SOCIETY OF ACTUARIES

Advanced Topics in General Insurance

Exam GIADV

Date: Thursday, November 4, 2021

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 8 questions numbered 1 through 8 with a total of 40 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Written-Answer Instructions

1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.

   a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 and σ^2 can be typed as sigma^2.

   b) In the Excel document formulas should be entered. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit. Rows can be inserted to the answer input area as required to provide space for your answer.

   c) Individual exams may provide additional directions that apply throughout the exam or to individual items.

2. The answer should be confined to the question as set.

3. Prior to uploading your Word and Excel files, each file should be saved and renamed with your five-digit candidate number in the filename.

4. The Word and Excel files that contain your answers must be uploaded before the five-minute upload period expires.

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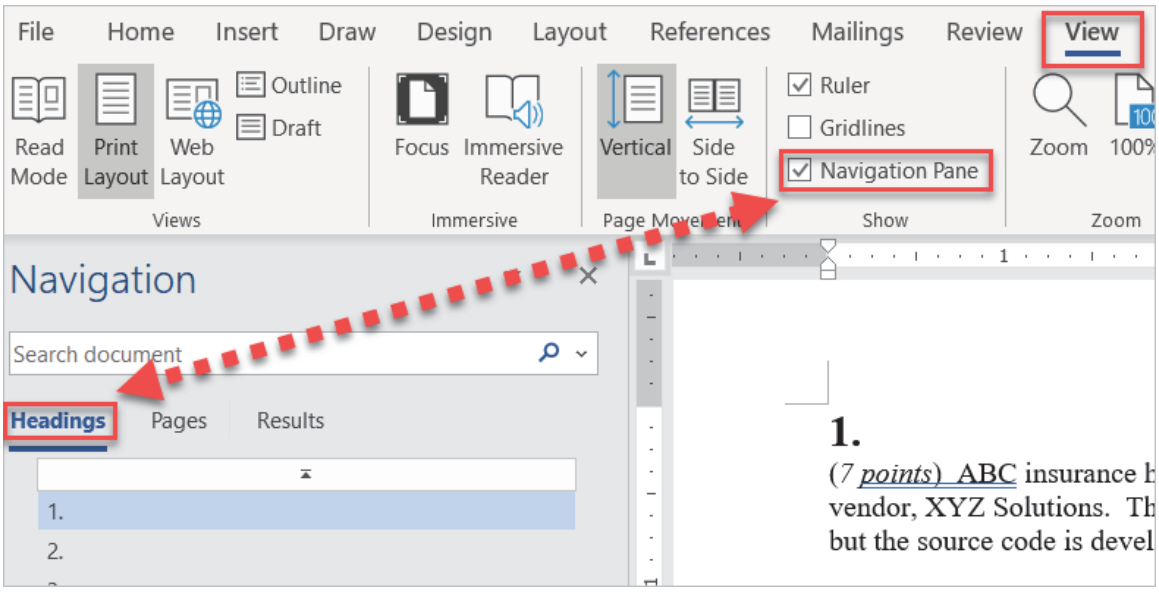
Navigation Instructions

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



1.

(*4 points*) You are modeling catastrophe losses.

The number of catastrophe losses, *n*, follows a negative binomial distribution with parameters $\alpha = 1$ and $p = 0.5$. This results in a distribution with:

- Mean 1
- Variance 2
- $\Pr(n) = (0.5)^{n+1}$, $n = 0, 1, 2, \ldots$

The loss size distribution is:

Loss Size (billions)	**Probability**
1	0.6
2	0.3
3	0.1

Loss sizes are independent of one another and independent of the number of losses.

Aggregate catastrophe losses have the following probability distribution for amounts below 5 billion:

Aggregate Losses (billions)	**Probability**
0	0.50000
1	0.15000
2	0.12000
3	0.08350
4	0.05055

(a) (*2 points*) Calculate the probability that aggregate losses will be 5 billion.

Provide the response for this part in the Excel spreadsheet.

(b) (*1.5 points*) Calculate the mean and coefficient of variation of aggregate catastrophe losses.

Provide the response for this part in the Excel spreadsheet.

1. Continued

(c) (*0.5 points*) Identify one disadvantage of using a recursive formula to calculate aggregate distribution probabilities.

Provide the response for this part in the Excel spreadsheet.

2.

(*4 points*) You are applying the Target Total Rate of Return model and are given the following information:

- The risk-free rate is 2%.
- The insurer's beta is 1.4.
- The market risk premium is 5%.
- Premiums are 900,000.
- Owner's equity is 500,000.
- Investable assets are 1,200,000.
- The insurer's investment return is 6%.

(a) (*1 point*) Calculate the target total rate of return.

Provide the response for this part in the Excel spreadsheet.

(b) (*1.5 points*) Calculate the underwriting profit margin.

Provide the response for this part in the Excel spreadsheet.

