on the diagonal of the triangle.

10. Continued

(b) Describe two considerations in the calculation of R_t values.

Commentary on Question:
There are more than two considerations. The model solution is an example of a full credit solution.

- The different trend rates that are associated with claims at differing limits
- Whether to use actual historical data, industry data, or a combination

(c) Explain why alternative methods should be considered based on the results from part (a).

Commentary on Question:
The model solution is an example of a full credit solution.

There are large differences in the estimated IBNR between the two methods.

(d) Identify two considerations when applying the increased limits factors approach.

Commentary on Question:
There are more than two considerations. The model solution is an example of a full credit solution.

- Treatment of ALAE
- Whether the factors are applicable to claims or premiums

11. **Learning Objectives:**
6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:
(6b) Develop rates for claims made contracts.

Sources:
Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:
This question required the candidate to respond in Excel for part (b). An example of a full credit solution for this part is in the Excel solutions spreadsheet. The solution in this file for part (b) is for explanatory purposes only.

Solution:
(a) Describe the following terms with respect to claims-made insurance:

(i) Step factor

(ii) Tail policy

(iii) Tail factor

Commentary on Question:
The model solution is an example of a full credit solution.

(i) Factor used to determine claims-made rate for non-mature claims-made policy and is expressed as a relativity to the mature claims-made rate.

(ii) Policy that covers claims that are reported after a claims-made policy has expired provided the claim arises from an incident that occurred during the period for which claims-made coverage was in effect.

(iii) Factor used to determine a tail policy rate dependent on number of years for which claims-made coverage was purchased and is expressed relative to the rate at a given claims-made maturity.

(b) Calculate tail factors for a claims-made policy for the following maturities:

(i) First year

(ii) Third year

(iii) Mature

11. Continued

Commentary on Question:
AY = accident year, RY = report year,
$C_{i,j}$ = claims incurred (%) for AY lag *i* that are reported in RY *j*

(i) First year
- Create an accident year lag by report year matrix filling in the $C_{i,j}$ values on the diagonal with the reporting pattern provided.
- Tail factor is calculated as the *sum of $C_{i,j}$ for RYs 2 to 5 divided by RY1 $C_{i,j}$*.

(ii) Third year
- Create an accident year lag by report year matrix filling in the $C_{i,j}$ values for the diagonal and 1 and 2 years below the diagonal. Each year is detrended by 5%.
- Tail factor is calculated as the *sum of $C_{i,j}$ for RYs 2 to 5 divided by the sum of the $C_{i,j}$ for RY1*.

(iii) Mature
- Create an accident year lag by report year matrix filling in the $C_{i,j}$ values for the diagonal and AY lags below the diagonal. Each year is detrended by 5%.
- Tail factor is calculated as the *sum of $C_{i,j}$ for RYs 2 to 5 divided by the sum of the $C_{i,j}$ for RY1*.

12. **Learning Objectives:**
6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:
(6a) Price for deductible options and increased limits.

Sources:
Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland
- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:
This question required the candidate to respond in Excel for parts (b) to (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (b) to (d) is for explanatory purposes only.

Solution:
(a) Describe two issues that should be investigated with respect to the industry data used in this analysis.

Commentary on Question:
There are more than two issues. The model solution is an example of a full credit solution.

- Is the industry distribution by territory, policy limit or type of business comparable to the business the company will be writing?
- Has any trend been applied to the claims data? If it was applied, what were the trending parameters and how were they selected?

(b) Calculate the observed increased limits factors (ILFs) for the following indemnity limits, relative to a basic indemnity limit of 1,000,000:

(i) 1,500,000

(ii) 2,500,000

(iii) 3,500,000

(iv) 5,000,000

- Combine first two rows of data table (so it represents range 0 to 1,000,000)
- For each row, calculate:
  A. Claims in interval = count × indemnity severity in interval × (1 + ALAE % of indemnity)
  B. Cumulative amount of claims from A

12. Continued

C. Claims by limit = cumulative amount from B + limit × sum of counts for all limits greater than the limit for the row + sum of ALAE for all limits greater than the limit for the row
D. Severity by limit = claims by limit from C ÷ total count over all limits
E. ILF = severity by limit from D ÷ severity for 1,000,000 limit from D

(c) Test the consistency of the ILFs calculated in part (b).

