SOA Research INSTITUTE

2017

Mortality and Longevity

Old Age Mortality Experience Study Report

OCTOBER | 2022

Old Age Mortality Experience Study Report

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Introduction

In the study of insured lives mortality rates and the development of industry mortality tables, the **Older Age** mortality (OAM) has been an area of significant judgment. This has led to adjustments in terms of slope and level of the mortality rates for the insurance industry over the past two decades. The most recent study of industry mortality conducted by the Society of Actuaries (SOA) Research Institute's Individual Life Experience Committee (ILEC) contained a significant increase in exposures and death claims, especially at the **Older Ages.** In a dataset as large as the ILEC mortality experience data, it is challenging to efficiently discover important drivers for mortality phenomena using traditional Pivot Table-oriented approaches.

Despite this increase in experience data, it did not do much to shed light into addressing many of the questions regarding the level and slope of mortality at these **Older Ages**. For example, a general tenet of mortality table development is that mortality rates are monotonically increasing as age and duration increase. For the insured mortality experience at ages above attained age 95, this concept appears to no longer hold when observing the raw mortality experience. In addition, the pattern of mortality appeared inconsistent with that of the general population.

To further explore the dynamics of the mortality pattern and observed sharp drop in the OAM rates, especially after age 95, the ILEC formed a subgroup to focus on the OAM (OAM Subgroup) to explore the pattern of mortality rates going forward. In so doing, the OAM Subgroup focused on attained ages 70 and above and, with more advanced modeling approaches and with supplemental population data, tried to further explain **Older Age** mortality and determine whether refinement in the judgments made in the development of the 2015 Valuation Basic Tables (2015 VBT) were needed. These judgments centered around (1) data completeness, (2) the relationship between insured and general population mortality; and (3) cohort impacts based on underwriting/issue year era, type of business, and other changes over time. This led the OAM Subgroup to ask the following questions:

Topic	2015 VBT Judgment for OAM	Questions to Research by OAM Subgroup
(1) Data completeness	The underlying data was credible up to attained age 95. Data beyond attained age 95 was sparse and determined to be of questionable accuracy. This was thought to possibly be a result of under and late reporting of deaths, especially for policies with lower face amounts.	With the additional exposure data at the **Older Ages**, is the sharp decreasing pattern of experience at the **Older Ages** driven by an underreporting of deaths or is it attributable to the uniqueness of the insured population?
(2) Relationship between insured and general population mortality	The actual experience was not given full credibility at the **Older Ages**, which demonstrated a significantly lower mortality rate and A/E than expected; and There was no forced convergence to population mortality, other than an assumed omega rate of 0.5.	Is there support to suggest that the insured mortality exhibits a different pattern of mortality than population, especially at ages above 95, when attempting to normalize the population data for socioeconomic equivalency?
(3) Cohort differences	There was no demonstrable cohort differentiation observed in the underlying data nor in other external consultant studies with similar exposure periods. There was no specific adjustment for the potential impacts of Stranger-Owned Life Insurance (STOLI) business and the potential impact it may have on the experience. There were no adjustments for the more recent trend of **Older Age** specific underwriting such as focusing on Activities of Daily Living and Instrumental Activities of Daily Living (IADLs), in which the underlying data did not specifically capture the impacts for the changes in underwriting practices of many carriers in the late 2000s.	With the additional exposure data at the **Older Ages**, is the sharp decreasing pattern of experience at the **Older Ages** driven by: a) changes in the exposure by mix of business; or b) the impact of the changing exposure by more recent, higher face amount policies? Is there further experience or analysis to better explain the impact STOLI business has had on the **Older Age** experience? Is there any evidence to suggest a difference in OAM experience between newer issue and underwriting eras and the older issue and underwriting eras?

The OAM Subgroup analyzed the experience using both Generalized Linear Modeling (GLM) techniques and machine-learning techniques, specifically gradient-boosted decision trees, to better explain the odd pattern or slope of mortality at the ages above 95 and to try to isolate the key drivers and interaction of drivers to the experience.

For purposes of this paper:

- **Age** refers to attained age, unless otherwise stated;
- **Older Age** is defined as attained ages 70 and above. Much of the analysis and mortality patterns shown throughout begin at attained ages 65 to better show changes in slope and to better allow for the blending in of other results. Age 70 was consistent with the definition used in the development of the SOA 2015 VBT and is often the age at which senior-specific underwriting begins; and
- **Oldest Ages** refers to attained ages over 95.

Executive Summary

In the study of insured lives mortality rates and the development of industry mortality tables, the **Older Age** mortality (OAM) has been an area of significant judgment. This has led to adjustment in terms of slope and level of the mortality rates for the insurance industry over the past two decades.

As further experience studies are performed for the industry, there is substantially more data at **Ages** 70 and above, especially at **Ages** 95 and above, than existed for the data used in the development of the most recent industry mortality table, the SOA 2015 Valuation Basic Table (2015 VBT). With this increased data, a working group of mortality experts was formed to discern whether this additional data provided further insights to the following questions:

*1) With the additional exposure data at the **Older Ages,** is the sharp decreasing pattern of experience at the **Older Ages** driven by an underreporting of deaths or is it attributable to the uniqueness of the insured population?*

*2) Is there support to suggest that the insured mortality exhibits a different pattern of mortality than population, especially at **Ages** above 95, when attempting to normalize the population data for socioeconomic equivalency?*

*3) With the additional exposure data at the **Older Ages**, is the sharp decreasing pattern of experience at the **Older Ages** driven by:*

a) Changes in the exposure by mix of business; or

b) The impact of the changing exposure by more recent, higher face amount policies?

*4) Is there further experience or analysis to better explain the impact STOLI (Stranger-Owned Life Insurance) business has had on the **Older Age** experience?*

5) Is there any evidence to suggest a difference in OAM experience between newer issue and underwriting eras and the older issue and underwriting eras?

Three approaches were utilized to research the questions posed by the OAM Subgroup on **Older Age** insured mortality:

1) Basic comparison of the actual mortality rates on a log-linear basis;

2) Generalized Linear Models (GLMs) using a significance level of 5%; and

3) Machine-learning models using a modern evolution of gradient-boosted decision trees to the ILEC dataset called catboost[1].

Analysis was performed by gender, issue age, attained age, issue year cohorts, smoking risk classification, benefit band, select vs ultimate period and interactions. The analysis focused both on the number of deaths, the actual exposure rates and the mortality rates.

[1] Catboost has innovations over traditional xgboost when it comes to modeling mortality data, specifically, it has more intelligent handling of categorical predictors such as gender, smoker status, underwriting class, face amount band, etc.

For this analysis, the OAM Subgroup utilized the ILEC data from the 2017 Individual Life Insurance Mortality Experience Report. This report included exposure and claims experience for exposure years 2009 to 2017. The number of contributing companies for this study varied by exposure year, from a low of 48 in 2009 to over 90 contributing companies in years 2014 and later. The OAM analysis focused on **Ages** 65 and above with the following exceptions:

- To exclude the effects of the risk selection process, experience data and exposures for lives in the first 20 policy durations[2] and experience from term policies in the post-level term period were excluded (unless otherwise noted).
- To remove the impact of changing exposure by face amount that would slant the weight towards more recent issue or underwriting eras and, in order to compare to U.S. population mortality, the OAM Subgroup focused the initial analysis on number of claims rather than dollar amount.

In addition to the ILEC data, analysis was also performed to compare to the U.S. population data from the Human Mortality Database (HMD) and population data normalized to reflect the highest socioeconomic decile cohort.

High-level findings

- Within this dataset, nearly 70% of the policies exposed and 77% of the deaths are associated to Unknown risk class or a Uni-smoke basis. Many of these policies were underwritten and issued prior to the fluid-based underwriting more commonly used today (e.g., prior to the 1980s) and in a period with very different behavior related to diet, exercise, and tobacco usage.
- For both males and females, the U.S. population mortality for the highest socioeconomic decile approaches the U.S. general mortality as **Age** increases, though there continues to be a differential even at the **Oldest Ages** (above **Age** 95) for both males and females.
- Smoker mortality exceeds non-smoker and unknown smoker mortality up to **Age** 95, while unknown smoker mortality exceeds non-smoker mortality up to around **Age** 90. For the earliest issue periods, unknown smoker mortality becomes less than non-smoker mortality as **Age** increases.
- Smoker mortality begins to converge with SED10 mortality, becoming close around **Age** 95. Unknown smoker mortality is similar to SED10 mortality up to **Ages** 85 to 90 and then becomes and stays lower after that. Non-smoker mortality is less than SED10 mortality up to around **Age** 85 and then follows the SED10 mortality fairly closely.
- The U.S. population exposure and deaths follow a similar pattern and relationship to the insured trend up to approximately **Age** 100. Beyond **Age** 100, the U.S. population exposure and deaths follow a similar, decreasing trend and the relationship between the two is generally proportional. This contrasts to the insured experience for both males and females, where the trend in insured exposure decreases more slowly than the trend of deaths and the mortality rate appears to deviate significantly from population mortality, decreasing sharply after **Age** 100.
- Overall, mortality rates follow the general pattern of being lower as face amount increases.

[2]The 20-year select period was chosen for simplicity given that **Older Ages** have shorter select periods and that longer select periods only apply to younger age males.

- When isolating to issue years 1995 and later, there is an odd hump to the mortality rates for the largest face amount bands in the early to mid-2000s; this is most pronounced for face amounts of $2.5 million and above. The pattern appears to normalize back to more expected levels for issue years 2010 and later.
- There appears to be differentiation by issue year period for the unknown smoking status and a potential that the odd phenomenon observed in the insured mortality at the Older Ages that is driven not by an improved number of deaths, but rather an overstatement of the exposures at these ages. This may be a result from the changes in the maturity ages and products over time and lack of good monitoring for removing expired or matured contracts.
- When analyzing the data by issue year period, there did seem to be some suggestion that the STOLI policies appear to show something going on between 2004 and 2007 for permanent business.
- When isolating to issue ages 70 and above, we observed a slight uptick in the mean mortality for face amounts above approximately $1M to $2.5M from the late 1990s until the late 2000s. Though this change in mortality was present for issue ages 70+, it was most acute for attained ages 65+ and issue ages < 70. Therefore, it does not appear that this differential by face amount explains the Older Age mortality patterns.
- By product, it appears that the shifts in the mortality rates are occurring in the UL/VL/ULSG/VLSG subsets. However, when analyzing the A/E, this is less apparent and the highest A/Es appear to be in the permanent and term plans. This is likely due to the analysis by count.
- By risk class structure, the shift in mortality appears prominent in the three-class non-smoker systems, which corresponds roughly with peak issue years for that system. This shift or uptick in mortality rate appears to be driven mostly by the residual risk classes irrespective of the total number of preferred classes. However, when one considers actual and model experience relative to the 2015 VBT, the phenomenon vanishes in this dataset. That suggests that this is due to a shift in the average issue age of exposures.
- By face amount band, the sharp increases in the average Age are also consistent with the increases in the face amount mortality in the early to late 2000s and most prominent for face amounts of $2.5 million and above.
- Focusing on issue ages 70 and above, the subsets with the most obvious shifts are above $2.5 million for the UL, ULSG and VLSG plans, with UL and ULSG being the most prominent.

Conclusion

The OAM Subgroup analysis did not draw any firm conclusions; however, the analysis does point to some potential areas for further study. Carriers may want to further investigate their own data quality, specifically around older policies which may have previously matured.

1) Data completeness. Though there is a significant increase in the exposure and claims data in the insured population at the Older Ages, the odd mortality pattern appears to be driven by not an underreporting of deaths, but rather an over-reporting of the exposures, more likely caused by maturing policies not taken off the books.

2) Relationship between insured and general population mortality. We did not observe strong support to suggest the insured mortality at the Older Ages exhibited a different pattern of mortality than the general U.S. population once normalized for socioeconomic equivalency.

3) Cohort differences. There is evidence to suggest differences between attained age mortality at the Older Ages from newer issue year cohorts at larger face amounts, though this seems to have normalized beginning around 2010. This is more difficult to observe when viewing an A/E analysis than the raw mortality rates or more advanced analysis with interactions.

Section 1: Background

For life insurance, the challenges with understanding both the level and the slope of mortality between **Ages** 70 and 95 are a result of many factors, including, but not limited to:

- Changes in underwriting and risk classification over time;
- Varying impact of mortality improvement;
- Extension of maturity age from 85, 90, 95, 100 and ultimately to **Age** 120 leading to changes in exposure at the **Oldest Ages**;
- Reporting challenges for unreported deaths and other policy terminations;
- Changes in type of business over time with a move from whole life insurance to more of a mix of interest sensitive and term insurance, leading to changes in average policy size and an increase in average issue age;
- Potential cohort impacts based on year of birth;
- Changes in policyholder behavior over time, including the impact of STOLI and lifetime no-lapse guarantees, both which extended persistency beyond historical experience;
- Increased replacement activity with the introduction of preferred risk selection and the potential anti-selective mortality for those less healthy lives that did not go through underwriting again; and
- Changes in the number of companies contributing data and lack of homogeneity in the underlying dataset year over year.

Historically, the mortality experience at the **Older Ages** lacked credibility beyond **Age** 95. Insured data has been unreliable past **Age** 95 due to suspected data quality issues. This was reflected in the development of the 2015 VBT where models were used up to **Age** 95 with extrapolations past that **Age** to an omega mortality rate of 0.5 beginning at **Age** 112[3].

Since the 2015 VBT was released, there has been an increase in the exposure of the **Older Age** risks, but, despite the increase in exposure, the emerging pattern of mortality rates continued to demonstrate a declining slope beyond age 95. With the most recent industry experience data available (2009 to 2017[4]), the Society of Actuaries Research Institute's ILEC noted the following in its 2017 Individual Life Mortality Experience Report, published in December of 2021[5]:

- In terms of the death claim amounts for **Ages** 65+, **Ages** 80-89 have the highest distribution and most experience, which is reasonable given a high mortality rate at these ages. At **Ages** 90+, there is less experience, which also makes sense given that there is less remaining exposure after **Age** 89.
- **Older Ages** seem to exhibit a fairly good fit to the VBT for this aggregate view as they all have smaller deviations from the 100% line than **Ages** less than 65.
- **Older Ages** demonstrate some level of decreasing A/E during the study period, but the slope is less pronounced than for **Ages** less than 65 and <$100,000.
- For low face amounts (less than $100,000), the actual to expected mortality level declines significantly by **Age**. One possible explanation is that the level of anti-selection is less pronounced at the **Older Ages**.
- For higher face amounts, there is a less clear relationship between A/E and **Age**.

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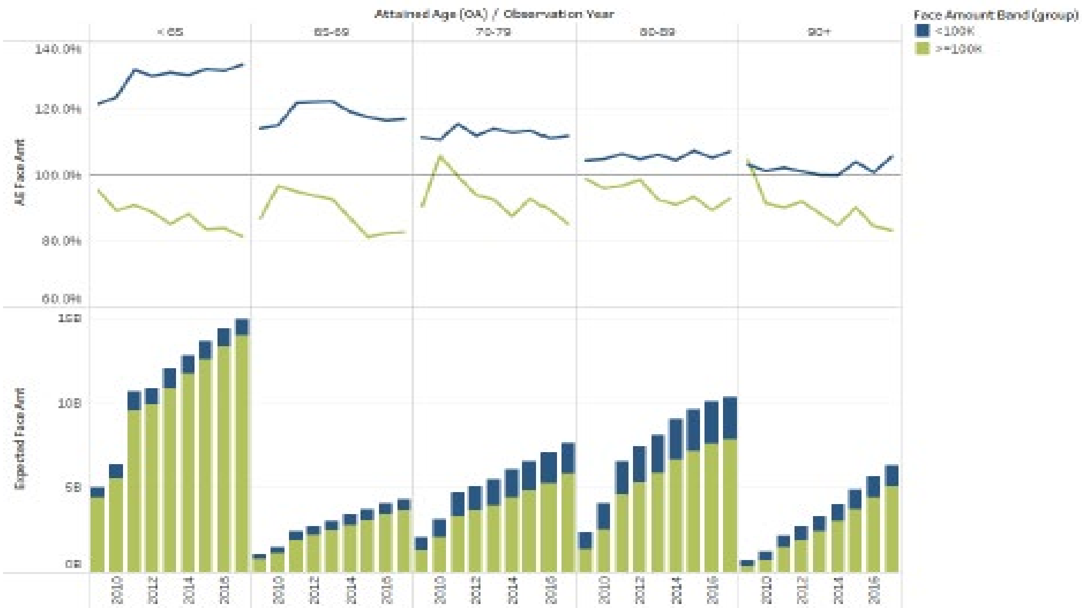
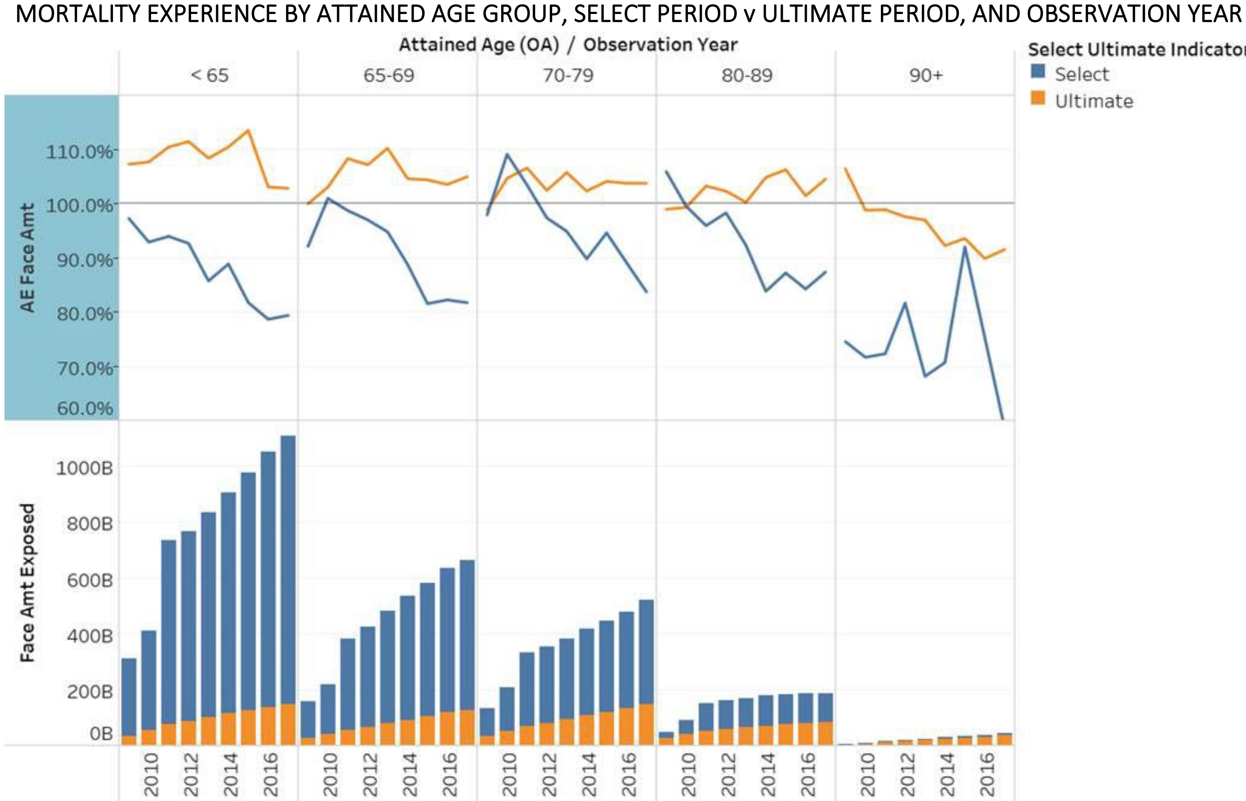
[3] 2015 Valuation Basic Report and Tables, April 2016, https://www.soa.org/resources/experience-studies/2015/2015-valuation-basic-tables.
[4] Data from 2018 was also available, but dismissed from the study as it comprised of a smaller subset of company contributions and exposure.
[5] https://www.soa.org/globalassets/assets/files/resources/research-report/2021/ilec-report-2017.pdf

- Overall, there appears to be less differentiation between low and high face amount mortality for Older Ages.
- These are demonstrated in figure 13 from the ILEC Individual Life Experience Report below.