An alternative method to obtain the underwriting profit margin (*UPM*) is via Fairley's CAPM model. The formula is

$$UPM = -kR_f + \beta_u[E(R_m) - R_f].$$

(c) (*1.5 points*) Identify three differences in how Fairley's model determines *UPM* compared to the Target Total Rate of Return model.

Provide the response for this part in the Excel spreadsheet.

3.

(*5 points*) You are given the following data extracted from a triangle of cumulative paid losses:

Accident Year	From (months)	To (months)	Increment	Diagonal Age	Accident Year Total
2017	0	12	2,500	48	5,000
2017	12	24	1,800	48	5,000
2017	24	36	500	48	5,000
2017	36	48	200	48	5,000
2018	0	12	4,100	36	7,000
2018	12	24	2,000	36	7,000
2018	24	36	900	36	7,000
2019	0	12	4,600	24	6,800
2019	12	24	2,200	24	6,800
2020	0	12	5,300	12	5,300

You apply Clark's stochastic reserving model using the LDF method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-x/\theta}$ where x is in months.

The maximum likelihood estimate of θ is 8.15473.

(a) (*2 points*) Calculate the maximum likelihood estimate of ultimate losses (*ULT*) for each of the four accident years.

Provide the response for this part in the Excel spreadsheet.

(b) (*1 point*) Estimate the scale factor, σ^2.

Provide the response for this part in the Excel spreadsheet.

(c) (*1 point*) Estimate the process standard deviation of the loss reserve for all accident years combined.

Provide the response for this part in the Excel spreadsheet.

3. Continued

In the Excel spreadsheet, there is a graph that shows the normalized residuals plotted against the increment age. It will be automatically populated with values once the work in parts (a) and (b) is completed.

(d) (*0.5 points*) Describe how the graph should appear if the model assumptions are satisfied.

Provide the response for this part in the Excel spreadsheet.

(e) (*0.5 points*) Determine if the model assumptions are satisfied, based on this graph.

Provide the response for this part in the Excel spreadsheet.

4.

(*9 points*) You are interested in determining a model for loss development. The triangle of loss data you are working with, by accident year (AY) and development year, is:

	Development Year						
AY	1	2	3	4	5	6	7
1	6,012	8,269	11,907	12,805	15,539	17,181	20,009
2	106	5,285	5,396	11,666	13,782	16,599	
3	4,410	8,992	14,873	16,141	19,735		
4	4,655	11,555	14,766	21,266			
5	2,092	9,565	16,836				
6	513	6,445					
7	1,557						

(a) (*1.5 points*) State the three statistical assumptions underlying the chain ladder model.

Provide the response for this part in the Excel spreadsheet.

(b) (*2 points*) Demonstrate that the test statistic suggested by Mack to test for a calendar year effect is equal to 2.

Provide the response for this part in the Excel spreadsheet.

A test statistic equal to 2 indicates that there is a calendar year effect and implies that one of the chain ladder assumptions does not hold.

(c) (*0.5 points*) Identify that assumption.

Provide the response for this part in the Excel spreadsheet.

As an alternative to the chain ladder model, you begin with the Cape Cod method where the expected emerged loss at each age is a development term $f(d)$. This is equivalent to the additive chain ladder model.

(d) (*1 point*) Calculate the development terms $f(1)$-$f(7)$ that minimize the sum of squared residuals.

Provide the response for this part in the Excel spreadsheet.

4. Continued

(e) (*1 point*) Estimate the loss that will emerge in the next calendar year for accident years 2-7 combined.

Provide the response for this part in the Excel spreadsheet.

You then add diagonal effects, so the expected emerged loss is a development term $f(d)$ times a diagonal effect $g(w + d)$. Without loss of generality, you assume that $g(8) = 1$.

(f) (*3 points*) Calculate the values of $f(1)$-$f(7)$ and $g(2)$-$g(7)$ that minimize the sum of squared residuals by fixing the f-values to estimate the g-values by linear regression, then fixing the g-values to estimate the next iteration of f-values by linear regression, and so on until consecutive g-values agree to two decimal places. Begin the iterative process with the f-values calculated in part (d).

Provide the response for this part in the Excel spreadsheet.

5.

(*4 points*) A property catastrophe reinsurance company with a portfolio of business, portfolio Q, is considering writing an additional portfolio of property catastrophe business. The selection is to be from one of portfolios U, V, or W.

You are given the following information for independent insured events:

Event	Probability	Loss to portfolio Q	Loss to portfolio U	Loss to portfolio V	Loss to portfolio W
1	0.045	200,000	85,000	80,000	110,000
2	0.025	320,000	5,000	120,000	20,000
3	0.020	125,000	360,000	45,000	75,000
4	0.010	750,000	30,000	250,000	215,000

The company uses the Marginal Variance (MV) method to calculate risk loads.

The MV risk load multiplier, λ, is 0.00002.

(a) (*1.5 points*) Calculate the following for each of the four portfolios:

(i) Expected loss

(ii) Variance

(iii) Coefficient of variation

Provide the response for this part in the Excel spreadsheet.

