2017 Actuarial Research Conference

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Session C5: Valuation of Unit-Linked Insurance

Saturday, July 29th, 2017

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Experience Studies: The Linear Force Distribution

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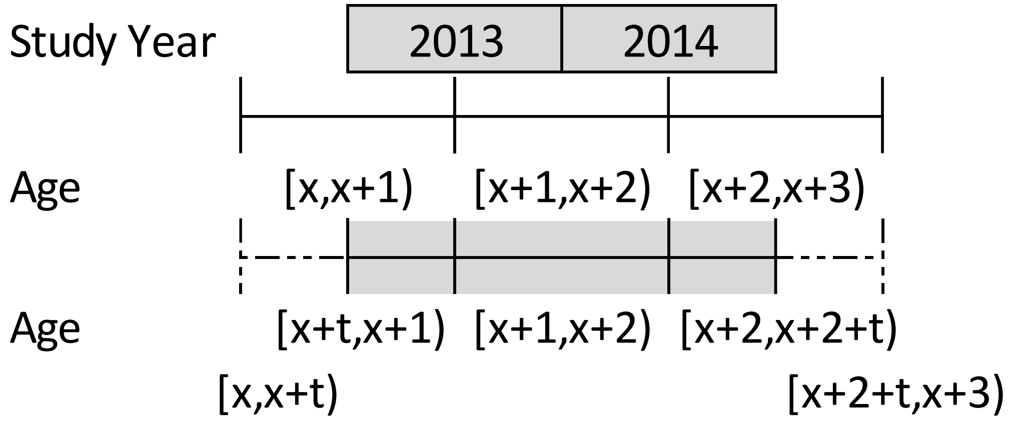
SOA Experience Study Calculations

- By David B. Atkinson & John K. McGarry, Oct. 2016.
  - www.soa.org/tables-calcs-tools/experience-study-tool/
- Basic Exposure and Rate calculations:
  - Individual records, Grouped data.
- Current Practice by Product/Study:
  - Life Mortality, Lapse, DI/LTC Incidence/Termination
- Three study methods:
  - Traditional Exposure, or Actuarial, Method,
  - Daily Exposure, or Exact, Method, and
  - Distributed Exposure Method.
- Linear Force Model used to test different methods.

Calendar-Year Mortality Studies

- At the start and end of a calendar-year study, ages and study years intersect to give partial ages. E.g. for a 2 year study 2013-2014, where t is the fractional year from age anniversary to year end.



- Where fractional exposure is calculated, by calendar year or quarter, to analyze trends or distributions, partial ages occur throughout the study period.