Figure 13 from ILEC Individual Life Experience Report
MORTALITY EXPERIENCE BY ATTAINED AGE GROUP, FACE AMOUNT, AND OBSERVATION YEAR



from ILEC Individual Life Experience Data
MORTALITY EXPERIENCE BY ATTAINED AGE GROUP, SELECT PERIOD v ULTIMATE PERIOD, AND OBSERVATION YEAR

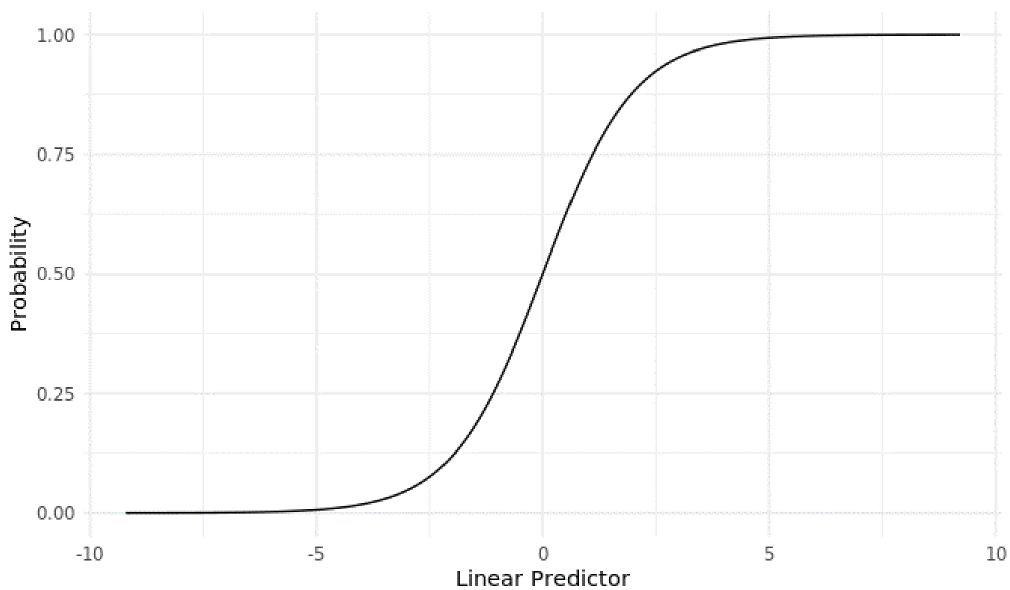


Section 2: Methodology

Three approaches were utilized to research the questions posed by the OAM Subgroup on **Older Age** insured mortality:

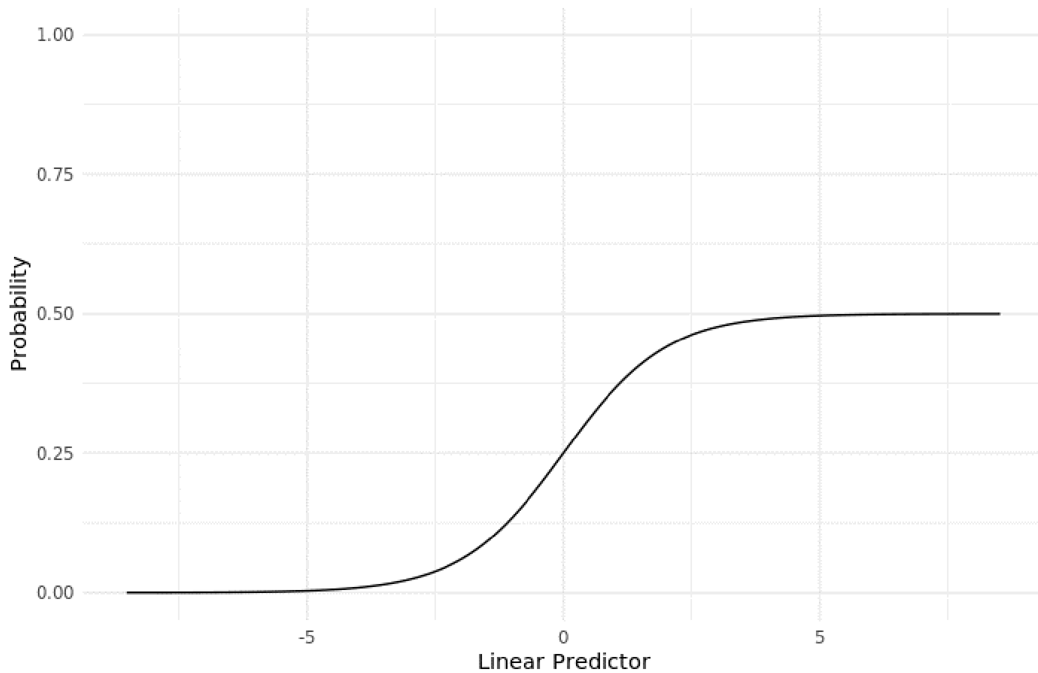
1. Basic comparison of the actual mortality rates on a log-linear basis
2. GLMs using a significance level of 5%
    - For each risk class, GLMs were calculated for **Ages** 65 to 95 using both a linear age and quadratic age model with binomial regression.
    - Binomial regression models a linear relationship between the log of the odds ratio, q/(1-q), and the predictor variables. This can be illustrated easily by plotting the inverse function and mapping the linear predictor to a probability, as in figure 2.1.

Figure 2.1
LINEAR PREDICTOR AND ASSOCIATED PROBABILITY



- To project the **Older Age** mortality and to determine the expected **Age** at death, it required mortality to be estimated past **Age** 95 up to **Age** 120. The binomial model estimates tended to the theoretical maximum probability of 1 as **Age** increases due to the odds ratio. However, other research shows that the **Oldest Ages** reach a limit of around 0.5 as reflected in the maximum mortality rate of 0.5 in the 2015 VBT from **Age** 112. For the 2015 VBT, the mortality rates over **Age** 95 were developed by extrapolation using a cubic polynomial up to **Age** 112. This analysis used an alternative approach, a modified odds ratio, $q/(q_{Max} - q)$, with a maximum probability of 0.5.

Figure 2.2
ADJUSTED ODDS RATIO BY PROBABILITY



With q_{max} = 0.5, the binomial regression was repeated for non-smokers resulting in mortality at **Age** 120 slightly below 0.5. The quadratic model is clearly a better fit as it more closely follows U.S. population mortality, while the linear model has a slower acceleration.

- In order to calculate the expected **Age** at death, the quadratic age model using standard binomial regression has been blended with that using the non-smoker modified binomial regression. The standard regression model is used up to **Age** 95 and the modified regression model from **Age** 100, and blended estimates for **Ages** 96-99.

3. Machine-learning models using a modern evolution of gradient-boosted decision trees to the ILEC dataset called catboost[6].
    - The model requires predictor variables (e.g., insurance plan, attained age, etc.), a response variable (raw death rate by count), a weighting variable (number of policies exposed), and a likelihood loss function to be specified as the optimization target (Poisson). In addition, an overfitting threshold was set at 0.05.
    - Analysis for fit of the forecast model was performed by **Age** and gender; **Age**, gender, and duration; and, finally, by **Age**, gender, duration and face amount band. For transparency and to aid further analysis by others, most code is included in Appendix A.

[6] Catboost has innovations over traditional xgboost when it comes to modeling mortality data, specifically, it has more intelligent handling of categorical predictors such as gender, smoker status, underwriting class, face amount band, etc.

Section 3: Data

For this analysis, the OAM Subgroup utilized the ILEC data for **Ages** 65 and above with the following exceptions:

- To exclude the effects of the risk selection process, experience data and exposures for lives in the first 20 policy durations and experience from term policies in the post-level term period were excluded (unless otherwise noted).
- To remove the impact of changing exposure by face amount that would slant the weight towards more recent issue or underwriting eras and, in order to compare to U.S. population mortality, the OAM Subgroup focused the initial analysis on number of claims rather than dollar amount.

In addition to the ILEC data, analysis was also performed to compare to the U.S. population data from the HMD and population data normalized to reflect the highest socioeconomic decile cohort, as identified in the recent SOA-sponsored study, "Mortality by Socioeconomic Category in the United States"[7] (https://www.soa.org/resources/research-reports/2020/us-mort-rate-socioeconomic/).

For purposes of the OAM Subgroup's basic comparison and regression analysis, the following terms and definitions were used:

Table 3.1
CODES AND ABBREVIATIONS

Code	Description
US	U.S. Human Mortality Database dataset
Insured	ILEC dataset
SED10	U.S. Highest Socioeconomic Decile (as determined in Mortality by Socioeconomic Category in the United States report)
F	Female gender
M	Male gender
N	Non-smoker
S	Smoker
U	Unknown smoker
D	Combined distinct smoker statuses (non-smoker and smoker)

[7] Magali Barbieri, Ph.D., University of California-Berkeley, published January, 2021.

Table 3.2

ILEC WEIGHTED EXPOSURE AND DEATHS BY SELECT & ULTIMATE STATUS

Select & Ultimate Status	Exposure by Count		Exposure by Amount		Claims		Issue Years
	# (1,000s)	%	Amount ($ billions)	%	# (1,000s)	Amount ($ millions)	
Select*	26,635	25.8%	6,292	71.7%	447	82,475	1989-2017
Ultimate	76,489	74.2%	2,482	28.3%	3,234	76,045	1913-2017
Total	**103,124**	**100.0%**	**8,774**	**100.0%**	**3, 681**	**158,520**	**1913-2017**

*Select Period = Policy Years 1-20, Ages 65-95 for all tables. Note: This select period was utilized for modeling simplification and is slightly different from that used in the 2015 VBT, which graded down by age.

As shown in table **3.3**, the exposure used in the analysis consisted of nearly 76.5 million policies and approximately 3.2 million deaths over the time period. Table **3.4** shows the gender breakdown and issue years by smoking status.

Table 3.3

ILEC WEIGHTED EXPOSURE AND DEATHS BY SMOKER STATUS

Smoking Status	Exposure by Count		Exposure by Amount		Claims		Issue Years
	# *(1,000s)*	%	Amount ($ billions)	%	# (1,000s)	Amount ($ millions)	
N	19,703	25.8%	1,570	63.3%	560	38,896	1981-1997
S	3,582	4.7%	175	7.1%	177	7,646	1981-1997
U	53,240	69.6%	736	29.7%	2,496	29,502	1913-1997
Total	**76,489**	**100.0%**	**2,482**	**100.0%**	**3,234**	**76,045**	**1913-1997**

Within this dataset, nearly 70% of the policies exposed and 77% of the deaths are associated to Unknown risk class or a Uni-smoke basis. Many of these policies were underwritten and issued prior to the fluid-based underwriting more commonly used today (e.g., prior to the 1980s) and in a period with very different behavior related to diet, exercise, and tobacco usage, to name a few. For the remaining smoker distinct policies, 15% are identified with smoking usage at time of policy issue. As the U.S. population data from the HMD do not contain a smoking status indicator, there is no direct comparison to the U.S. population for the smoker usage at these **Older Ages**.

By product, the distinct smoker statuses are made up of 57% traditional life, 40% universal/variable life and 3% term life, of which, overall, 9% has a preferred structure. The unknown smoker status has 99% traditional life and 1% term life.

As a point of comparison, the OAM Subgroup compared the ILEC exposure and claims mix by **Age**, gender and smoking status to the general population where the U.S. population data was from the HMD by gender for calendar years 2009 to 2017. For this analysis, in order to remove the impact of the risk selection process, the insured data excludes durations up to and including year 20 and post-level term data. Table 3.4 shows the exposure differences between the U.S. population and U.S. insured mortality for attained ages 65 to 95.

Table 3.4

U.S. POPULATION COMPARED TO ILEC EXPOSURE AND DEATHS BY GENDER (AGE 65-95)

U.S. Population	Exposure (1000s)	Exposure (%)	# Deaths (1000s)	Insured Population	Exposure (1000s)	Exposure (%)	# Deaths (1000s)
F	228,189	56.0%	8,493	F	29,496	38.6%	1,102
M	179,519	44.0%	7,691	M	46,993	61.4%	2,132
Total	**407,708**	**100.0%**	**16,184**	**Total**	**76.489**	**100.0%**	**3,234**

Female lives represent 56% of the general population exposure but less than 40% of the insured population exposure at the **Older Ages**. Female deaths represent 52% of the general population and 32% for the insured claims.

For the machine-learning models, a training set was developed from the ILEC data by randomly selecting 70% of the rows for issue years prior to 2017, with the balance of pre-2017 serving as a test dataset. The GLM was fitted from **Ages** 65 to 95. The data was then grouped into five-year issue age and attained age groups as follows:

Age Predictor	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
Issue Age	<70	70-74	75-79	80-84	85-89	90-94	95-99
Attained Age	65-69	70-74	75-79	80-84	85-89	90-95	96+

Observation year 2017 rows were used as a forecasting subset. The testing and forecast subsets are intended to check whether there are any problems at a high level with the model. The dataset was also appended with the following four derived columns:

- Actual_Qx_Count: the ratio of number of deaths by policies exposed, 0 for rows with no policies exposed;
- Actual_Qx_Amount: the ratio of death claim amount by amount exposed, 0 for rows with no amount exposed;
- AE_Count: actual-to-expected ratio by count versus the 2015 VBT; and
- AE_Amount: actual-to-expected ratio by amount versus the 2015 VBT.

Table 3.5
DATA TRAIN-TEST-FORECAST STATISTICS

			By Count				By Amount			
Subset	Observation Year	Row Count	# Deaths	% Deaths	Policies Exposed	% Policies Exposed	Death Claim Amount ($ billions)	% Death Claim Amount	Amount Exposed ($ trillions)	% Amount Exposed
Training	2009-2016	5,757,160	2,357,026	61.3%	63,825,655	61.0%	$97.52	59.2%	$5.24	58.9%
Test	2009-2016	2,466,937	1,014,804	26.4%	27,331,973	26.1%	40.84	24.8%	$2.25	25.3%
Forecast	2017	1,404,677	476,085	12.4%	13,516,813	12.9%	26.30	16.0%	$1.41	15.8%

By amount, the split is biased more toward the forecast subset as a result of the increasing average face amount of policies over time. For example, the average amount exposed in 2009 is $59,414, while the same for 2015 is $104,144.

Section 4: Analysis – Basic Comparison

As a first step, the OAM Subgroup looked to answer the question about whether there is evidence to suggest that the insured mortality exhibits a different pattern of mortality than population, after attempting to normalize the population for socioeconomic equivalency. We compared the insured population to the general U.S. population (US), as well as that for the highest socioeconomic decile for the U.S. population (SED10) by:

A. Life expectancy at **Age** 65 by gender – Table 4.1
B. Exposure and number of deaths by **Age** – Figures 4.1 and 4.2;
C. Mortality by gender and **Age** – Figure 4.3;
D. Mortality by gender, smoking status and **Age** – Figure 4.4
E. Mortality by gender, smoking status and benefit band – Figure 4.5

For subsections 4.1 – 4.5, each of the graphic figures includes the **Age** 95 vertical line, while mortality charts include a horizontal line for the omega rate ($\log_{10}(0.5) = -0.3$).

4.1 LIFE EXPECTANCY AT 65 BY GENDER

For determination of the life expectancy, the U.S. and SED10 mortality rates are the actual mortality rates up to **Age** 120 up to the omega rate (0.5). The Insured expectation of life is estimated up to **Age** 95, the quadratic age model (m4), from **Age** 100, a non-smoker quadratic age model with modified odds ratio to enforce the omega rate, assuming that smoker status mortality converges on non-smoker mortality at **Age** 100, for ages 96-99, blended mortality. This is shown in table 4.1 below.

Table 4.1
LIFE EXPECTANCY AT AGE 65

Sex	Populations				Smoker Type			Distinct Smoker		
	US	SED10	Ins	Ins-SED10	D	U	U-D	S	N	N-S
F	20.2	21.5	21.0	-0.5	21.2	20.9	-0.3	17.0	22.2	-5.1
M	17.6	19.0	19.0	-0.0	18.8	19.0	0.2	14.8	19.7	-4.9
All	18.5	19.9	19.7	-0.2	19.9	19.7	-0.2	15.9	20.8	-4.9
F-M	2.6	2.5	2.0		2.3	1.9		2.2	2.2	

Observations:

- At **Age** 65, insured lives are expected to live 1.2 years longer than the lives in the general population but two-tenths of a year less than the highest socio-economic decile.
- Smokers are expected to live five years less than non-smokers.
- Females are expected to live two years longer than males in the insured population, about half a year less than the differential in the general population (overall and in the SED10 population).