Using the ILFs from part (b), calculate the marginal rates as the difference of successive ILFs divided by the corresponding difference in successive limits. ILFs are consistent if the marginal rates are decreasing for increasing limits.

(d) Recommend an ILF for a 2,000,000 indemnity limit. Justify your recommendation.

Using trial and error, a value between 1.068 (ILF at 1.5 million) and 1.135 (ILF at 2.5 million) was selected such that the marginal rates are decreasing. The graph should show a smooth curve increasing at a decreasing rate.

13. **Learning Objectives:**

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5c) Analyze and describe the various types of reinsurance.

(5j) Understand the application of a reinstatement premium.

Sources:

Basics of Reinsurance Pricing, Clark

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 10: A Reinsurance Primer

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the total losses recoverable under each treaty.

Surplus Share (4 lines with 1,000 retained line)

For each property & loss combination:

- Surplus cover = minimum of [4 × 1,000 and maximum of (0 and insured value – 1,000)]
- Recoverable from surplus = loss × (surplus cover / insured value)

Total recoverable from surplus = 6,620

For each loss, retention after surplus = loss – recoverable from surplus share.

Per Risk Excess of Loss (2,000 excess 1,000)

For each loss:

- Recoverable = minimum of [2,000 and maximum of (0 and retention after surplus – 1,000)]

Total recoverable from per risk excess of loss = 2,500

For each loss, retention after surplus and excess = retention after surplus – recoverable from per risk excess of loss.

13. Continued

Catastrophe (6,000 in excess of 4,000)

- Recoverable = minimum of [6,000 and maximum of (0 and sum of retention after surplus and excess for all losses – 4,000)] = 1,080

(b) Calculate the reinstatement premium for the catastrophe treaty.

Reinstatement premium = annual premium × pro-rata provision × recoverable from catastrophe treaty / catastrophe cover = 600 × 125% × 1,080 / 6,000 = 135

(c) Calculate the amount retained by ABC for each claim.

Commentary on Question:
This question included an AAD for a per risk excess of loss reinsurance treaty. There are two readings in the syllabus resources that discuss an AAD: the Clark reinsurance pricing paper and Chapter 10 from the Friedland text. Each reading has a different presentation of an AAD based on a different interpretation. The Clark reading interpreted the AAD as the reinsurer providing coverage after the insured has retained the AAD in the reinsured layer. The Friedland reading interpreted the AAD as the insured's maximum cumulative total retention below the attachment point. Both interpretations were considered acceptable to receive full credit. The model solution includes both interpretations.

Based on Clark, for each claim:

- Retention below attachment = minimum of (2,000 and the claim amount)
- Retention above limit = maximum of (0 and the claim amount – (6,000 + 2,000))
- Reinsurance before AAD = minimum of [6,000 and maximum of (0 and the claim amount – attachment)]
- Cumulative reinsurance before AAD = accumulation of reinsurance before AAD starting with claim 1 going sequentially to claim 5
- Cumulative reinsurance AAD = minimum of (10,000 and cumulative reinsurance before AAD)
- AAD from claim = incremental amounts by claim using cumulative reinsurance AAD
- Retained by ABC = retention below attachment + retention above limit + AAD from claim

Based on Friedland, for each claim:

- Retention below attachment before AAD = minimum of (2,000 and the claim amount)
- Retention above limit = maximum of (0 and the claim amount – (6,000 + 2,000))

13. Continued

- Cumulative retention below attachment before AAD = accumulation of retention below attachment before AAD starting with claim 1 going sequentially to claim 5
- Cumulative retention below attachment capped at AAD = minimum of (10,000 and cumulative retention below attachment before AAD)
- Retention below attachment capped at AAD = incremental amounts by claim using cumulative retention below attachment capped at AAD
- Retained by ABC = retention below attachment capped at AAD + retention above limit