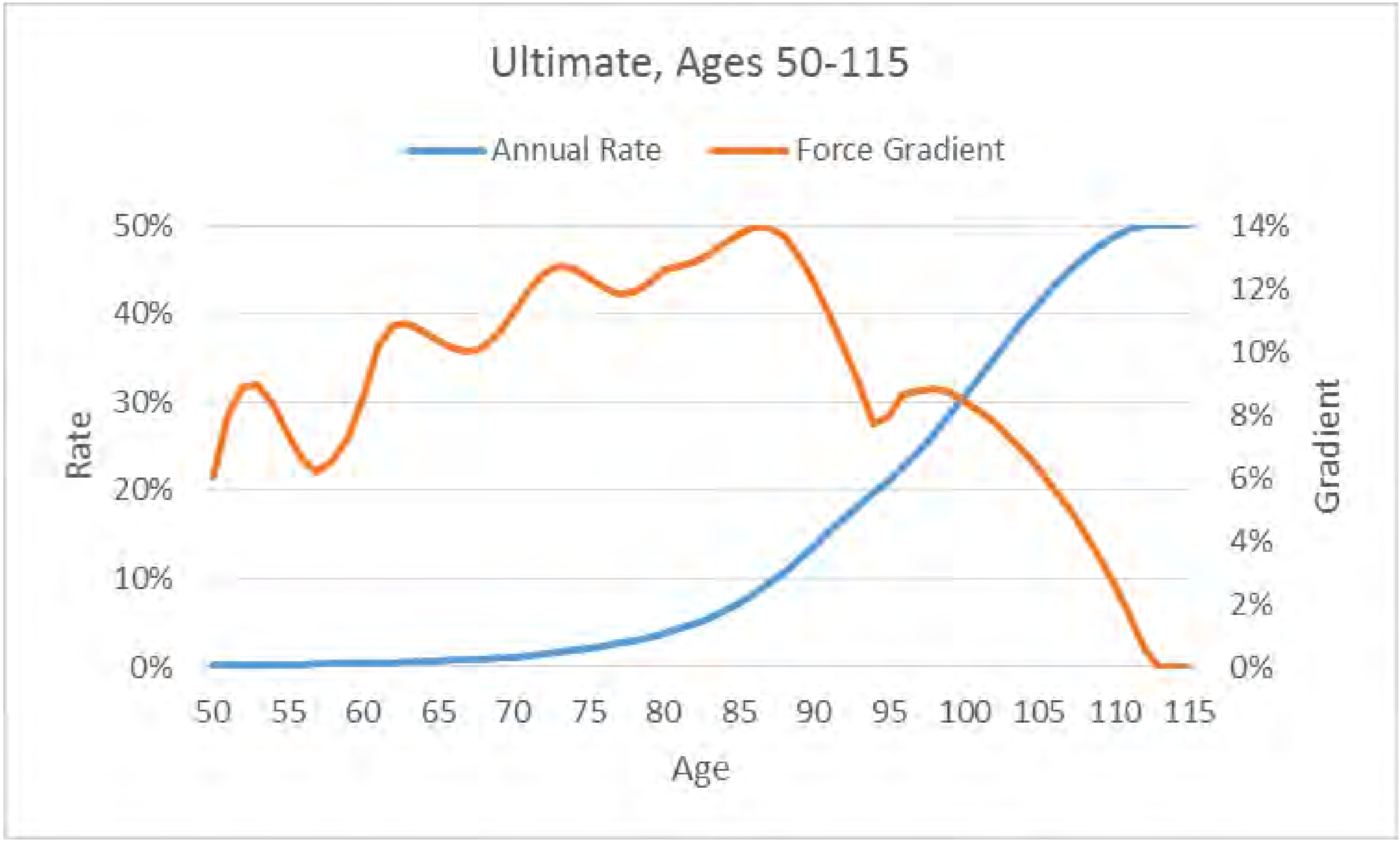
Calendar-Year Mortality Studies

- For partial ages, the study methods assume deaths are proportional to time spent in the year, giving an implicit distribution of deaths.
- The difference between the implicit and actual distributions may distort the rates calculated in the study.
- For small rates or roughly uniform deaths, these distortions will not be material.
- The rates for older ages and early durations may have significant distortions.