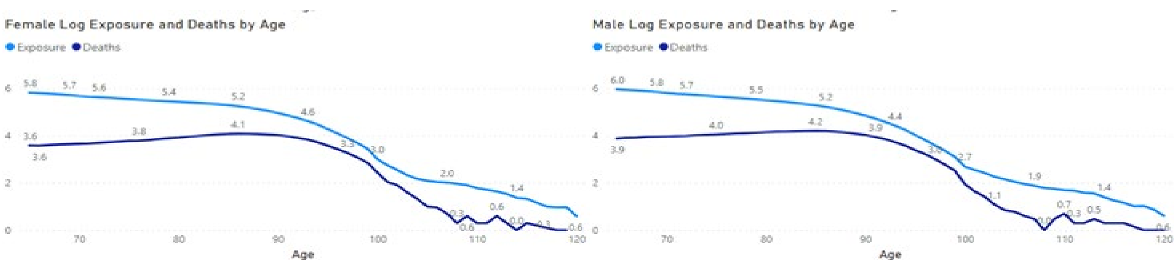
4.2 EXPOSURE AND NUMBER OF DEATHS BY AGE

Figure 4.1

LOG OF EXPOSURE AND LOG OF DEATHS - INSURED VERSUS U.S. POPULATION

Figure 4.2

LOG OF EXPOSURE AND LOG OF DEATHS - FEMALE VERSUS MALE



Observations:

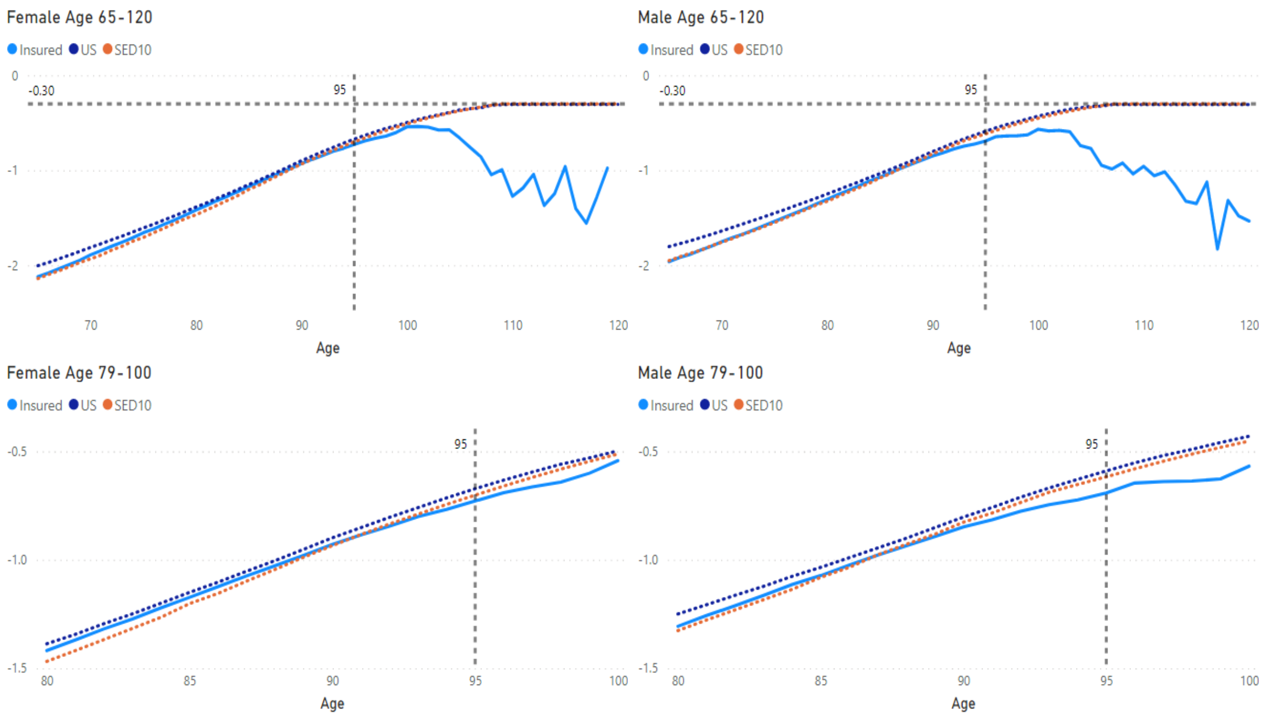
- The trend in insured exposure decreases more slowly than the trend of deaths.
- The U.S. population exposure and deaths follow a similar pattern and relationship to the insured trend up to approximately **Age** 100. Beyond **Age** 100, the U.S. population exposure and deaths follow a similar, decreasing trend. The charts above show the exposure and deaths for the entire insured and U.S. populations.

4.3 ACTUAL MORTALITY BY GENDER AND AGE

The OAM Subgroup compared the attained age subset of the insured data to the population data for **Ages** 65 to 120 for both males and females. To better observe the patterns and differences at the **Oldest Ages**, two comparisons, one for **Ages** 65 to 120 and the other for **Ages** 79 to 100, are shown in figure 4.3 below.

Figure 4.3

MORTALITY BY GENDER – INSURED VERSUS U.S. POPULATION (LOG SCALE)



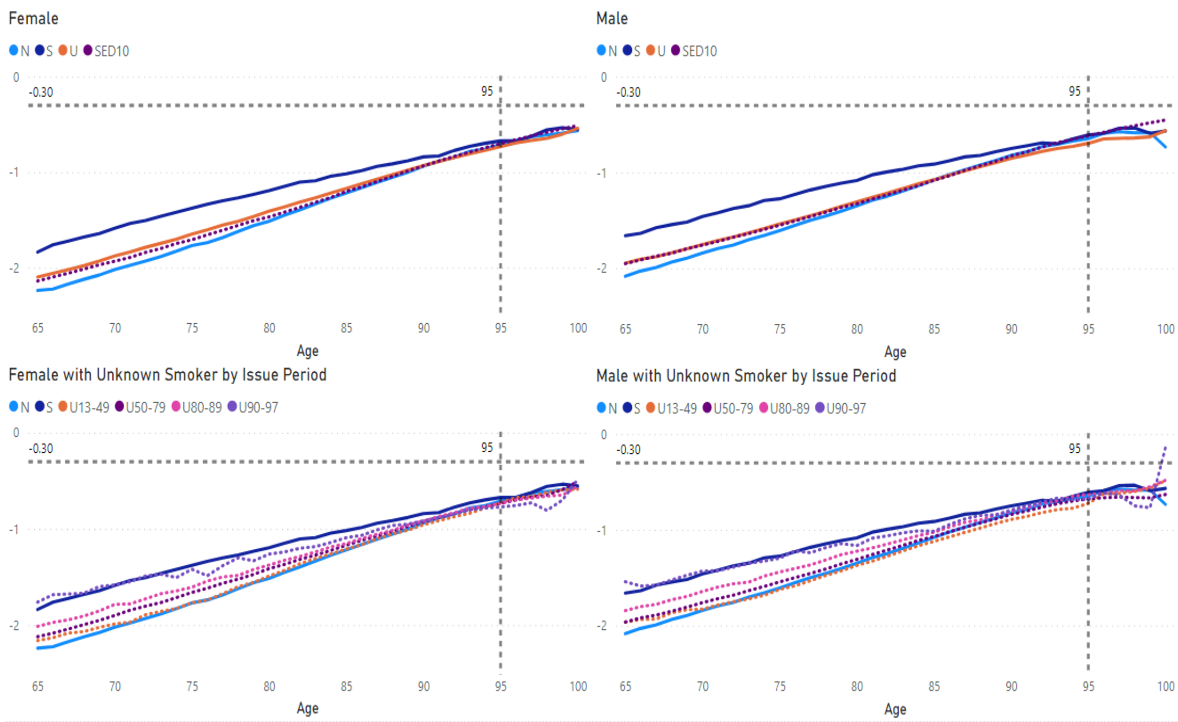
Observations:

- For both males and females, the SED10 mortality approaches the U.S. general mortality as attained age increases, though there continues to be a differential even at the **Oldest Ages**.
- Female insured mortality is slightly higher than SED10 mortality at **Age** 65, approaches U.S. mortality around **Age** 85, then diverges and becomes less than SED10 mortality.
- Male insured mortality closely follows SED10 mortality up to around **Age** 85 and then becomes less and exhibits a very different pattern beginning around **Age** 90.
- For male risks only, there appears to be step increases in the mortality at **Age** 96 for male unknown smoker risks and at **Age** 100 for both male non-smoker and male unknown smoker risks. This may be due to changes in maximum maturity ages over time, especially for whole of life policies, which commonly matured at **Age** 95 before maturity ages began extending to **Age** 100 and beyond around the same time as the move to smoker-distinct risk classification in the early to mid-1980s. This same phenomenon is not directly observable in the female mortality.
- The insured mortality, for both males and females, appears to deviate significantly from population mortality and decrease sharply after **Age** 100.

4.4 ACTUAL MORTALITY BY GENDER, SMOKER STATUS, AGE, AND ISSUE YEAR GROUP

The OAM Subgroup compared the mortality rates for the attained age subset of the insured data to the population data for **Ages** 65 to 100 for both males and females by smoking status. To discern whether there were cohort effects with the unknown smoker status, this subset was further divided by issue periods 1913-1949, 1950-1979, 1980-1989, and 1990-1997 and is shown in figure 4.4 below.

Figure 4.4
MORTALITY BY GENDER AND SMOKER STATUS – INSURED VERSUS U.S. POPULATION



Observations:

When layering in smoking classification to the age and gender analysis, we find:

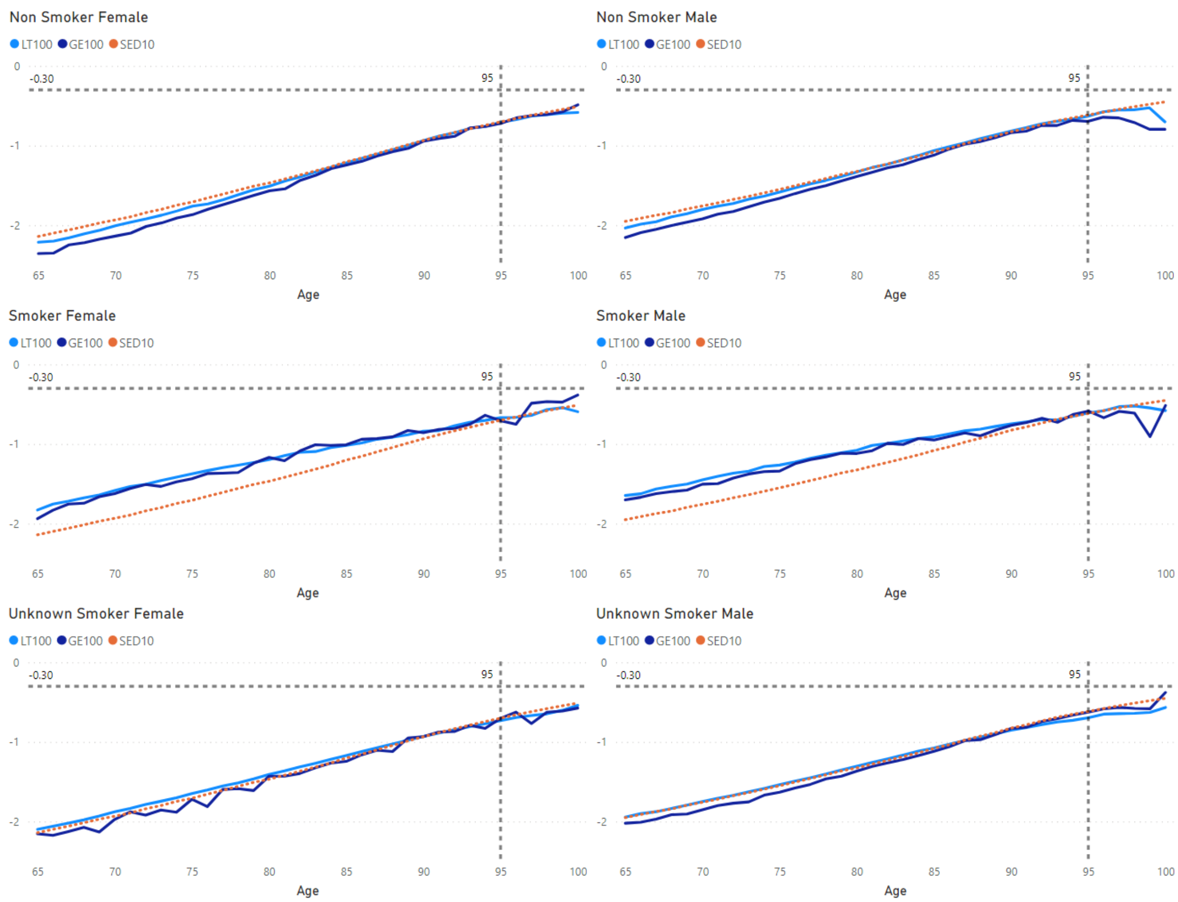
- Smoker mortality begins to converge with SED10 mortality, becoming close around **Age** 95. Unknown smoker mortality is similar to SED10 mortality up to **Age** 85 to 90 and then becomes and stays lower after that. Non-smoker mortality is less than SED10 mortality up to around **Age** 85 and then follows the SED10 mortality fairly closely.
- Smoker mortality exceeds non-smoker and unknown smoker mortality up to **Age** 95, while unknown smoker mortality exceeds non-smoker mortality up to around **Age** 90.
- Unknown smoker mortality by issue period increases from around non-smoker mortality as issue period increases. Issue period 1990-97 is similar to smoker mortality.
- For the earliest issue periods, unknown smoker mortality becomes less than non-smoker mortality as **Age** increases.

4.5 ACTUAL MORTALITY BY GENDER, SMOKER STATUS, BENEFIT BAND AND AGE

Additional analysis was performed to try to determine if there was a cohort effect due to the size of the amount of insurance. For this comparison, we analyzed the data by gender, smoker status and insurance amounts less than $100,000 (LT100), and that with insurance amounts of $100,000 and above (GE100). Comparisons were made to the SED10 population cohort.

Figure 4.5

MORTALITY BY GENDER, SMOKER STATUS AND INSURED BENEFIT BAND VERSUS U.S. POPULATION



Observations:

- For the LT100 cohort:
  - Both the male and female non-smoker mortality is lower than SED10 up until **Ages** 80 or 85, when they appear to converge with SED10 mortality before deviating away (lower) than SED10 at **Ages** 100 and above.
  - The smoker mortality is higher than SED10 until it converges with SED10 mortality near **Age** 95 and ultimately becomes lower than SED10 prior to **Age** 100. This pattern is observed in both the male and female mortality.
  - The unknown or uni-smoke smoker mortality was higher than or very similar to SED10 mortality for most attained **Ages**, but converges with and then drops lower than the SED10 mortality near age 90 before. The deviation from SED10 mortality is more pronounced for male lives than for female, especially after **Age** 95.

- For the GE100 cohort:
    - The non-smoker mortality follows a similar pattern as for the LT100 cohort, for both female and male lives up to **Age** 95. However, at **Ages** 95 and above, the female mortality does not show a departure from the SED10 mortality and continues to show convergence or even a slight increase in relation to SED10. This is not observed for the male lives.
    - The smoker mortality shows a similar pattern as for the LT100 cohort for both female and male lives up to **Age** 95 (higher mortality than the SED10 before converging). However, the female mortality decreases below the SED10 around **Age** 96 before a sharp increase at **Age** 97 at which point it remains higher than the SED10 mortality. The male mortality shows a similar pattern to the LT100 cohort with a steady increasing deviation to lower mortality rates than the SED10 after **Age** 95, and a sharp decrease at **Age** 99 before moving closer to the SED10 mortality near **Age** 100.
    - The unknown smoker mortality shows a similar pattern as the LT100 cohort for unknown smokers; however, it actually converges and, in some cases, exceeds the LT100 mortality starting around **Age** 90. The divergence is more pronounced for male risks than for females.

Section 5: Analysis – Regression Models

Binomial Generalized Linear Models were used to fit insured and U.S. population mortality. Significance was assessed using a statistical significance level of 5%. An actual versus fitted analysis was performed to further understand any differences in mortality patterns between the insured and general populations.

The regression models for the insured mortality were then expanded to analyze the linear and quadratic age A/E by smoking status and gender to see if some of the differences observed were driven by the changes in the mix of business between smoker distinct and uni-smoke business.

5.1 REGRESSION MODELS

The ILEC data were subdivided into six subsets, one for each combination of gender and smoker status in the Insured population. The U.S. population data were subdivided by gender. On each subset, a pair of binomial GLMs with custom link function were fitted, one with a linear term for attained age and the other with quadratic terms for attained age.

Table 5.1

REGRESSION MODELS

Dataset	Class	Linear Age Models					A/E	AIC	Quadratic Age Models			
		A/E	AIC	Deviance	Intercept	Age			Deviance	Intercept	Age	Age^2
US	F	100%	29,250	28,804	-12.0096	0.1115	100%	907	459	-6.1365	-0.0361	0.0009
	M	100%	30,095	29.653	-10.8635	0.1013	100%	656	211	-4.5779	-0.0592	0.0010
Insured	NF	100%	860	528	-13.7988	0.1301	100%	474	140	-9.4042	0.0196	0.0007
	NM	100%	804	460	-12.8669	0.1231	100%	524	177	-9.7475	0.0435	0.0005
	SF	100%	384	83	-10.1834	0.0936	100%	342	39	-12.6770	0.1574	-0.0004
	SM	100%	399	97	-9.7300	0.0915	100%	344	40	-12.4205	0.1614	-0.0004
	UF	100%	655	286	-12.3820	0.1149	100%	524	153	-10.9128	0.0785	0.0002
	UM	100%	1,787	1,394	-11.6606	0.1090	100%	1,698	1,303	-10.8357	0.0885	0.0001

Table 5.1 summarized the following key characteristics of the GLMs:

- The actual mortality rate (Qx) is the response variable. The models automatically fit to have a 100% A/E fit overall.
- The Akaike Information Criterion (AIC) helps in avoiding irrelevant predictors that would increase the complexity of the model unnecessarily. Selecting the model with the smallest AIC results in the best fit. Within a subset, it is possible to compare the linear and quadratic age terms to select the better model. If the difference in AIC is significant with respect to an asymptotic Chi-square test with one degree of freedom (since one (quadratic) term was added), then we can conclude that the quadratic terms model fits better than the linear terms model. This is true for all models presented.
- Deviance in this table is the residual deviance from the GLM. Residual deviance is twice the difference between the log-likelihood of a model, which fits the data perfectly, and the log-likelihood of this model. Rather than test goodness-of-fit with this statistic, we qualitatively assess goodness-of-fit in figures 5.1-5.3.

Table 5.1 also tells us about the broad attained age patterns in each subset. For example, in the linear age models, the age coefficient approximately expresses mortality slope, with higher values suggesting a steeper overall slope. Non-smokers have steeper slopes, smokers have flatter slopes, and uni-smoke Insured lives have average slopes between these. Of interest in the linear age models is how the closeness of the uni-smoke Insured lives slopes and intercepts with the analogous coefficients from the U.S. population. Statistical tests to confirm this phenomenon are outside the scope of this paper.

Gleaning meaning from the quadratic terms models is more challenging. The linear terms are no longer directly comparable. The sign of the quadratic terms provides a sense of the shape on the edges. The terms for uni-smoke and non-smoker Insured lives are positive and suggest an upward trend at the edges, while the negative quadratic term for the smoker model suggests a downward curve at the edges. Illustrations in figures 5.1-5.3 of the quadratic term models are more informative than the table output.

For these models, care should be taken when interpreting intercept terms. Because attained age was not recentered, the intercept term corresponds to attained age 0. Interpreting it without context is technically an extrapolation of the model to **Age** 0.

Further assessment of model fit can be found in the residual analysis in figures 5.1, 5.2, and 5.3.

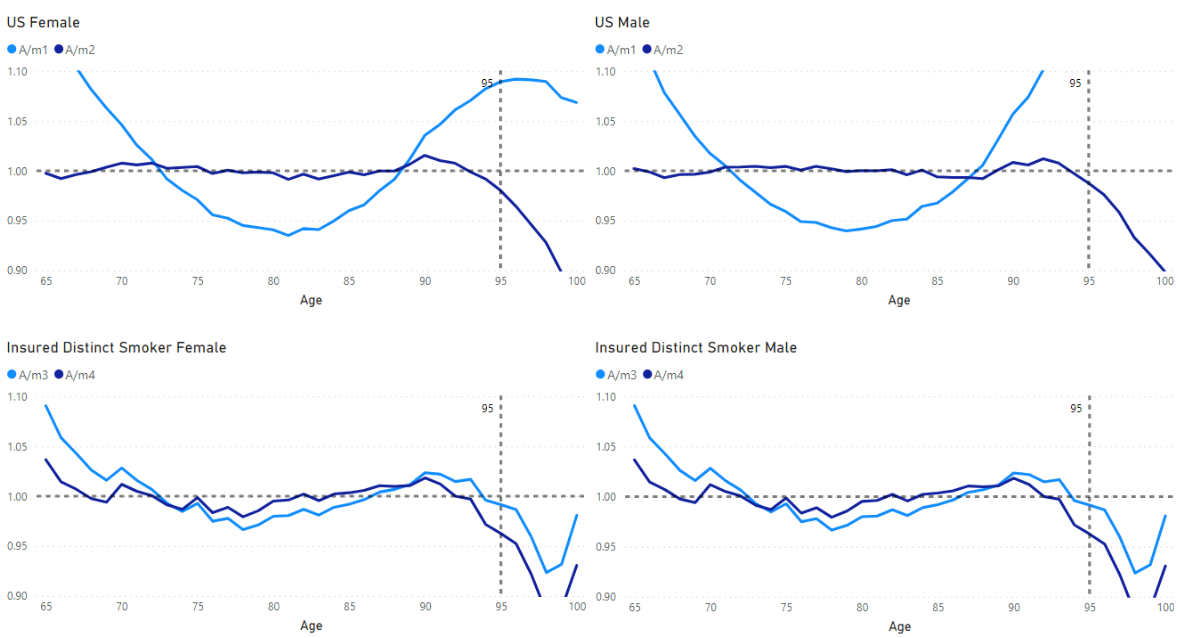
In addition to the codes used in the basic comparison analysis above, the regression models also use the following codes and abbreviations as shown in table 5.2 below.