(b) (*1 point*) Recommend which portfolio the reinsurance company should add if it wants to minimize the size of the total risk load. Justify your answer.

Provide the response for this part in the Excel spreadsheet.

The company calculates renewal risk loads using the Covariance Share method.

(c) (*1.5 points*) Calculate the renewal risk loads for portfolio Q and the portfolio you recommended be added in part (b).

Provide the response for this part in the Excel spreadsheet.

6.

(*5 points*) You are calculating a risk margin for the insurance liabilities of an insurer writing only one line of business using the methodology set out in "A Framework for Assessing Risk Margins" by Marshall et al.

They categorize risk into the following sources of uncertainty:

I. Independent Risk
II. Internal Systemic Risk
III. External Systemic Risk

(a) (*1.5 points*) Identify the source of uncertainty to which each of the following belongs:

(i) uncertainty from changes to the process of setting up case reserves

(ii) insurance process too complex for any model to fully capture

(iii) unavailability of data required to conduct a credible valuation

(iv) randomness associated with the insurance process compromising the ability to select appropriate parameters

(v) uncertainty of claim costs arising from catastrophes

(vi) pure effect of the randomness associated with the insurance process

ANSWER:

(i)
(ii)
(iii)
(iv)
(v)
(vi)

(b) (*1 point*) Provide two reasons why stochastic modeling techniques do not enable a complete analysis of all sources of uncertainty.

ANSWER:

6. Continued

The risk margin is to be calculated at the 75% adequacy level and is to be based on the following information:

Type of Liabilities	Amount of Liabilities	Coefficients of Variation		
		Independent Risk	Internal Systemic Risk	External Systemic Risk
Outstanding Claims Liabilities (CL)	9,000	6%	5%	3%
Premium Liabilities (PL)	6,000	4%	3%	8%

	Independent Risk	Internal Systemic Risk	External Systemic Risk
Correlation between CL and PL	0%	50%	25%

- The sources of uncertainty are assumed to be mutually independent.
- Claims are assumed to be normally distributed.
- The z-value of the 75th percentile of the normal distribution is 0.674.

(c) (*1.5 points*) Calculate the coefficient of variation for each risk source for the total insurance liabilities.

Provide the response for this part in the Excel spreadsheet.

(d) (*1 point*) Calculate the amount of the risk margin for the total insurance liabilities at the 75% adequacy level.

Provide the response for this part in the Excel spreadsheet.

7.

(*4 points*) You are given the following information with respect to a retrospective rating plan:

- The expected loss, E, is 100.
- The total expenses, e, are 30.
- The loss conversion factor, C, is 1.25.
- The basic premium, b, is 15.
- The minimum premium, H, is 15.
- Losses for the risk follow a normal distribution.

Table M		
r	$\phi(r)$	$\psi(r)$
0.00		
0.20		0.00
0.40		
0.60		0.03
0.80		
1.00		0.16
1.20	0.08	
1.40		
1.60	0.01	
1.80		
2.00	0.00	

(a) (*2 points*) Determine the following values:

(i) net insurance charge, I

(ii) maximum premium, G

Provide the response for this part in the Excel spreadsheet.

(b) (*2 points*) Fill in the missing values in Table M.

Provide the response for this part in the Excel spreadsheet.

8.

(*5 points*) A reinsurer is pricing a property per risk excess treaty for accident year 2022 covering the layer 3,000,000 excess of 1,000,000.

You are given the following information:

Loss ID	Accident Year	Untrended Loss Evaluated as of 12/31/2020
1	2018	1,100,000
2	2018	2,100,000
3	2019	900,000
4	2019	1,400,000
5	2020	800,000
6	2020	1,800,000

- All losses of at least 500,000 are shown.
- On level subject premium is 5,000,000 for each year from 2018-2020.
- Loss trend is 5% per year.
- The insured value of each property is 6,000,000.
- Accident year development factors applicable to losses in the layer 3,000,000 excess of 1,000,000:

12-Ultimate	1.50
24-Ultimate	1.25
36-Ultimate	1.00

(a) (*2 points*) Estimate the experience rating loss cost as a percentage of the subject premium.

Provide the response for this part in the Excel spreadsheet.

(b) (*0.5 points*) Define free cover.

Provide the response for this part in the Excel spreadsheet.

8. Continued

The following exposure factors are considered appropriate for pricing this treaty:

Percent of Insured Value	Exposure Factor
10%	30%
20%	50%
30%	61%
40%	69%
50%	75%
60%	80%
70%	85%
80%	89%
90%	92%
100%	95%
110%	98%
120%	100%

(c) (*2 points*) Calculate a revised loss cost as a percentage of the subject premium using these exposure factors to estimate the cost of free cover.

Provide the response for this part in the Excel spreadsheet.

(d) (*0.5 points*) Assess whether the loss cost percentage you calculated in part (c) would be appropriate for pricing coverage on properties with insured values of 12 million.

Provide the response for this part in the Excel spreadsheet.

****END OF EXAMINATION****