Increase in the Force of Mortality

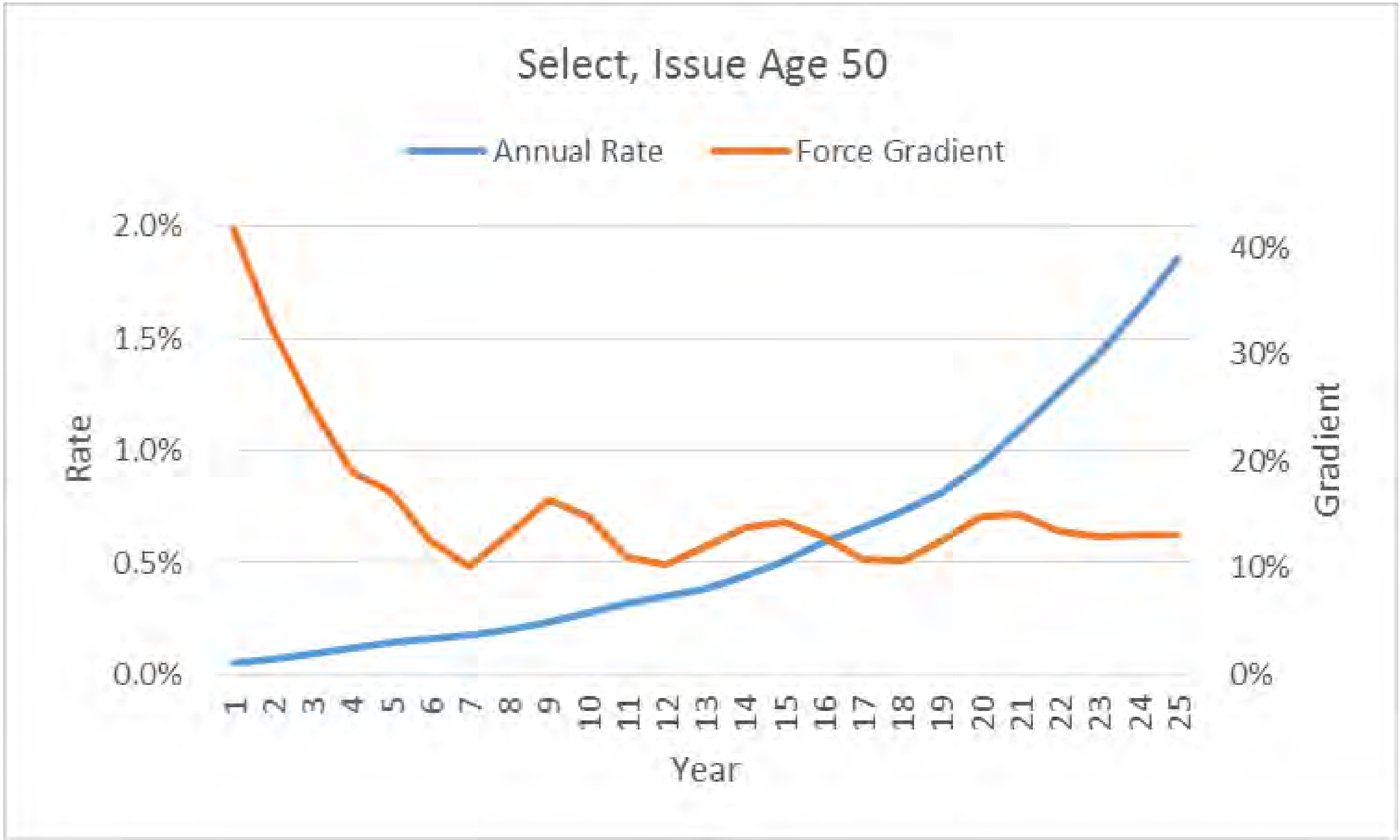
- As mortality is continuous, the distribution of deaths is determined by the increase in the force of mortality over the year.
- The increase in force for a given age is derived from the rates for the prior and following ages.
- The relative increase in force, i.e. the increase in force divided by the average force, or "gradient", Δ_x, gives the distribution independent of size of the rates across the age range.
- Industry table: VBT 2015 M NS ANB