Table 5.2
ADDITIONAL REGRESSION CODES AND ABBREVIATIONS FOR FIGURES 5.1-5.3

Code	Description
m1	U.S. linear age regression model by gender
m2	U.S. quadratic age regression model by gender
m3	Insured linear age regression model by gender and smoker
m4	Insured quadratic age regression model by gender and smoker
A	Actual data point or experience

5.2 ACTUAL VERSUS MODELED MORTALITY BY GENDER AND AGE

Figure 5.1
MODEL A/E* BY GENDER



**E = expected basis using either m1, m2, m3 or m4*

Observations:

- For the U.S. population mortality:
  - U.S. linear age A/E (m1) is parabolic.
  - U.S. quadratic age A/E (m2) is mostly flat up to **Age** 87 and then accelerates.
- For this Basic Comparison analysis, the insured models utilize smoker distinct experience only. On this basis:
  - Insured linear age A/E (m3) is parabolic up to **Age** 90, but sinusoidal up to **Age** 95.
  - Insured quadratic age A/E (m4) is similar to m3 but less pronounced.
- The quadratic models for the U.S. population tend to show a better fit and more consistency in pattern across most ages over the linear model; however, both diverge around **Age** 90 (linear) and 95 (quadratic). Both the linear and quadratic models show similar patterns for the insured smoker distinct mortality, but also show a decreasing A/E at **Ages** in the mid-90s with significant declines after **Age** 95 for both female and male lives. This suggests gender alone does not explain the mortality phenomenon at the **Oldest Ages**.

5.3 ACTUAL VERSUS MODELED MORTALITY BY GENDER, SMOKER STATUS AND AGE

In the following plots, the dotted lines represent the 95% prediction interval from the underlying GLM.

Figure 5.2

INSURED LINEAR AGE A/E BY SMOKER AND GENDER

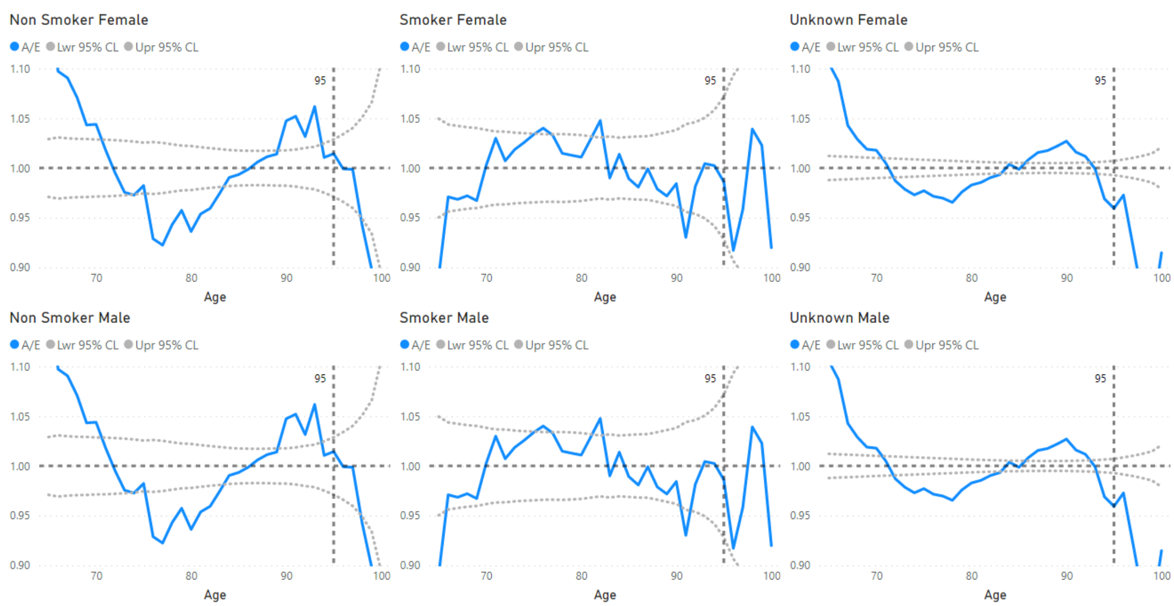
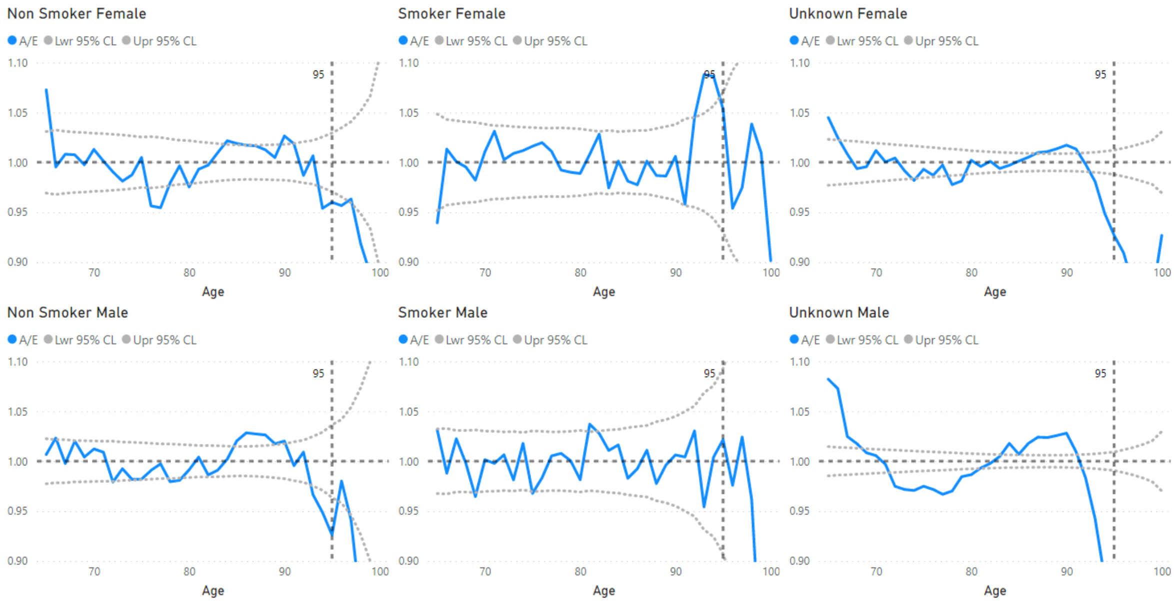


Figure 5.3

INSURED QUADRATIC AGE A/E BY SMOKER STATUS AND GENDER



Observations:

- The non-smoker and unknown smoker linear age A/Es are sinusoidal.
- The non-smoker and unknown smoker quadratic age A/Es are sinusoidal but less pronounced than m3.
- The smoker linear age A/E is parabolic, indicating unmodeled residual effects.
- The smoker quadratic age A/E is flat.
- It appears likely that the smoker risks could fit better with a quadratic model, but NS and U appear to be more sinusoidal / cubic.
- Regardless of whether using a linear or quadratic age model, the unknown smoker male risks actual to model experience was often outside the 95% confidence interval; a similar, but less pronounced, pattern was observed for the unknown smoker female risks.
- For non-smoker risks (both male and female), the actual to model ratios were within the 95% confidence interval more consistently with the quadratic regression model; however, as **Age** approaches 100, the actual to model results for the linear regression are less startling.
- The unknown smoker status shows a similar increase in the A/E in the late 80s and early 90s as the pattern observed in the underlying mortality rates; however, it then exhibits a significant drop in A/E outside the confidence interval after the early 90s.

Based on the observations detailed in the Basic Comparison and the Regression Models, there appears to be differentiation by issue year period for the unknown smoking status, and a potential that the odd phenomenon observed in the insured mortality at the **Older Ages** is driven, not by an improved number of deaths, but rather an overstatement of the exposures at these ages. This may be a result from the changes in the maturity ages and products over time and lack of good monitoring for removing expired or matured contracts.

When analyzing the data by issue year period, there did seem to be some suggestion that STOLI policies might appear to show something going on between 2004 and 2007 for permanent business.

Section 6: Analysis – Machine-Learning Models

The regression models in Section 5 suggested potential variation due to issue year cohort and smoking status, as well as a potential exposure challenge at the **Oldest Ages**. To further the understanding and to identify interaction between variables, a machine-learning model leveraging boosted decision trees was built (See Appendix A: Catboost Models and Information for further description).

6.1 FIT TESTS AND VALIDATION

Fit tests and validation were performed by measuring actual to fitted death rates across three groupings:

1) By Attained Age and Gender
2) By Duration and Gender
3) By Duration and Face Amount Band

In these plots, the 95% confidence interval using the normal approximation to the binomial are the red bars, the blue line is the model, and the black dots are the actual death rates.

Figure 6.1
ACTUAL VS FITTED DEATH RATES BY ATTAINED AGE, GENDER AND SUBSET

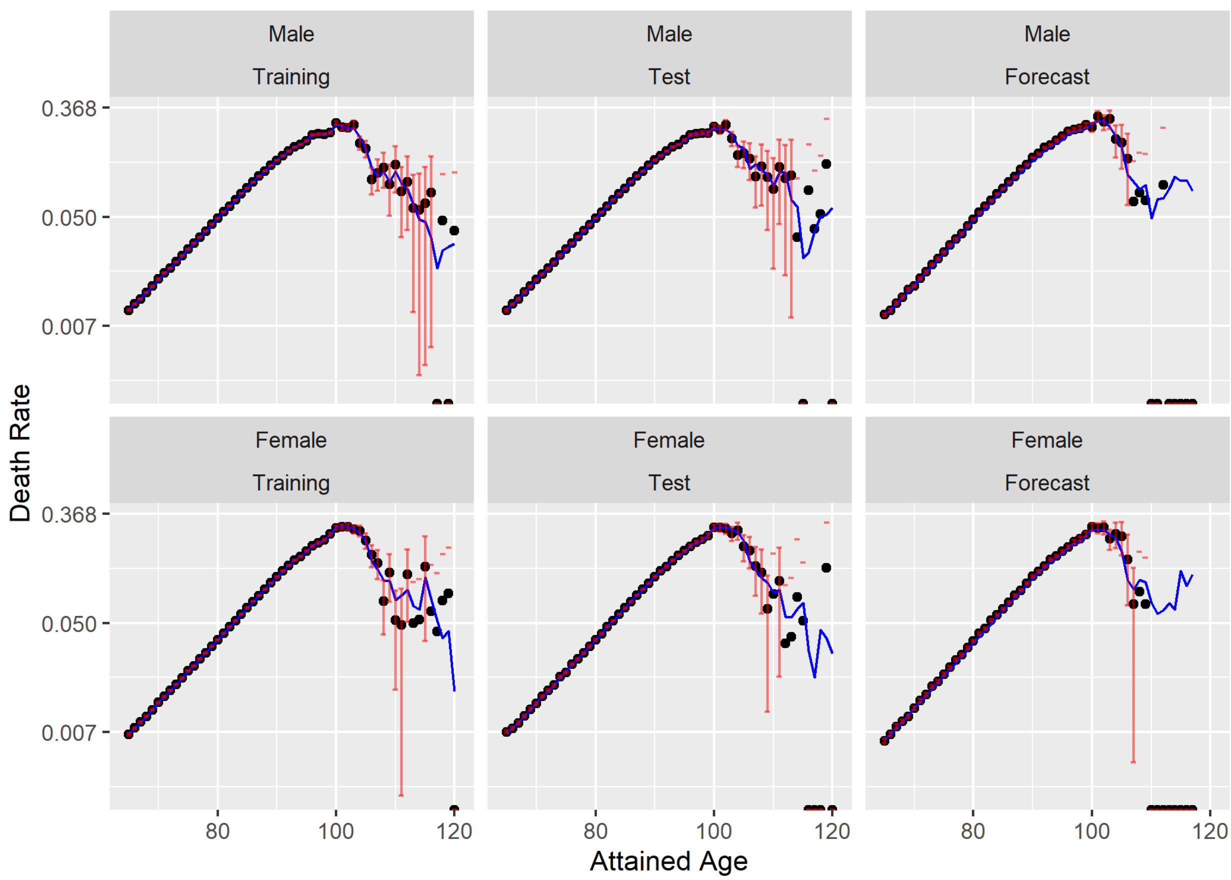


Figure 6.2
ACTUAL VS FITTED DEATH RATES BY DURATION, GENDER AND SUBSET

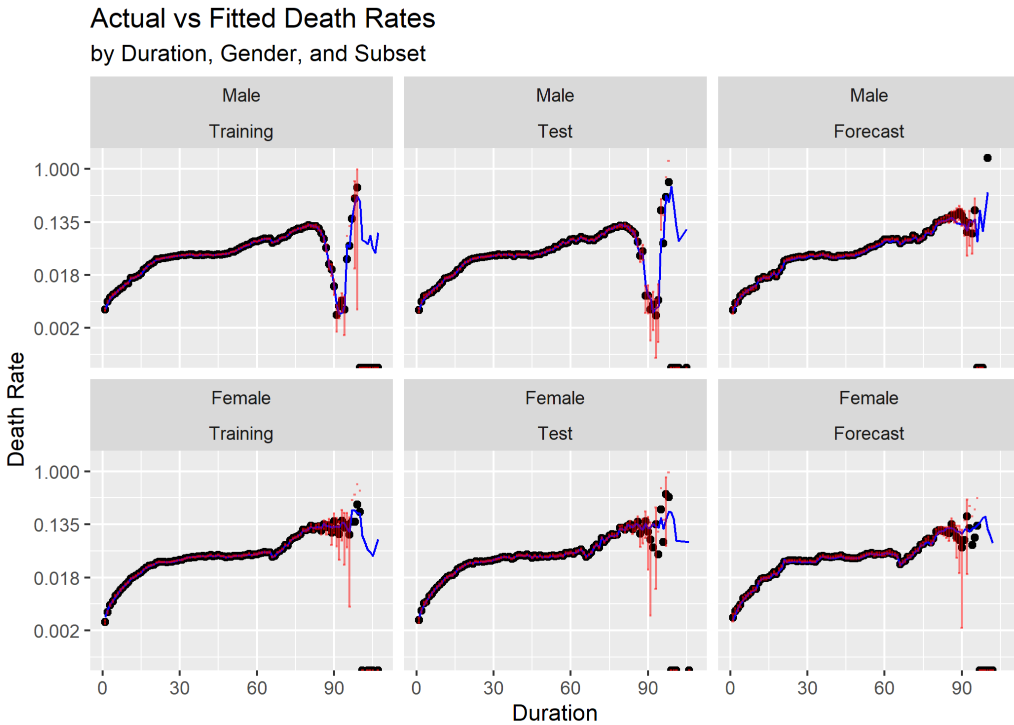
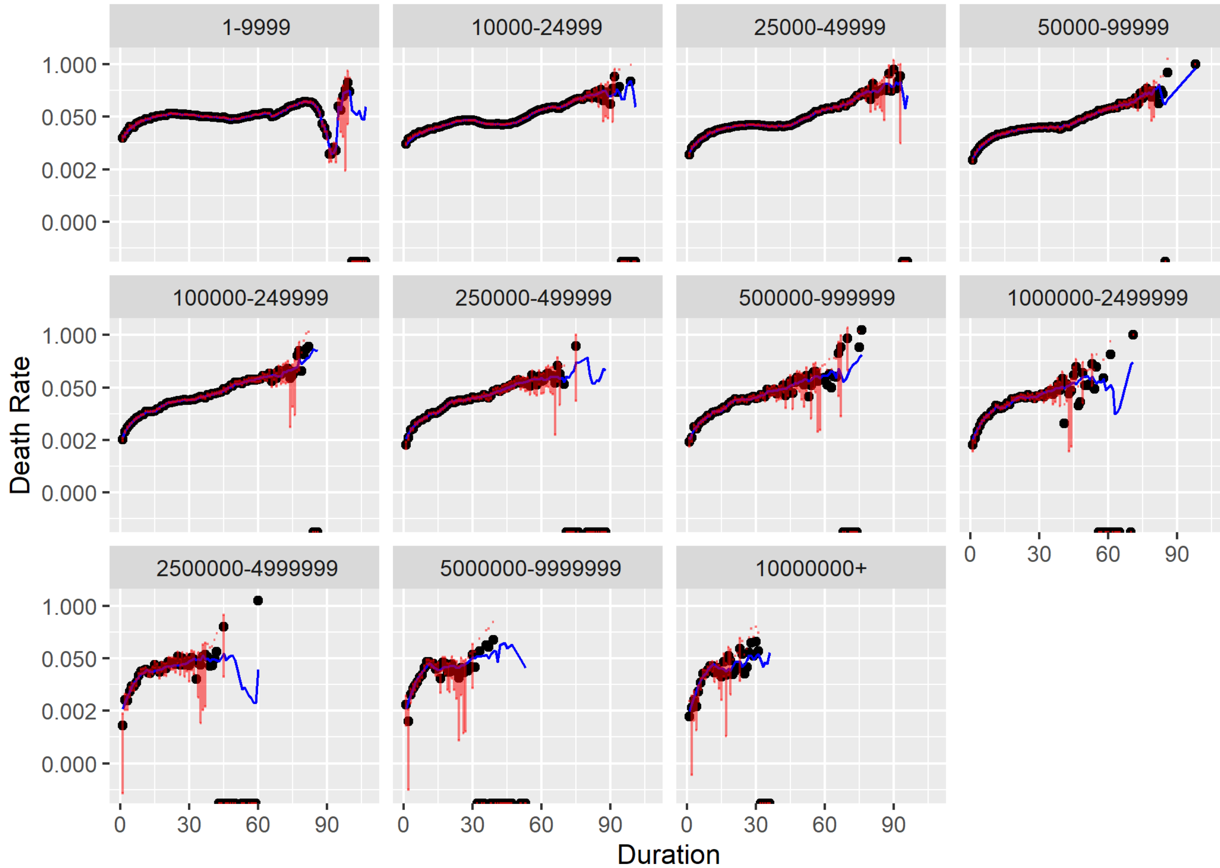


Figure 6.3
ACTUAL VS FITTED DEATH RATES BY DURATION AND FACE AMOUNT BAND



The model tends to fit well across the subsets. Even where data gets thin, the model tends to make a reasonable effort, provided that the model is allowed to run long enough to capture variation in every corner of the data.

6.2 FEATURE IMPORTANCE

Feature importance tables are a commonly presented collection of measures of the relative importance of a feature in a dataset. For a given pair of leaves of a decision tree, the feature importance of a feature or group of features is the mean square deviation of the predicted values in the leaves against the average value across the leaves. If more than one feature is used to reach a pair of leaves, this variation is divided equally across features. The total importance is the sum across all constituent trees.

A characteristic of this allocation rule is that features higher in a tree will gather more of the feature importance statistics propagating up from the leaves than will features typically appearing lower in the trees. Thus, a feature will be higher in the table of relative importance if a) it more frequently appears early in the tree and b) if a given split has wider deviation from the unsplit mean.

Boosted decision trees rely on sequentially fitted decision trees to fit a model and explain variation in the outcome. From a mathematical point of view, a decision tree is a step function. This setup is ideal for categorical outcomes. However, decision trees must work much harder to reproduce a continuous effect, such as the slope of mortality by attained age. Bear these facts in mind when interpreting the feature importance table.

Table 6.1
ESTIMATED MODEL FEATURE IMPORTANCE

Feature	Importance
Attained_Age	58.8%
Issue_Age	9.3%
Smoker_Status	6.4%
Select_Ultimate_Indicator	5.3%
Duration	4.8%
Gender	3.5%
Preferred_Indicator	2.7%
Issue_Year	2.4%
Age_Basis	1.9%
Face_Amount_Band	1.9%
Number_Of_Preferred_Classes	0.8%
Preferred_Class	0.7%
SOA_Post_level_Term_Indicator	0.6%
Insurance_Plan	0.5%
Observation_Year	0.2%
SOA_Guaranteed_Level_Term_Period	0.1%
SOA_Anticipated_Level_Term_Period	0.0%

The foregoing table shows the relative importance of features in the model.

An initial read of the table would be surprising to an experienced actuary, as gender seems too low on the list of important features among other issues. This would be misleading. Because attained age has a smoothly increasing impact on mortality on average, the decision tree must create many more splits for its step function to approximate attained age effects. This has the effect of amplifying the importance of attained age and, therefore, muting the apparent impact of other variables. A dive into SHAP values reinforces this point of view.

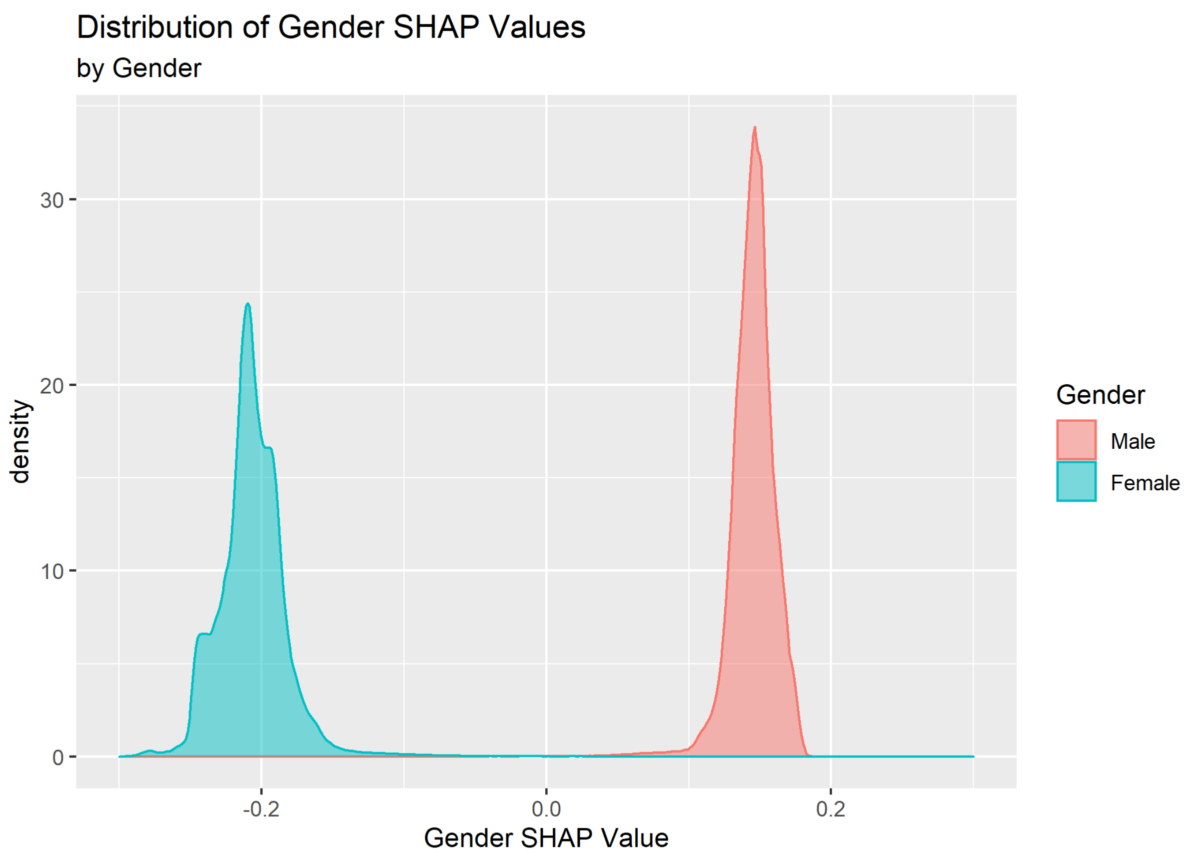
6.3 SHAP VALUE REVIEW

SHAP values can be provided for each row. While the description of SHAP values and how they are computed is outside our scope, they can be thought of as a decomposition of a predicted value into its components by feature. For example, the SHAP values for a row in the training data are a vector of the contributions of each feature to the predicted outcome. It is analogous to the coefficient of a predictor variable in a main-effects-only regression model.

Plotting SHAP values on such a large dataset requires careful consideration. SHAP values are often presented as point plots. The underlying training data have nearly six million rows. Plotting that many data elements on a plot is generally ill-advised without special formatting or processing.

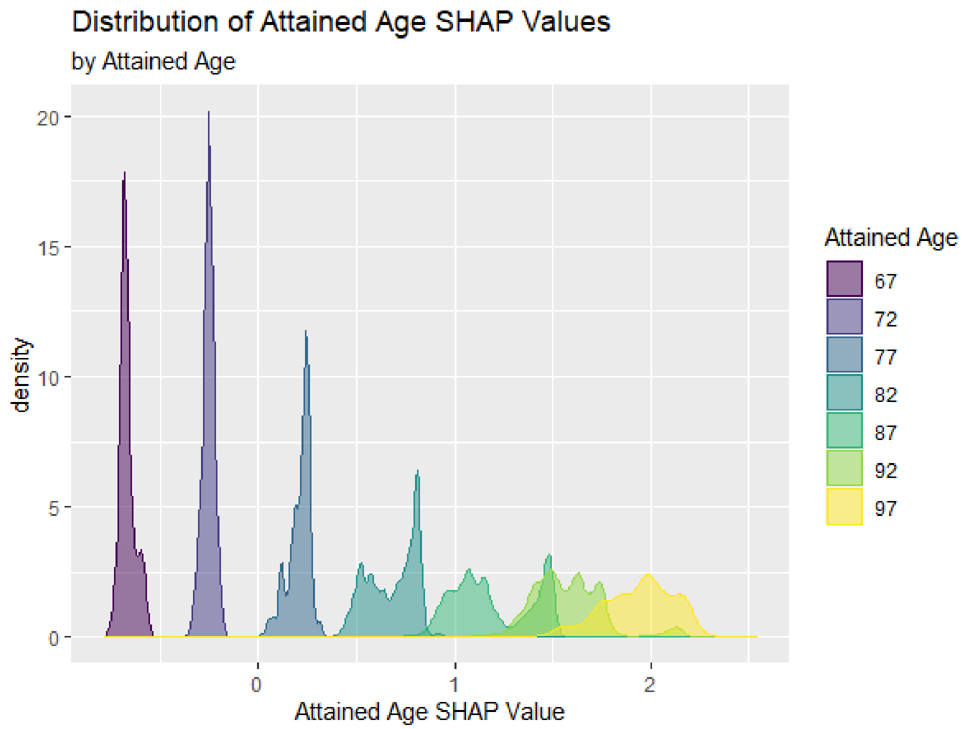
We saw that gender was low in the feature importance table. Let’s see what happens in the SHAP values.

Figure 6.4
DISTRIBUTION OF GENDER SHAP VALUES BY GENDER



Compare this with the distribution of attained age SHAP values.

Figure 6.5
DISTRIBUTION OF ATTAINED AGE SHAP VALUES BY ATTAINED AGE



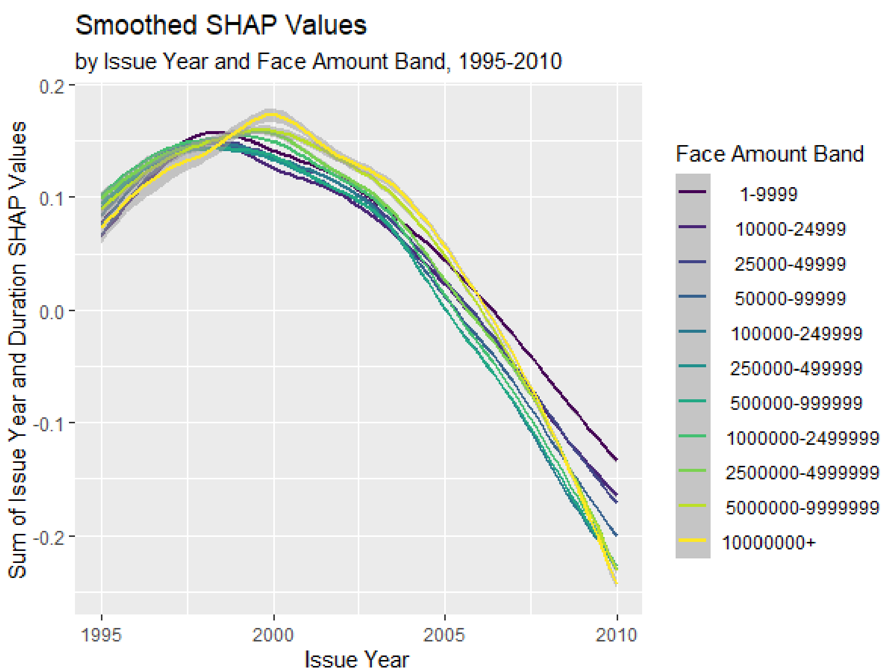
There was far more overall variation to the contributions of **Age** to mortality variation than for gender. An interesting aside is that there is evidence for multimodality in higher **Ages**, especially attained age 87.

The point of this comparison was that feature importance should not be used in isolation for assessment of variable importance or worthiness of inclusion. Both **Age**, a feature with high relative importance, and gender, a feature with seemingly minor importance, are both needed.

As a prelude to some of the analysis that follows, consider what we find if we look at smoothed SHAP values by issue year and face amount band.

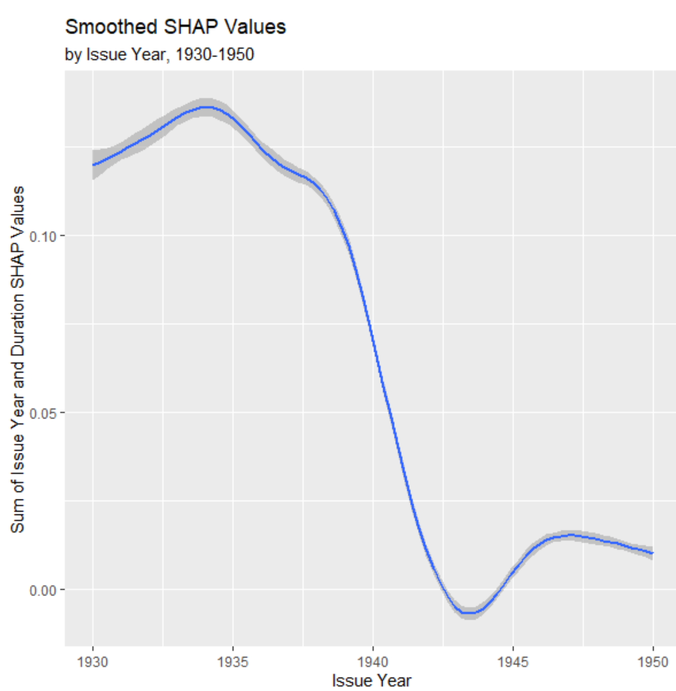
In the next figure, we see smoothed SHAP values by issue year and face amount band, where the plotted values are the sum of the SHAP values for issue year and for duration. The colors from darker to lighter represent increasing face amount bands. Dashing is intended to further allow one to see the patterns. Due to socioeconomic and underwriting differences by face amount, we would expect mortality to be lower with increasing face amount, all else equal. Starting in the late 1990s, the average SHAP value for higher face amounts breaks this pattern. The average SHAP values for higher face amounts rise above the lines for lower face amounts.

Figure 6.6
SMOOTHED SHAP VALUES BY ISSUE YEAR AND FACE AMOUNT BAND, 1995-2010



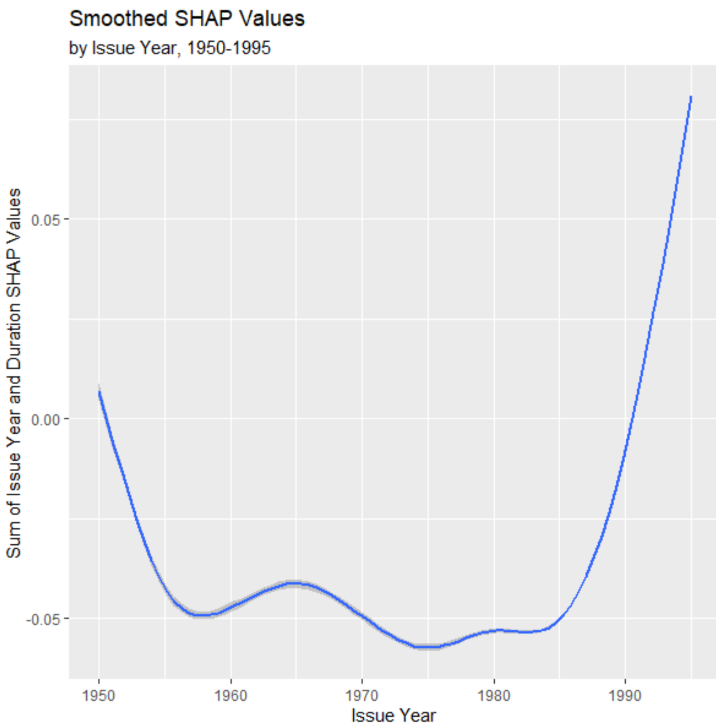
Looking to the much older issue years in the next figure, one can see that issue years from the Great Depression appear to have higher mortality than the issue years immediately following World War II.

Figure 6.7
SMOOTHED SHAP VALUES BY ISSUE YEAR, 1930-1950



Finally, in the next figure, the issue years in between exhibit phenomena worthy of further research, but which are not readily answered from the data itself.

Figure 6.8
SMOOTHED SHAP VALUES BY ISSUE YEAR, 1950-1995



6.4 INTERACTION TERMS

In addition to single feature importance, analysis was performed to understand the most impactful interaction terms of features. The most impactful interactions are issue age with attained age, age with issue year, and issue age and duration.

Table 6.2

TOP 20 FEATURE INTERACTIONS

Rank	Feature Pairs		Score
	Feature 1	Feature 2	
1	Issue_Age	Attained_Age	7.49
2	Issue_Age	Issue_Year	6.09
3	Attained_Age	Issue_Year	5.89
4	Issue_Age	Duration	4.22
5	Duration	Attained_Age	4.00
6	Observation_Year	Attained_Age	3.81
7	Observation_Year	Issue_Age	3.50
8	Duration	Issue_Year	3.22
9	Observation_Year	Issue_Year	3.15
10	Attained_Age	Face_Amount_Band	2.82
11	Issue_Age	Face_Amount_Band	2.45
12	Face_Amount_Band	Issue_Year	2.43
13	Observation_Year	Duration	2.15
14	Smoker_Status	Issue_Age	1.45
15	Insurance_Plan	Attained_Age	1.45
16	Smoker_Status	Attained_Age	1.43
17	Duration	Face_Amount_Band	1.34
18	Insurance_Plan	Issue_Age	1.34
19	Observation_Year	Face_Amount_Band	1.25
20	Insurance_Plan	Issue_Year	1.15

Issue year turned up to be a key interaction term. This was consistent with the OAM Subgroup hypothesis that the mortality experience was influenced by the different issue and underwriting eras. Additional analysis was then performed to identify the significant two-way interactions of variables with issue year. Table 6.3 shows the top interactions.

Table 6.3
FEATURE INTERACTIONS WITH ISSUE YEAR

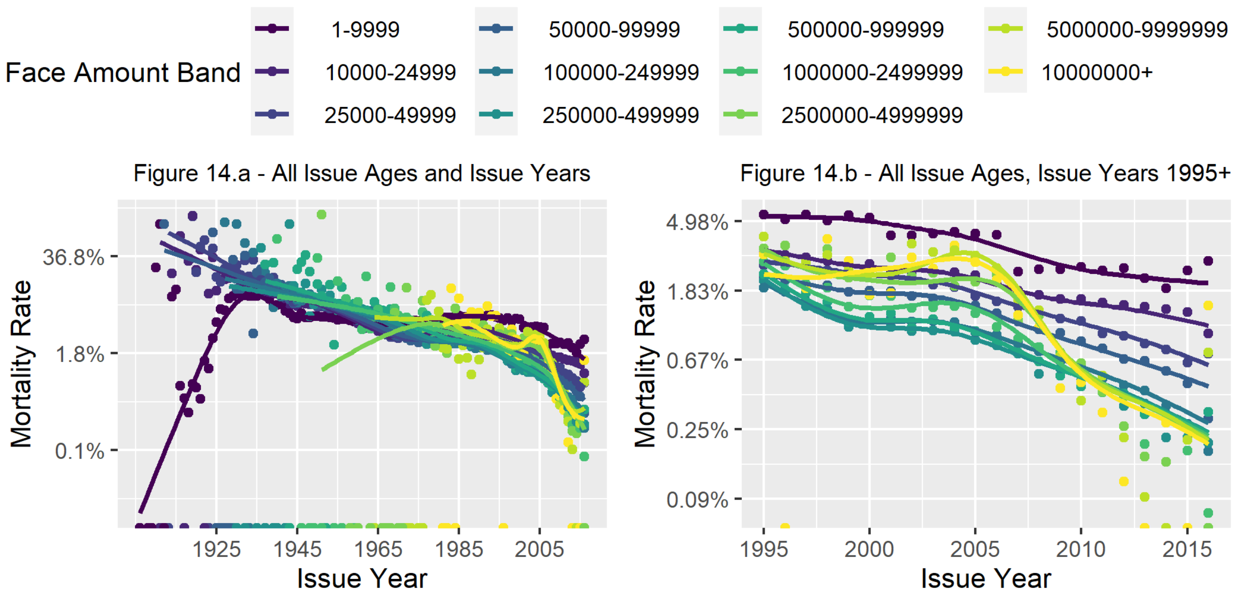
Rank	Feature Pairs		Score
	Feature 1	Feature 2	
2	Issue_Age	Issue_Year	6.09
3	Attained_Age	Issue_Year	5.89
8	Duration	Issue_Year	3.22
9	Observation_Year	Issue_Year	3.15
12	Face_Amount_Band	Issue_Year	2.43
20	Insurance_Plan	Issue_Year	1.15
22	Smoker_Status	Issue_Year	0.68
30	Issue_Year	SOA_Post_level_Term_Indicator	0.48
39	Issue_Year	Preferred_Class	0.38
50	Preferred_Indicator	Issue_Year	0.18
51	Issue_Year	SOA_Guaranteed_Level_Term_Period	0.17
58	Issue_Year	Number_Of_Preferred_Classes	0.13
76	Issue_Year	SOA_Anticipated_Level_Term_Period	0.05

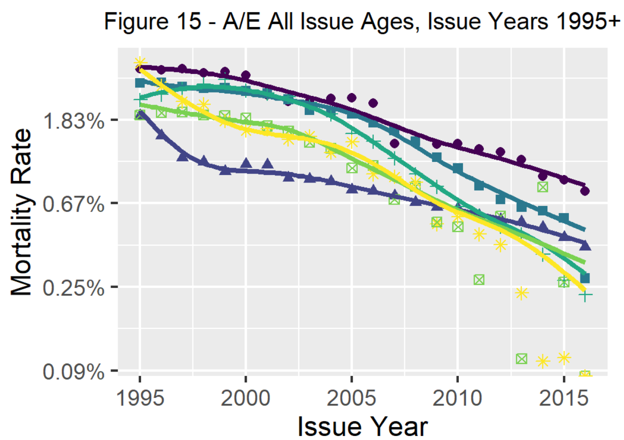
The most significant interactions that warranted further exploration were those with age, duration, observation year, face amount band, and insurance plan.