Gradients



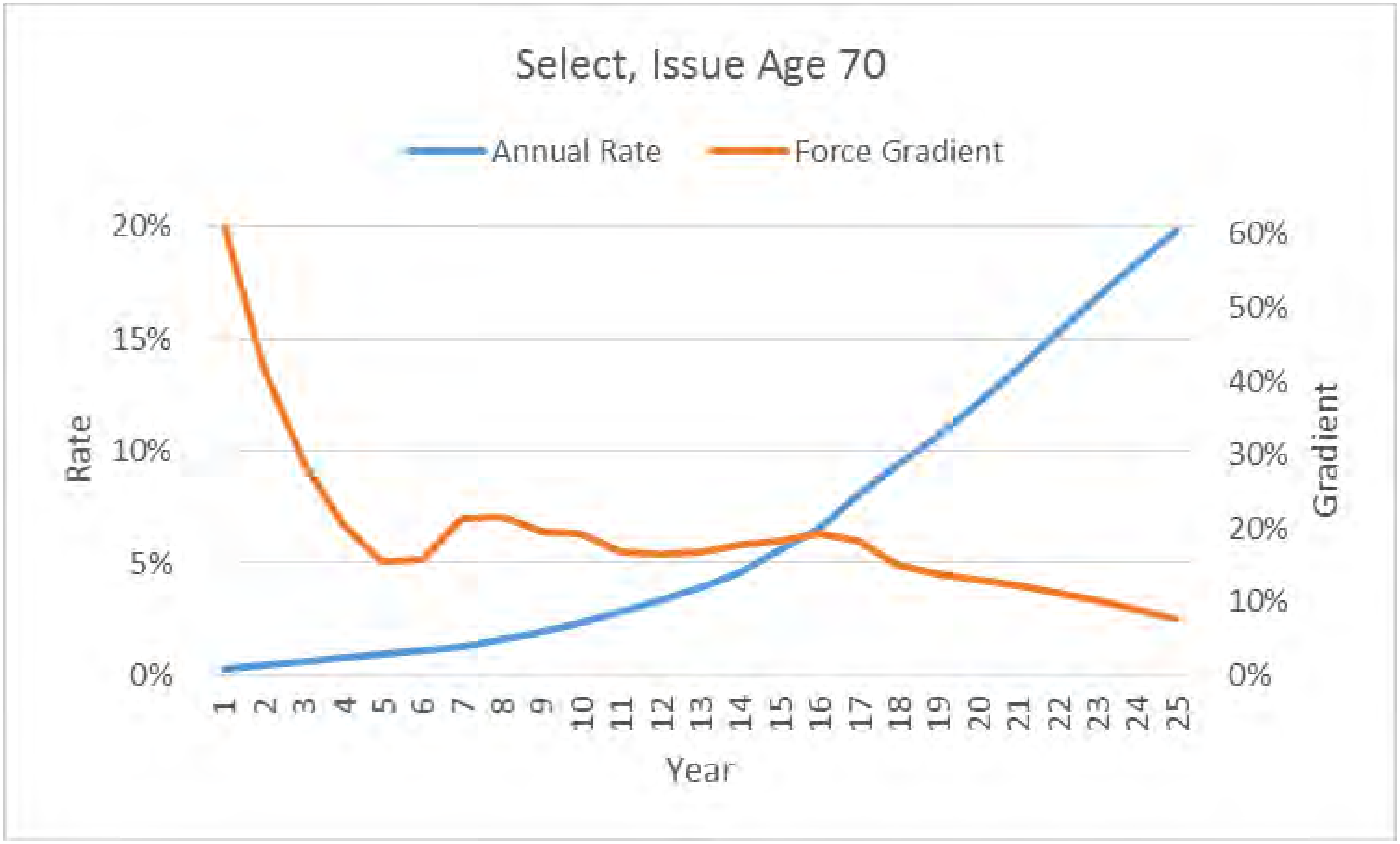
Ultimate, Ages 50-115
Annual Rate
Force Gradient
50%
40%
30%
20%
10%
0%
Rate
14%
12%
10%
8%
6%
4%
2%
0%
Gradient
50 55 60 65 70 75 80 85 90 95 100 105 110 115
Age

Gradients



Select, Issue Age 50
Annual Rate
Force Gradient
2.0%
1.5%
1.0%
0.5%
0.0%
Rate
40%
30%
20%
10%
0%
Gradient
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Year