Figures 14 through 21 graphically represent the plots of the actual to model mortality rates by issue year and face amount band for the subsets shown below. For each subset, the graphs are provided for all issue years and those for 1995+. In addition to the model and actual mortality rates, the model A/E and actual A/E where the E is the 2015 VBT are also provided:

a) By issue year and face amount band
b) By issue year and face amount band for issue years 1995+
c) By issue year and face amount band for issue ages 70+
d) By issue year and face amount band for issue ages 70+ for issue years 1995+
e) By issue year and insurance plan for issue years 1995+
f) By issue year and risk class structure

For each of the figures below, the dots represent the actual mortality rates, and the lines are the smoothed versions of the modeled rates using a GAM smoother.





Observations:

- Overall, mortality rates follow the general pattern of being lower as face amount increases.
- When isolating to issue years 1995 and later, there is an odd hump to the mortality rates for the largest face amount bands in the early to mid-2000s; this is most pronounced for face amounts of $2.5 million and above. The pattern appears to normalize back to more expected levels for issue years 2010 and later.
- The A/E for the higher face amounts does not exhibit this mortality rate pattern, which suggests that the mortality "hump" is already built in to the 2015 VBT.

It appears likely that the smoker risks could fit better with a quadratic model, but NS and U appear to be

The sharp departure of a monotonic mortality pattern at the higher face amounts for issue years in the early to mid-2000s led to a question as to whether this was driven by the increased prominence of STOLI business entered into during that similar time period and whether that could be a driver in the **Older Age** mortality patterns. Figures 16.c, 16.d and 17 further look at the analysis for issue ages 70 and above.

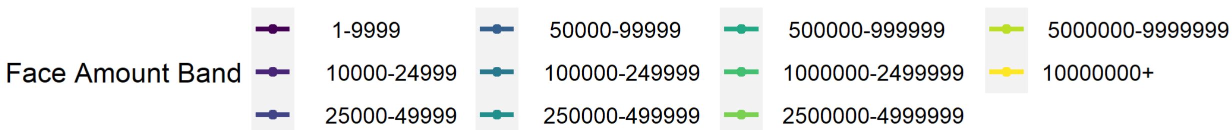


Figure 16.c - Issue Ages 70+, All Issue Years

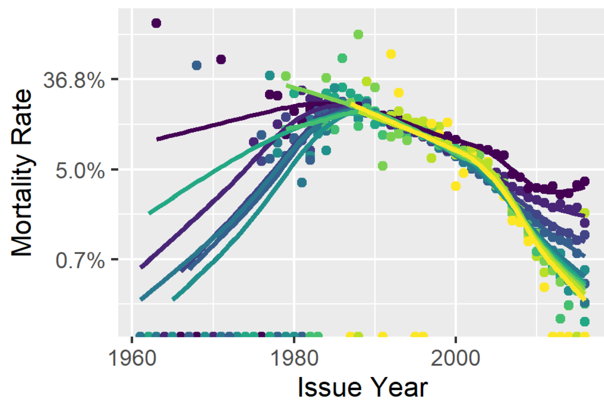


Figure 16.d - Issue Ages 70+, Issue Years 1995+

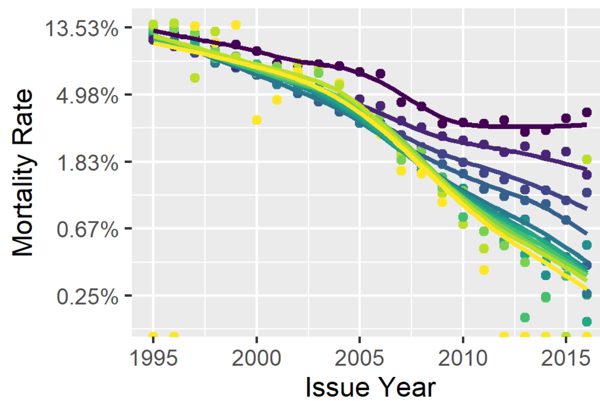
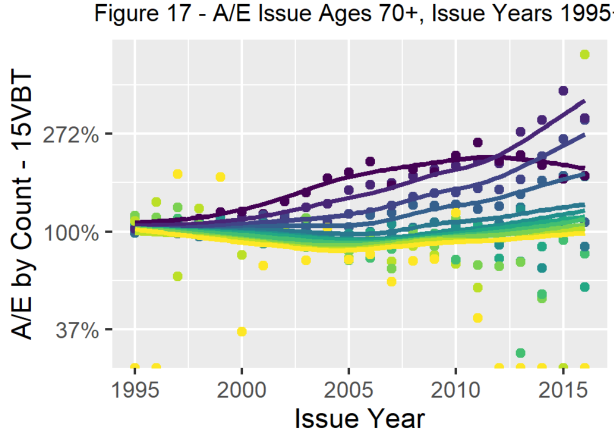


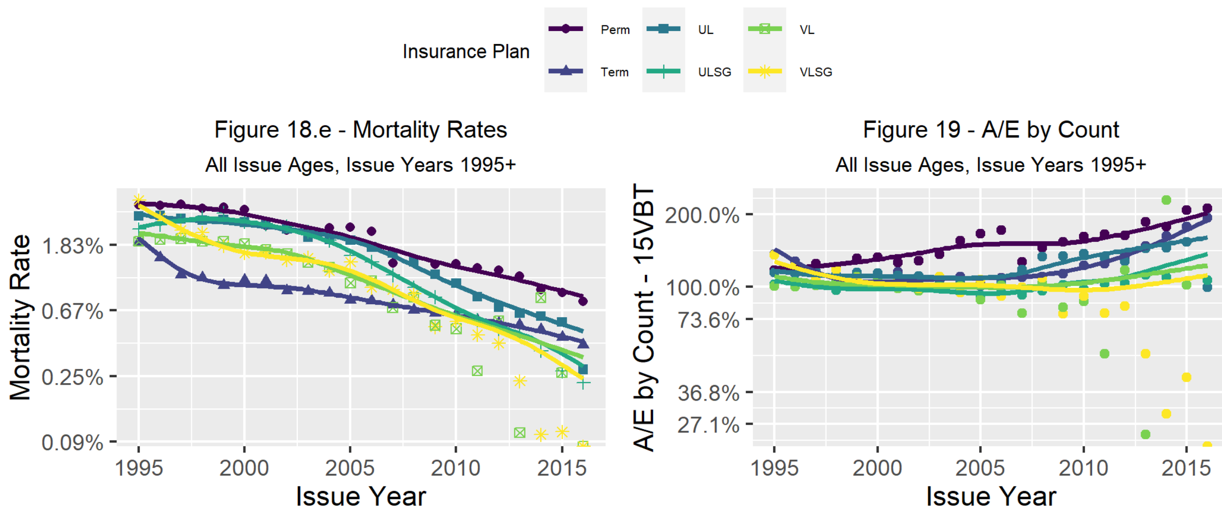
Figure 17 - A/E Issue Ages 70+, Issue Years 1995+

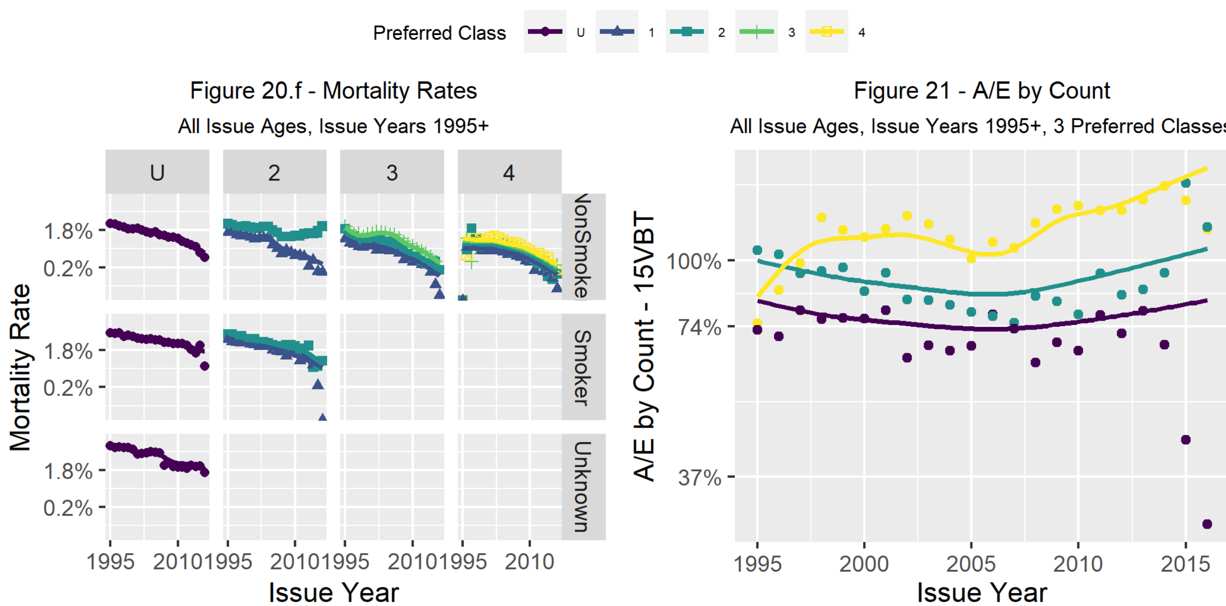


Observations:

- When isolating to issue ages 70 and above, we do continue to observe a slight uptick in the mean mortality for face amounts above approximately $1M to $2.5M from the late 1990s until the late 2000s. Though this change in mortality was present for issue ages 70+, it was most acute for Ages 65+ and issue ages < 70.
- Therefore, it does not appear that this differential by face amount explains the Older Age mortality patterns.

The OAM Subgroup then reviewed the experience by plan type and risk class structure to see how the changes in the mix of business could be influencing the mortality rates. Figures 18.e and 19 show the mortality and A/E experience, respectively, by plan type for issue years 1995 and later. Figures 20.f and 21 show the mortality and A/E experience, respectively, by risk class structure or number of risk classes. For these analyses, we only focused on issue years 1995 and later.





Observations:

- By product, it appears that the shifts in the mortality rates are occurring in the UL/VL/ULSG/VLSG subsets. However, when analyzing the A/E, this is less apparent and the highest A/Es appear to be in the permanent and term plans. This is likely due to the analysis by count.
- By risk class structure, the shift in mortality appears prominent in the three-class non-smoker systems, which corresponds roughly with peak issue years for that system. This shift or uptick in mortality rate appears to be driven mostly by the residual risk classes irrespective of the total number of preferred classes.

- However, when one considers actual and model experience relative to the 2015 VBT, the phenomenon vanishes in this dataset. That suggests that this is due to a shift in average age of exposures.

Based on the above observations, further analysis was performed to determine if insights could be drawn from examining the changes in exposures and average issue age over time. Exposure trends and average issue ages for this analysis went back to issue years 1990 and later. Figures 22 and 23 show the exposure trends for average issue age by face amount band since issue year 1990. Figures 24 and 25 show the exposure trends for average issue age by issue year and face amount grouping for the various insurance plan types.

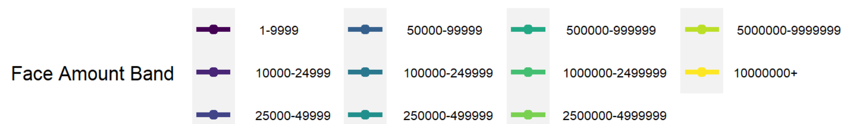


Figure 22 - Exposure Trend by Face Amount Band
All Issue Ages, Issue Years 1990+

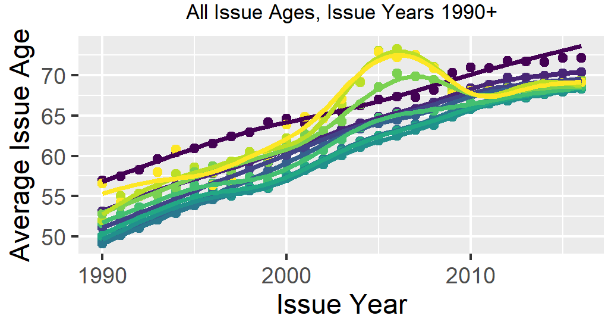
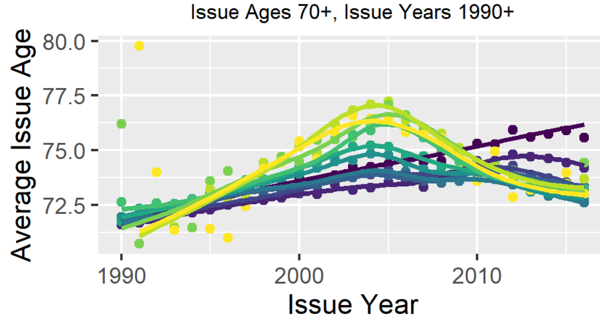


Figure 23 - Exposure Trend by Face Amount Band
Issue Ages 70+, Issue Years 1990+



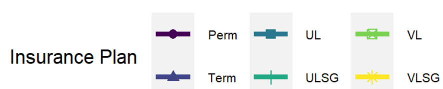


Figure 24 - Exposure Trend by Plan Type and Face Amount Group
All Issue Ages, Issue Years 1990+

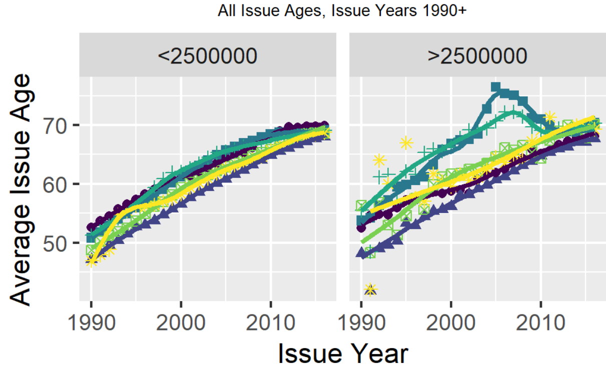
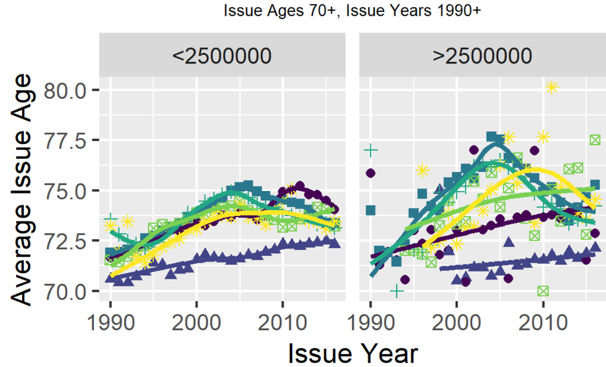


Figure 25 - Exposure Trend by Plan Type and Face Amount Group
Issue Ages 70+, Issue Years 1990+



Observations:

- By face amount bands, the sharp increases in the average age are also consistent with the increases in the face amount mortality in the early to late 2000s and most prominent for face amounts $2.5 million and above.
- Focusing on issue ages 70 and above, the subsets with the most obvious shifts above $2.5 million for the UL, USSG and VLSG plan, with the UL and ULSG the most prominent.

Section 7: Overall Conclusions

The basic comparisons and regression analyses suggest that there are exposure issues in the underlying insured experience data that are not necessarily observed just by analyzing the A/E relative to the 2015 VBT nor by grouping all products, risk classes or face amounts above $1 million together. The non-monotonic pattern of mortality and very low A/Es for the insurance experience is highly indicative of a problem with the overall exposures at these Oldest Ages.

The boosted decision trees provided output that suggests clues as to where to focus further analysis of the Older Age mortality. While the results were too opaque to yield conclusive insights, the analysis does suggest more focus should be placed on the interactions of issue year and certain other variables. When focusing on these interactions, unusual shifts in the average of mortality or exposure characteristics were observed, especially for certain products.

There is nothing to suggest that the insured mortality at Ages above 95 does not follow a similar pattern as the general population, especially when normalized for the highest socioeconomic group.

To answer the questions set out by the OAM Subgroup, we reached the following conclusions:

1) **Data Completeness.** Though there is a significant increase in the exposure and claims data in the insured population at the Older Ages, the odd mortality pattern appears to be driven in large part by an over-reporting of the exposures, more likely caused by maturing policies not taken off the books. This suggests something more complicated than the underreporting of deaths that has been noted in prior table development efforts.
2) **Relationship between insured and general population mortality.** We did not observe strong support to suggest the insured mortality at the Older Ages exhibited a different pattern of mortality than the general U.S. population, once normalized for socioeconomic equivalency.
3) **Cohort differences.** There is evidence to suggest differences between attained age mortality at the Older Ages from newer issue year cohorts at larger face amounts, though this seems to have normalized beginning around 2010. This is more difficult to observe when viewing an A/E analysis than the raw mortality rates and more advanced analysis with interactions. While there does seem to be some evidence to suggest potential STOLI policies between 2004 and 2007 for permanent business exhibit a different mortality pattern from the other blocks of policies, and that shifts in the mix of business do have an impact on the mortality rates, there was nothing conclusive to indicate either of these are the drivers to explain the Older Age mortality patterns. The data was not at a granular enough level to perform more in-depth analysis.

While the OAM Subgroup analysis did not draw any firm conclusions, the analysis does point to some potential areas for further study. Carriers may want to further investigate their own data quality, specifically around older policies which may have previously matured.

Section 8: Acknowledgments

The authors' deepest gratitude goes to those without whose efforts this project could not have come to fruition: the volunteers who generously shared their wisdom, insights, advice, guidance, and arm's-length review of this study prior to publication. Any opinions expressed may not reflect their opinions nor those of their employers. Any errors belong to the authors alone.

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Appendix A: Catboost Models and Information

The following code and text are from the R markdown containing the analysis around the catboost models of the **Older Age** data. Some of the embedded narrative has been edited to be coherent as an appendix to this document, even though the output has been stripped out.

A. DATA LOAD AND PREP

The data.table package is used for efficient data management. The ggplot2 and lemon packages are used for plotting. The packages, Flextable and Officedown, are used for creating tables. The package, TAM, has convenient weighted-moment functions.

The catboost library is used to run the boosted decision trees. It is similar in character to xgboost. However, catboost has better native handling of categorical predictors, which is the predominant feature type in this dataset. The library also has faster tree algorithms and better protections against overfitting.

The training subset is limited to a random sample of 70% of rows prior to 2017. The subset of test data is in the remaining 30% of rows prior to 2017. Observation year 2017 rows are used as a forecasting subset. For our purposes, the testing and forecast subsets are intended to check whether there are any problems at a high level with the model.

For speed, GPU is used for fitting. The GPUs used here are an nVidia Geforce RTX 2080 Ti and an nVidia Geforce GTX 1080 Ti, both having 11GB of VRAM. Most of the VRAM was used.

Four derived columns are appended to the dataset. For this first round, only one is used.

- Actual_Qx_Count: the ratio of number of deaths by policies exposed, 0 for rows with no policies exposed
- Actual_Qx_Amount: the ratio of death claim amount by amount exposed, 0 for rows with no amount exposed
- AE_Count: actual-to-expected ratio by count versus the 2015VBT
- AE_Amount: actual-to-expected ratio by amount versus the 2015VBT

```
ilec.dat <- readRDS('ilec.dat.rds')

ilec.dat <- ilec.dat[Observation_Year <= 2017]
ilec.dat[,Actual_Qx_Count:=ifelse(Policies_Exposed>0,
                                  Number_Of_Deaths/Policies_Exposed,
                                  0)]
ilec.dat[,Actual_Qx_Amount:=ifelse(Amount_Exposed>0,
                                   Death_Claim_Amount/Amount_Exposed,
                                   0)]

ilec.dat[,AE_Count:=ifelse(Expected_Death_QX2015VBT_by_Policy>0,
                           Number_Of_Deaths/Expected_Death_QX2015VBT_by_Policy,
                           0)]
ilec.dat[,AE_Amount:=ifelse(Expected_Death_QX2015VBT_by_Amount>0,
                            Death_Claim_Amount/Expected_Death_QX2015VBT_by_Amount,
                            0)]

set.seed(123)
ilec.dat[,IsTraining:=Observation_Year < 2017
         & runif(nrow(.SD)) <= training.fraction]
ilec.dat[,Subset:=factor(ifelse(IsTraining,
                                "Training",
```

```
                                ifelse(Observation_Year < 2017,
                                       "Test",
                                       "Forecast")),
                         levels=c("Training","Test","Forecast"))]

ilec.dat[,Attained_Age_Grp:=cut(Attained_Age,
                                breaks=c(0,69,74,79,84,89,95,120),
                                labels=c('65-69','70-74',
                                         '75-79','80-84',
                                         '85-89','90-95',
                                         '96+'))]

ilec.dat[,Issue_Age_Grp:=cut(Issue_Age,
                             breaks=c(-1,69,74,79,84,89,94,99),
                             labels=c("<70","70-74",
                                      "75-79","80-84",
                                      "85-89","90-94",
                                      "95-99"))]

ilec.dat[,IY_Grp_1:=cut(Issue_Year,
                        breaks=c(1900,1989,1999,2009,2017),
                        labels=c('1900-1989',
                                 '1990-1999',
                                 '2000-2009',
                                 '2010-2017'))]
```

The following creates a quick summary of the train-test-forecast split. We see that, by amount, the split is biased more toward the forecast subset. This reflects the increasing average face amount of policies over time. Average amount exposed in 2009 is 59,414, while the same for 2015 is 104,144.

```
ilec.dat.summary <- ilec.dat[,.(`Row Count`=.N,
            `Number Of Deaths`=sum(Number_Of_Deaths),
            `Policies Exposed`=sum(Policies_Exposed),
            `Death Claim Amount`=sum(Death_Claim_Amount)/1e9,
            `Amount Exposed`=sum(Amount_Exposed)/1e12),
         by=.(Subset,
              `Observation Year`=ifelse(Observation_Year==2017,
                                        '2017',
                                        '2009-2016'))]

ilec.dat.summary[,`:=`(`Number Of Deaths (%)`=100*`Number Of Deaths`/sum(`Number Of D
eaths`),
                       `Policies Exposed (%)`=100*`Policies Exposed`/sum(`Policies Ex
posed`),
                       `Death Claim Amount (%)`=100*`Death Claim Amount`/sum(`Death C
laim Amount`),
                       `Amount Exposed (%)`=100*`Amount Exposed`/sum(`Amount Exposed`
))]

setcolorder(ilec.dat.summary,
            c(1,2,3,4,8,5,9,6,10,7,11))

tbl.summary <- ilec.dat.summary %>%
  flextable(col_keys = names(ilec.dat.summary)) %>%
```

```
border_remove() %>%
# Format header
add_header_row(values=c("","By Count","By Amount"),
               colwidths = c(3,4,4)) %>%
add_header_row(
  values="Data Train-Test-Forecast Statistics",
  colwidths = 11
) %>%
align(i=2,j=4:7,align="center",part="header") %>%
align(i=2,j=8:11,align="center",part="header") %>%
align(i=1,j=1:11,align="center",part="header") %>%
hline_top(
          border=officer::fp_border(
            color="black",
            style="solid",
            width=2
          ),
          part="header") %>%
surround(
  i=1,
  j=1:11,
  border.bottom = officer::fp_border(
    color="black",
    style="solid",
    width=2
  ),
  part="header"
) %>%
surround(
  i=2,
  j=4:7,
  border.bottom = officer::fp_border(
    color="black",
    style="solid",
    width=2
  ),
  part="header"
) %>%
surround(
  i=2,
  j=8:11,
  border.bottom = officer::fp_border(
    color="black",
    style="solid",
    width=2
  ),
  part="header"
) %>%
surround(
  i=3,
  j=1:11,
  border.bottom = officer::fp_border(
    color="black",
    style="solid",
    width=2
```

```
  ),
  part="header"
) %>%
# Body
# Format columns
colformat_double(
  j=8,
  suffix="B",
  digits=2
) %>%
colformat_double(
  j=10,
  suffix="T",
  digits=2
) %>%
colformat_double(
  j=c(5,7,9,11),
  suffix="%",
  digits=1
) %>%
border_inner_h(
  border=officer::fp_border(
    color="black",
    style="solid",
    width=1
  ),
  part="body"
) %>%
surround(
  j=4,
  border.left = officer::fp_border(
    color="black",
    style="solid",
    width=2
  )
) %>%
surround(
  j=8,
  border.left = officer::fp_border(
    color="black",
    style="solid",
    width=2
  )
) %>%
surround(
  i=2:3,
  j=4,
  border.left = officer::fp_border(
    color="black",
    style="solid",
    width=2
  ),
  part="header"
) %>%
surround(
```

```
    i=2:3,
    j=8,
    border.left = officer::fp_border(
      color="black",
      style="solid",
      width=2
    ),
    part="header"
  ) %>%
  bold(part="header")

if(!knitr::pandoc_to("docx")) {
  tbl.summary
} else {
  tbl.summary %>% fontsize(size=8,part="all")
}

if(saveTables)
  tbl.summary %>% fontsize(size=8,part="all") %>% save_as_docx(path="datastats.docx")

#rm(ilec.dat.summary,tbl.summary)
```

Catboost requires that the data be converted into its own format. Three items are specified:

- data for predictor variables, e.g., insurance plan, attained age, etc.
- label for response variable, in this case raw death rate by count
- weight for the weighting variable, in this case policies exposed

A parameters object also needs to be specified. The loss function is “Poisson,” which uses a Poisson likelihood as the optimization target. Also supplied is an overfitting threshold of 0.05. Task type will be either CPU or GPU.

```
train_pool <- catboost.load_pool(
  data=ilec.dat[IsTraining==TRUE,2:18],
  label = ilec.dat[IsTraining==TRUE,Actual_Qx_Count],
  weight = ilec.dat[IsTraining==TRUE,Policies_Exposed])

test_pool <- catboost.load_pool(
  data=ilec.dat[IsTraining==FALSE & Observation_Year < 2017,2:18],
  label = ilec.dat[IsTraining==FALSE & Observation_Year < 2017,
                   Actual_Qx_Count],
  weight = ilec.dat[IsTraining==FALSE & Observation_Year < 2017,
                    Policies_Exposed])

forecast_pool <- catboost.load_pool(
  data=ilec.dat[Observation_Year == 2017,2:18],
  label = ilec.dat[Observation_Year == 2017,Actual_Qx_Count],
  weight = ilec.dat[Observation_Year == 2017,Policies_Exposed])

fit_params <- list(iterations = 2000,
                   task_type=cb.task.type,
                   loss_function = 'Poisson',
                   verbose=100,
                   od_pval=0.05)
```

B. MODELING

The model here is a catboost model. The main attractive innovation of catboost over xgboost is the handling of categorical predictors. In xgboost, the traditional method has been to use one-hot encoding by default. For a category with n levels, one-hot encoding creates a column for each level, with 1 where the level is present and 0 otherwise. For example, gender would be expanded to two columns. A column for males with 1 whenever the row has gender of male, and 0 otherwise. Females would have the complementary definition.

Catboost uses a different approach to encoding categorical predictors. The original paper provides a reasonable description for the technically-inclined on how their method addresses shortcomings of prior encoding strategies.

```
if(useCached)
{
  cb.model <- catboost.load_model(model_path = "oa.mod.cbm")
} else {
  cb.model <- catboost.train(learn_pool=train_pool,
                             test_pool = test_pool,
                             params=fit_params)

  catboost.save_model(cb.model,
                      model_path = 'oa.mod.cbm')
}

train_pred <- catboost.predict(cb.model,train_pool)
test_pred <- catboost.predict(cb.model,test_pool)
forecast_pred <- catboost.predict(cb.model,forecast_pool)

ilec.dat[IsTraining==TRUE,
         `:=`(Predicted_Deaths=Policies_Exposed*exp(train_pred))]
ilec.dat[IsTraining==FALSE & Observation_Year < 2017,
         `:=`(Predicted_Deaths=Policies_Exposed*exp(test_pred))]
ilec.dat[Observation_Year == 2017,
         `:=`(Predicted_Deaths=Policies_Exposed*exp(forecast_pred))]
```

C. HIGH-LEVEL FIT CHECKS

C.1 ACTUAL VERSUS FITTED DEATH RATES BY ATTAINED AGE, AND SUBSET

```
p <- ggplot(
  data=ilec.dat[,.(Number_Of_Deaths=sum(Number_Of_Deaths),
                   Policies_Exposed=sum(Policies_Exposed),
                   Predicted_Deaths=sum(Predicted_Deaths)),
                by=.(Attained_Age,Gender,Subset)],
      aes(x=Attained_Age)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed)) +
  geom_line(aes(y=Predicted_Deaths/Policies_Exposed),color="blue") +
  geom_errorbar(aes(ymin=qnorm(0.025,
                               Number_Of_Deaths,
                               sqrt(Number_Of_Deaths*
                                      (Policies_Exposed -
Number_Of_Deaths)/Policies_Exposed))
                      /Policies_Exposed,
                    ymax=qnorm(0.975,
                               Number_Of_Deaths,
                               sqrt(Number_Of_Deaths*
```

```
                                      (Policies_Exposed - 
Number_Of_Deaths)/Policies_Exposed))
                      /Policies_Exposed),
                color="red", alpha=0.5) +
  facet_wrap(vars(Gender,Subset),nrow=2) +
  scale_y_continuous(trans="log",name="Death Rate",labels = 
scales::label_number(accuracy = 0.001)) +
  scale_x_continuous(name="Attained Age")
print(p +
  ggtitle("Actual vs Fitted Death Rates",
          subtitle="by Attained Age, Gender, and Subset"))

if(saveFigures)
  ggsave("figure11.png",
         plot=p,
         device="png",
         width=7,
         height=5,
         units="in")
```

C.2 ACTUAL VERSUS FITTED DEATH RATES BY DURATION, GENDER AND SUBSET

```
p <- ggplot(data=ilec.dat[,.(Number_Of_Deaths=sum(Number_Of_Deaths),
                                        Policies_Exposed=sum(Policies_Exposed),
                                        Predicted_Deaths=sum(Predicted_Deaths)),
                          by=.(Duration,Gender,Subset)][Policies_Exposed >0],
       aes(x=Duration)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed)) +
  geom_line(aes(y=Predicted_Deaths/Policies_Exposed),color="blue") +
  geom_errorbar(aes(ymin=qnorm(0.025,
                               Number_Of_Deaths,
                               sqrt(Number_Of_Deaths*
                                      (Policies_Exposed - Number_Of_Deaths)/Policies_
Exposed))
                      /Policies_Exposed,
                    ymax=qnorm(0.975,
                               Number_Of_Deaths,
                               sqrt(Number_Of_Deaths*
                                      (Policies_Exposed - Number_Of_Deaths)/Policies_
Exposed))
                      /Policies_Exposed),
                color="red", alpha=0.5) +
  facet_wrap(vars(Gender,Subset),nrow=2) +
  scale_y_continuous(trans="log",name="Death Rate",labels = scales::label_number(accu
racy = 0.001)) +
  scale_x_continuous(name="Duration") +
  ggtitle("Actual vs Fitted Death Rates",
          subtitle="by Duration, Gender, and Subset")
print(p +
  ggtitle("Actual vs Fitted Death Rates",
          subtitle="by Duration, Gender, and Subset"))
```

```
if(saveFigures)
  ggsave("figure12.png",
         plot=p,
         device="png",
         width=7,
         height=5,
         units="in")
```

C.3 ACTUAL VERSUS FITTED DEATH RATES BY DURATION AND FACE AMOUNT BAND

```
p <- ggplot(data=ilec.dat[Subset=="Training",.(Number_Of_Deaths=sum(Number_Of_Deaths)
,
                                        Policies_Exposed=sum(Policies_Exposed),
                                        Predicted_Deaths=sum(Predicted_Deaths)),
                     by=.(Duration,Face_Amount_Band)][Policies_Exposed >0],
       aes(x=Duration)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed)) +
  geom_line(aes(y=Predicted_Deaths/Policies_Exposed),color="blue") +
  geom_errorbar(aes(ymin=qnorm(0.025,
                               Number_Of_Deaths,
                               sqrt(Number_Of_Deaths*
                                      (Policies_Exposed - Number_Of_Deaths)/Policies_
Exposed))
                      /Policies_Exposed,
                    ymax=qnorm(0.975,
                               Number_Of_Deaths,
                               sqrt(Number_Of_Deaths*
                                      (Policies_Exposed - Number_Of_Deaths)/Policies_
Exposed))
                      /Policies_Exposed),
                color="red", alpha=0.5) +
  facet_wrap(vars(Face_Amount_Band),nrow=3) +
  scale_y_continuous(trans="log",name="Death Rate",labels = scales::label_number(accu
racy = 0.001)) +
  scale_x_continuous(name="Duration")

print(p +
  ggtitle("Actual vs Fitted Death Rates",
          subtitle=" by Duration and Face Amount Band, Training Data"))

if(saveFigures)
  ggsave("figure13.png",
         plot=p,
         device="png",
         width=7,
         height=5,
         units="in")
```

D. FEATURE IMPORTANCE

```
if(!useCached)
{
  feature_importance <- data.table(
    catboost.get_feature_importance(cb.model,
```

```
                                    train_pool),
    keep.rownames = T)

  setnames(feature_importance,c("rn","V1"),c("Variable","Importance"))
  saveRDS(feature_importance,'feature_importance.rds')
} else
  feature_importance <- readRDS("feature_importance.rds")

feat.sum <- feature_importance[order(-Importance)] %>%
    setnames("Variable","Feature") %>%
    flextable() %>%
    border_remove() %>%
    add_header_row(values="Estimated Model Feature Importance",
                   colwidths = 2) %>%
    align(i=1,align="center",part="header") %>%
    align(i=2,j=1,align="left",part="header") %>%
    align(i=2,j=2,align="right",part="header") %>%
    hline_top(border=officer::fp_border(
        color="black",
        style="solid",
        width=2
      ),
      part="header") %>%
    surround(i=1,
             border.bottom = officer::fp_border(
               color="black",
               style="solid",
               width=2
             ),
             part="header") %>%
    surround(i=2,
             border.bottom = officer::fp_border(
               color="black",
               style="solid",
               width=2
             ),
             part="header") %>%
    border_inner_h(
      border = officer::fp_border(
        color="black",
        style="solid",
        width=1
      ),
      part="body"
    ) %>%
    colformat_double(
      j=2,
      suffix="%",
      digits=1
    ) %>%
    hline_bottom(
      border=officer::fp_border(
        color="black",
        style="solid",
        width=2
```

```
      ),
      part="body"
    ) %>%
    bold(part="header") %>%
    autofit()

feat.sum

if(saveTables)
  feat.sum %>% save_as_docx(path="featsum.docx")
```

E. SHAP VALUE REVIEW

```
if(!useCached)
{
  shaps <- catboost.get_feature_importance(cb.model,
                                           train_pool,
                                           type="ShapValues")

  colnames(shaps) <- c(names(ilec.dat)[2:18],"BIAS")
  shaps <- data.table(shaps)

  # Format that is understandable by SHAPforxgboost plotting
  BIAS0 <- shaps[,.(BIAS)]
  shaps[,BIAS:=NULL]
  imp <- colMeans(abs(shaps))

  imp <- unlist(
    lapply(shaps,
           function(l)
             sum(l * ilec.dat[IsTraining==TRUE,
                              Policies_Exposed])/sum(ilec.dat[IsTraining==TRUE,
                                                              Policies_Exposed])
           )
    )

  shaps.plot <- list(shap_score=shaps,
                     mean_shap_score=imp[order(imp,decreasing = T)],
                     BIAS0=BIAS0)

  #ilec.dat.shaps <- cbind(ilec.dat[IsTraining==TRUE,c(2:22,27,32,54)],shaps)

  shaps_int <- as.data.table(catboost.get_feature_importance(cb.model,
                                           train_pool,
                                           type="Interaction"))

  shaps_int[,`:=`(F1 = names(ilec.dat)[feature1_index+2],
                  F2 = names(ilec.dat)[feature2_index+2])]
  shaps_int[,Rank:=1:nrow(.SD)]
  setcolorder(shaps_int,c("Rank","F1","F2","score"))
  shaps_int[,`:=`(feature1_index=NULL,feature2_index=NULL)]

  rm(shaps,BIAS0,imp)

  saveRDS(shaps.plot,'shaps.plot.rds')
```

```
  saveRDS(shaps_int,'shaps_int.rds')
} else {
  shaps.plot <- readRDS("shaps.plot.rds")
  shaps_int <- readRDS("shaps_int.rds")
}
```

E.1 SHAP VALUE DISTRIBUTION – GENDER

```
if(!useCached)
{
  shaps <- catboost.get_feature_importance(cb.model,
                                           train_pool,
                                           type="ShapValues")

  colnames(shaps) <- c(names(ilec.dat)[2:18],"BIAS")
  shaps <- data.table(shaps)

  # Format that is understandable by SHAPforxgboost plotting
  BIAS0 <- shaps[,.(BIAS)]
  shaps[,BIAS:=NULL]
  imp <- colMeans(abs(shaps))

  imp <- unlist(
    lapply(shaps,
           function(l)
             sum(l * ilec.dat[IsTraining==TRUE,
                              Policies_Exposed])/sum(ilec.dat[IsTraining==TRUE,
                                                              Policies_Exposed])
           )
    )

  shaps.plot <- list(shap_score=shaps,
                     mean_shap_score=imp[order(imp,decreasing = T)],
                     BIAS0=BIAS0)

  #ilec.dat.shaps <- cbind(ilec.dat[IsTraining==TRUE,c(2:22,27,32,54)],shaps)

  shaps_int <- as.data.table(catboost.get_feature_importance(cb.model,
                                           train_pool,
                                           type="Interaction"))

  shaps_int[,`:=`(F1 = names(ilec.dat)[feature1_index+2],
                  F2 = names(ilec.dat)[feature2_index+2])]
  shaps_int[,Rank:=1:nrow(.SD)]
  setcolorder(shaps_int,c("Rank","F1","F2","score"))
  shaps_int[,`:=`(feature1_index=NULL,feature2_index=NULL)]

  rm(shaps,BIAS0,imp)

  saveRDS(shaps.plot,'shaps.plot.rds')
  saveRDS(shaps_int,'shaps_int.rds')
} else {
  shaps.plot <- readRDS("shaps.plot.rds")
  shaps_int <- readRDS("shaps_int.rds")
}
```

```
names(shaps.plot$shap_score) <- paste0(names(shaps.plot$shap_score),"_shap")
shaps.plot.sum <- cbind(
  ilec.dat[Subset=="Training",
           .(Gender,Policies_Exposed)],
  shaps.plot$shap_score[,
                        .(Gender_shap)])

shaps.plot.sum.means <- shaps.plot.sum[,
                                .(Gender_shap_mean=sum(Gender_shap*Policies_Exposed)/
                                    sum(Policies_Exposed)),
                                by=.(Gender)]

p <- ggplot(data=shaps.plot.sum,
       aes(x=Gender_shap,y=..density..,color=Gender,fill=Gender)) +
  geom_density(alpha = 0.5) +
  scale_x_continuous(name="Gender SHAP Value",limits=c(-.3,.3)) +
  ggtitle("Distribution of Gender SHAP Values",
        subtitle="by Gender")

p

if(saveFigures)
  ggsave("dist_gendershap.png",
         plot=p,
         device="png",
         width=7,
         height=5,
         units="in")
```

E.2 SHAP VALUE DISTRIBUTION – ATTAINED AGE

```
shaps.plot.sum <- cbind(
  ilec.dat[Subset=="Training",
           .(Attained_Age,Policies_Exposed)],
  shaps.plot$shap_score[,
                        .(Attained_Age_shap)])

p <- ggplot(data=shaps.plot.sum,
       aes(x=Attained_Age_shap,y=..density..)) +
  geom_density(alpha = 0.5) +
  scale_x_continuous(name="Attained Age SHAP Value") +
  ggtitle("Distribution of Attained Age SHAP Values")

p

if(saveFigures)
  ggsave("dist_attainedageshap.png",
         plot=p,
         device="png",
         width=7,
         height=5,
         units="
```

F. INFLUENTIAL 2-WAY INTERACTIONS

F.1 TOP 20

```
shaps_int.tab <- head(shaps_int,20) %>%
  setnames(c("F1","F2","score"),
           c("Feature 1","Feature 2","Score")) %>%
  flextable() %>%
  add_header_row(
    values=c("","Feature Pairs",""),
    colwidths = c(1,2,1)
  ) %>%
  add_header_row(
    values="Top 20 Feature Interactions",
    colwidths = 4
  ) %>%
  align(
    i=1:2,
    align="center",
    part="header"
  ) %>%
  bold(part="header") %>%
  border_inner_h(
    border=officer::fp_border(
      color="black",
      style="solid",
      width=1
    ),
    part="body"
  ) %>%
  colformat_double(
    digits=2
  ) %>%
  width(j=2:3,
        1.6)

shaps_int.tab

if(saveTables)
  shaps_int.tab %>% save_as_docx(path="shapsint.docx")
```

F.2 INTERACTIONS WITH ISSUE YEAR

```
shaps_int.tab <- shaps_int[F1 == "Issue_Year" | F2 == "Issue_Year"] %>%
  setnames(c("F1","F2","score"),
           c("Feature 1","Feature 2","Score")) %>%
  flextable() %>%
  add_header_row(
    values=c("","Feature Pairs",""),
    colwidths = c(1,2,1)
  ) %>%
  add_header_row(
    values="Feature Interactions with Issue Year",
    colwidths = 4
  ) %>%
```

```
  align(
    i=1:2,
    align="center",
    part="header"
  ) %>%
  bold(part="header") %>%
  border_inner_h(
    border=officer::fp_border(
      color="black",
      style="solid",
      width=1
    ),
    part="body"
  ) %>%
  colformat_double(
    digits=2
  ) %>%
  width(j=2,
        1.6) %>%
  width(j=3,
        2.9)

shaps_int.tab

if(saveTables)
  shaps_int.tab %>% save_as_docx(path="shapsint_iy.docx")
```

G. PLOTS

The following code produces plots of the actual and model mortality rates by issue year and face amount band for specific subsets. The dots are the actual rates. The lines are smoothed versions of the modeled rates.

G.1 MODEL AND ACTUAL DEATH RATES BY COUNT BY ISSUE YEAR AND FACE AMOUNT BAND

```
p1 <- ggplot(data=ilec.dat[IsTraining==TRUE,.(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed,color=Face_Amount_Band)) +
  geom_smooth(aes(y=Predicted_Deaths/Policies_Exposed,color=Face_Amount_Band,weight=P
olicies_Exposed),
              method="gam",
              se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "Mortality Rate") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1900,2017,10)) +
  scale_color_viridis_d(name="Face Amount Band")

p1 +
  ggtitle(label="Model and Actual Death Rates by Count",
          subtitle = "By issue year and face amount band")

if(saveFigures)
  ggsave("iy_death_rates_fa.png",
```

```
     plot=p1,
     device="png",
     width=7,
     height=5,
     units="in")

p2 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                             Issue_Year >= 1995,.(Number_Of_Deaths=sum(Number_Of_Deat
hs),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed,color=Face_Amount_Band)) +
  geom_smooth(aes(y=Predicted_Deaths/Policies_Exposed,color=Face_Amount_Band,weight=P
olicies_Exposed),
              method="gam",
              se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "Mortality Rate") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,5)) +
  scale_color_viridis_d(name="Face Amount Band")
```

G.2 MODEL AND ACTUAL DEATH RATES BY COUNT BY ISSUE YEAR AND FACE AMOUNT BAND, ISSUE YEARS 1995+

```
p2 +
  ggtitle(label="Model and Actual Death Rates by Count",
          subtitle = "By issue year and face amount band, issue year 1995+")

if(saveFigures)
  ggsave("iy_death_rates_fa_ia95plus.png",
     plot=p2,
     device="png",
     width=7,
     height=5,
     units="in")

p3 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                             Issue_Age >= 70,.(Number_Of_Deaths=sum(Number_Of_Deaths)
,
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed,color=Face_Amount_Band)) +
  geom_smooth(aes(y=Predicted_Deaths/Policies_Exposed,color=Face_Amount_Band,weight=P
olicies_Exposed),
              method="gam",
              se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "Mortality Rate") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1900,2017,10)) +
  scale_color_viridis_d(name="Face Amount Band")
```

G.3 MODEL AND ACTUAL DEATH RATES BY COUNT BY ISSUE YEAR AND FACE AMOUNT BAND, ISSUE AGES 70+

```
p3 +
  ggtitle(label="Model and Actual Death Rates by Count",
          subtitle = "By issue year and face amount band, issue age 70+")

if(saveFigures)
  ggsave("iy_death_rates_fa_iy70plus.png",
     plot=p3,
     device="png",
     width=7,
     height=5,
     units="in")
```

G.4 MODEL AND ACTUAL DEATH RATES BY COUNT BY ISSUE YEAR AND FACE AMOUNT BAND, ISSUE AGES 70+, ISSUE YEARS 1995+

```
p4 <- ggplot(data=ilec.dat[IsTraining==TRUE & Issue_Age >= 70 & Issue_Year >= 1995,.(
Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed,color=Face_Amount_Band)) +
  geom_smooth(aes(y=Predicted_Deaths/Policies_Exposed,weight=Policies_Exposed,color=F
ace_Amount_Band),
              se=F,
              method="gam") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,5)) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "Mortality Rate") +
  scale_color_viridis_d(name="Face Amount Band")

p4 +
  ggtitle(label="Model and Actual Death Rates by Count",
          subtitle = "By issue year and face amount band, issue age 70+, issue year 1
995+")

if(saveFigures)
  ggsave("iy_death_rates_fa_iy95plus_ia70plus.png",
     plot=p4,
     device="png",
     width=7,
     height=5,
     units="in")
```

G.5 MODEL AND ACTUAL DEATH RATES BY COUNT BY ISSUE YEAR AND PLAN, ISSUE YEARS 1995+

```
pmm1 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                               Insurance_Plan != "Other" &
                               Issue_Year >= 1995,
                             .(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
```

```
                        Predicted_Deaths=sum(Predicted_Deaths)),
                     by=.(Insurance_Plan,Issue_Year)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed,color=Insurance_Plan,shape=Insur
ance_Plan)) +
  geom_smooth(aes(y=Predicted_Deaths/Policies_Exposed,color=Insurance_Plan,weight=Pol
icies_Exposed),
              method="gam",se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "Mortality Rate") +
  scale_color_viridis_d(name="Insurance Plan") +
  scale_shape_discrete(name="Insurance Plan") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,5))

pmm1 +
  ggtitle(label="Model and Actual Death Rates by Count",
          subtitle = "By issue year and plan, Issue years 1995+")

if(saveFigures)
  ggsave("ae_by_iy_ip_iy95plus.png",
         plot=pmm1,
         device="png",
         width=7,
         height=5,
         units="in")
```

G.6 MODEL AND ACTUAL DEATH RATES BY COUNT BY ISSUE YEAR AND RISK CLASS STRUCTURE, ISSUE YEARS 1995+

```
pmm2 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                               Issue_Year >= 1995,
                             .(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths)),
                     by=.(Smoker_Status,Number_Of_Preferred_Classes,Preferred_Class,
Issue_Year)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Policies_Exposed,color=Preferred_Class,shape=Pref
erred_Class)) +
  geom_smooth(aes(y=Predicted_Deaths/Policies_Exposed,color=Preferred_Class,weight=Po
licies_Exposed),
              method="gam",se=F) +
  scale_color_viridis_d(name="Preferred Class") +
  facet_grid(rows=vars(Smoker_Status),cols=vars(Number_Of_Preferred_Classes)) +
  scale_y_continuous(trans="log",labels=scales::percent, limits = c(NA,.1),name = "Mo
rtality Rate") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,10)) +
  scale_shape_discrete(name="Preferred Class")

pmm2  +
  ggtitle(label="Model and Actual Death Rates by Count",
          subtitle = "By issue year and risk class structure")

if(saveFigures)
```

```
  ggsave("ae_by_iy_pref_iy95plus.png",
         plot=pmm2,
         device="png",
         width=7,
         height=5,
         units="in")
```

G.7 MODEL AND ACTUAL A/E RATIOS BY COUNT BY ISSUE YEAR AND FACE AMOUNT BAND, ISSUE YEARS 1995+

```
pm1 <- ggplot(data=ilec.dat[IsTraining==TRUE & Issue_Year >= 1995,.(Number_Of_Deaths=
sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Face_Amo
unt_Band)) +
  geom_smooth(aes(y=Predicted_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Face_Am
ount_Band,weight=Policies_Exposed),
              method="gam",
              se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "A/E by Count - 15VBT"
) +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,5)) +
  scale_color_viridis_d(name="Face Amount Band")

pm1 +
  ggtitle(label="Model and Actual A/E Ratios by Count",
          subtitle = "By issue year and face amount band, issue year 1995+")

if(saveFigures)
  ggsave("ae_by_iy_fa.png",
         plot=pm1,
         device="png",
         width=7,
         height=5,
         units = "in")

pm1a <- ggplot(data=ilec.dat[IsTraining==TRUE &
                               Issue_Year >= 1995 &
                               Issue_Age >= 70,
                             .(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Face_Amo
unt_Band)) +
  geom_smooth(aes(y=Predicted_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Face_Am
ount_Band,weight=Policies_Exposed),
```

```
              method="gam",
              se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "A/E by Count - 15VBT"
) +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,5)) +
  scale_color_viridis_d(name="Face Amount Band")

pm1a +
  ggtitle(label="Model and Actual A/E Ratios by Count",
          subtitle = "By issue year and face amount band, issue year 1995+, issue age
70+")

if(saveFigures)
  ggsave("ae_by_iy_fa_ia70plus_iy95plus.png",
         plot=pm1a,
         device="png",
         width=7,
         height=5,
         units = "in")
```

G.8 MODEL AND ACTUAL A/E BY COUNT BY ISSUE YEAR AND PLAN, ISSUE YEARS 1995+

```
pm2 <- ggplot(data=ilec.dat[IsTraining==TRUE & Issue_Year >= 1995 & Insurance_Plan !=
'Other',.(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy)),
                     by=.(Issue_Year,Insurance_Plan)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Insuranc
e_Plan)) +
  geom_smooth(aes(y=Predicted_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Insuran
ce_Plan,weight=Policies_Exposed),
              method="gam",
              se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "A/E by Count - 15VBT"
) +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,5)) +
  scale_color_viridis_d(name="Insurance Plan")

pm2 +
  ggtitle(label="Model and Actual A/E Ratios by Count",
          subtitle = "By issue year and plan, issue year 1995+")

if(saveFigures)
  ggsave("ae_by_iy_plan.png",
         plot=pm2,
         device="png",
         width=7,
         height=5,
```

```
        units = "in")
```

G.9 MODEL AND ACTUAL A/E RATIOS BY COUNT BY ISSUE YEAR AND PREFERRED CLASS FOR 3-RISK CLASS STRUCTURE, ISSUE YEARS 1995+

```
pm3 <- ggplot(data=ilec.dat[IsTraining==TRUE & Issue_Year >= 1995 & Number_Of_Preferr
ed_Classes == 3,.(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy)),
                     by=.(Issue_Year,Preferred_Class)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Number_Of_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Preferre
d_Class)) +
  geom_smooth(aes(y=Predicted_Deaths/Expected_Death_QX2015VBT_by_Policy,color=Preferr
ed_Class,weight=Policies_Exposed),
              method="gam",
              se=F) +
  scale_y_continuous(trans="log",labels=scales::percent,name = "A/E by Count - 15VBT"
) +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1995,2017,5)) +
  scale_color_viridis_d(name="Preferred Class System")

pm3 +
  ggtitle(label="Model and Actual A/E Ratios by Count",
          subtitle = "By issue year and preferred class, three-class structure, issue
year 1995+")

if(saveFigures)
  ggsave("ae_by_iy_pref.png",
        plot=pm3,
        device="png",
        width=7,
        height=5,
        units = "in")
```

G.10 EXPOSURE TREND FOR AVERAGE ISSUE AGE BY ISSUE YEAR AND FACE AMOUNT BAND, ISSUE YEARS 1990+

```
paa1 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                               Issue_Year >= 1990,
                             .(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy),
                        Avg_Issue_Age=sum(Issue_Age*Policies_Exposed)/sum(Policies_Ex
posed)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Avg_Issue_Age,color=Face_Amount_Band)) +
```

```
  geom_smooth(aes(y=Avg_Issue_Age,color=Face_Amount_Band,weight=Policies_Exposed),
              method="gam",
              se=F) +
  scale_color_viridis_d(name="Face Amount Band") +
  scale_y_continuous(name="Average Issue Age") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1990,2017,5))

paa1 +
  ggtitle("Exposure Trend","Average Issue Age by Issue Year and Face Amount Band")

if(saveFigures)
  ggsave("aa_iy_fa.png",
         plot=paa1,
         device="png",
         width=7,
         height=5,
         units="in")
```

G.11 EXPOSURE TREND FOR AVERAGE ISSUE AGE BY ISSUE YEAR AND GROUPED FACE AMOUNT BAND BY INSURANCE PLAN, ISSUE YEARS 1990+

```
paa2 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                               Issue_Year >= 1990 &
                               Insurance_Plan != "Other",
                             .(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy),
                        Avg_Issue_Age=sum(Issue_Age*Policies_Exposed)/sum(Policies_Ex
posed)),
                     by=.(Issue_Year,Insurance_Plan,
                          Face_2=ifelse(Face_Amount_Band %in% c(" 2500000-4999999","
5000000-9999999","10000000+"),">2500000","<2500000"))],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Avg_Issue_Age,color=Insurance_Plan, shape=Insurance_Plan)) +
  geom_smooth(aes(y=Avg_Issue_Age,color=Insurance_Plan,weight=Policies_Exposed),
              method="gam",
              se=F) +
  facet_wrap(facets = vars(Face_2)) +
  scale_y_continuous(name="Average Issue Age") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1990,2017,5)) +
  scale_color_viridis_d(name="Insurance Plan") +
  scale_shape_discrete(name="Insurance Plan")

paa2 +
  ggtitle("Exposure Trend","Average Issue Age by Issue Year and Grouped Face Amount B
and")

if(saveFigures)
  ggsave("aa_iy_fa2.png",
         plot=paa1,
```

```
         device="png",
         width=7,
         height=5,
         units="in")
```

G.12 EXPOSURE TREND FOR AVERAGE ISSUE AGE BY ISSUE YEAR AND FACE AMOUNT BAND BY INSURANCE PLAN, ISSUE AGES 70+, ISSUE YEARS 1990+

```
paa3 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                               Issue_Year >= 1990 &
                               Issue_Age >= 70,
                             .(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy),
                        Avg_Issue_Age=sum(Issue_Age*Policies_Exposed)/sum(Policies_Ex
posed)),
                     by=.(Issue_Year,Face_Amount_Band)],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Avg_Issue_Age,color=Face_Amount_Band)) +
  geom_smooth(aes(y=Avg_Issue_Age,color=Face_Amount_Band,weight=Policies_Exposed),
              method="gam",
              se=F) +
  scale_y_continuous(name="Average Issue Age") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1990,2017,5)) +
  scale_color_viridis_d(name="Face Amount Band")

paa3 +
  ggtitle("Exposure Trend","Average Issue Age by Issue Year and Face Amount Band")

if(saveFigures)
  ggsave("aa_by_iy_fa_iy90plus_ia70+.png",
         device="png",
         plot=paa3,
         width=7,
         height=5,
         units="in")
```

G.13 EXPOSURE TREND FOR AVERAGE ISSUE AGE BY ISSUE YEAR AND GROUPED FACE AMOUNT BAND, ISSUE AGES 70+, ISSUE YEARS 1990+

```
paa4 <- ggplot(data=ilec.dat[IsTraining==TRUE &
                       Issue_Year >= 1990 &
                       Insurance_Plan != "Other"  &
                       Issue_Age >= 70,
                     .(Number_Of_Deaths=sum(Number_Of_Deaths),
                        Policies_Exposed=sum(Policies_Exposed),
                        Predicted_Deaths=sum(Predicted_Deaths),
                        Expected_Death_QX2015VBT_by_Policy=sum(Expected_Death_QX2015V
BT_by_Policy),
                        Avg_Issue_Age=sum(Issue_Age*Policies_Exposed)/sum(Policies_Ex
posed)),
```

```
                     by=.(Issue_Year,Insurance_Plan,
                          Face_2=ifelse(Face_Amount_Band %in% c(" 2500000-4999999","
5000000-9999999","10000000+"),">2500000","<2500000"))],
       aes(x=Issue_Year)) +
  geom_point(aes(y=Avg_Issue_Age,color=Insurance_Plan, shape=Insurance_Plan)) +
  geom_smooth(aes(y=Avg_Issue_Age,color=Insurance_Plan,weight=Policies_Exposed),
              method="gam",
              se=F) +
  facet_wrap(facets = vars(Face_2)) +
  scale_y_continuous(name="Average Issue Age") +
  scale_x_continuous(name="Issue Year",
                     breaks=seq(1990,2017,5)) +
  scale_color_viridis_d(name="Insurance Plan") +
  scale_shape_discrete(name="Insurance Plan")

paa4 +
  ggtitle("Exposure Trend","Average Issue Age by Issue Year and Grouped Face Amount B
and")

if(saveFigures)
  ggsave("aa_by_iy_gfa_iy90plus_ia70+.png",
         device="png",
         plot=paa4,
         width=7,
         height=5,
         units="in")

in")
```

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