Gradients

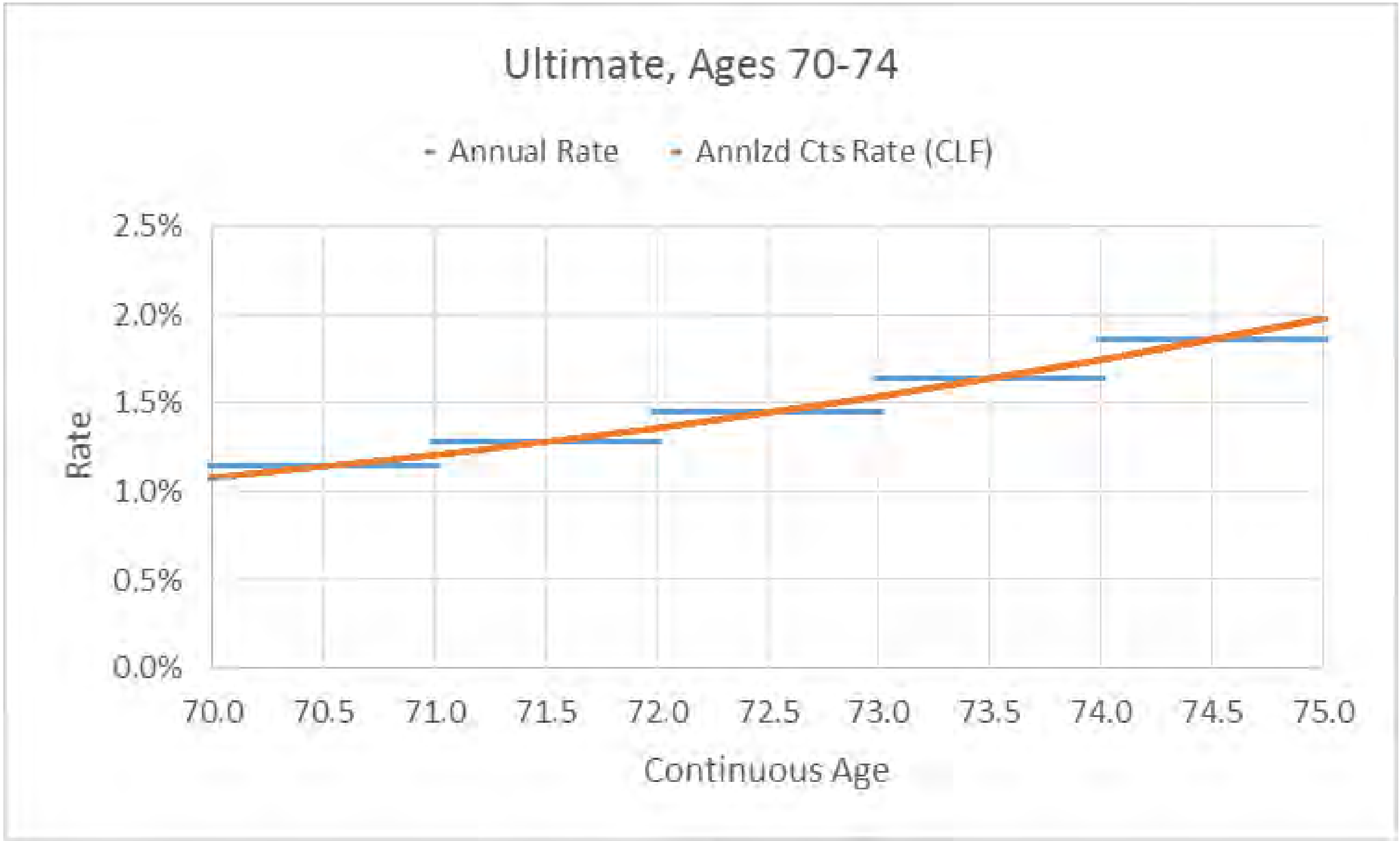


Select, Issue Age 70
Annual Rate
Force Gradient
Rate
20%
15%
10%
5%
0%
Gradient
60%
50%
40%
30%
20%
10%
0%
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Year

Linear Force Distribution

- The force at an exact age $x+t$ is interpolated assuming the force changes linearly from:
  - The force at exact age x (Boundary):
    - there is continuity from age to age, but the sum of the force over age x is not consistent with the rate for age x.
  - The average force at age $x + ½$ (Centered):
    - the sum of the force is consistent with the rate, but there are discontinuities from age to age, i.e. the force at exact age x is not well defined.
  - The average force at age $x + \mathrm{T}$ (Exact):
    - where time T is such that sum of the force is consistent with the rate, and there is continuity from age to age.
- Sample ages from VBT 2015 M NS ANB

Annualized Rates (Centered)



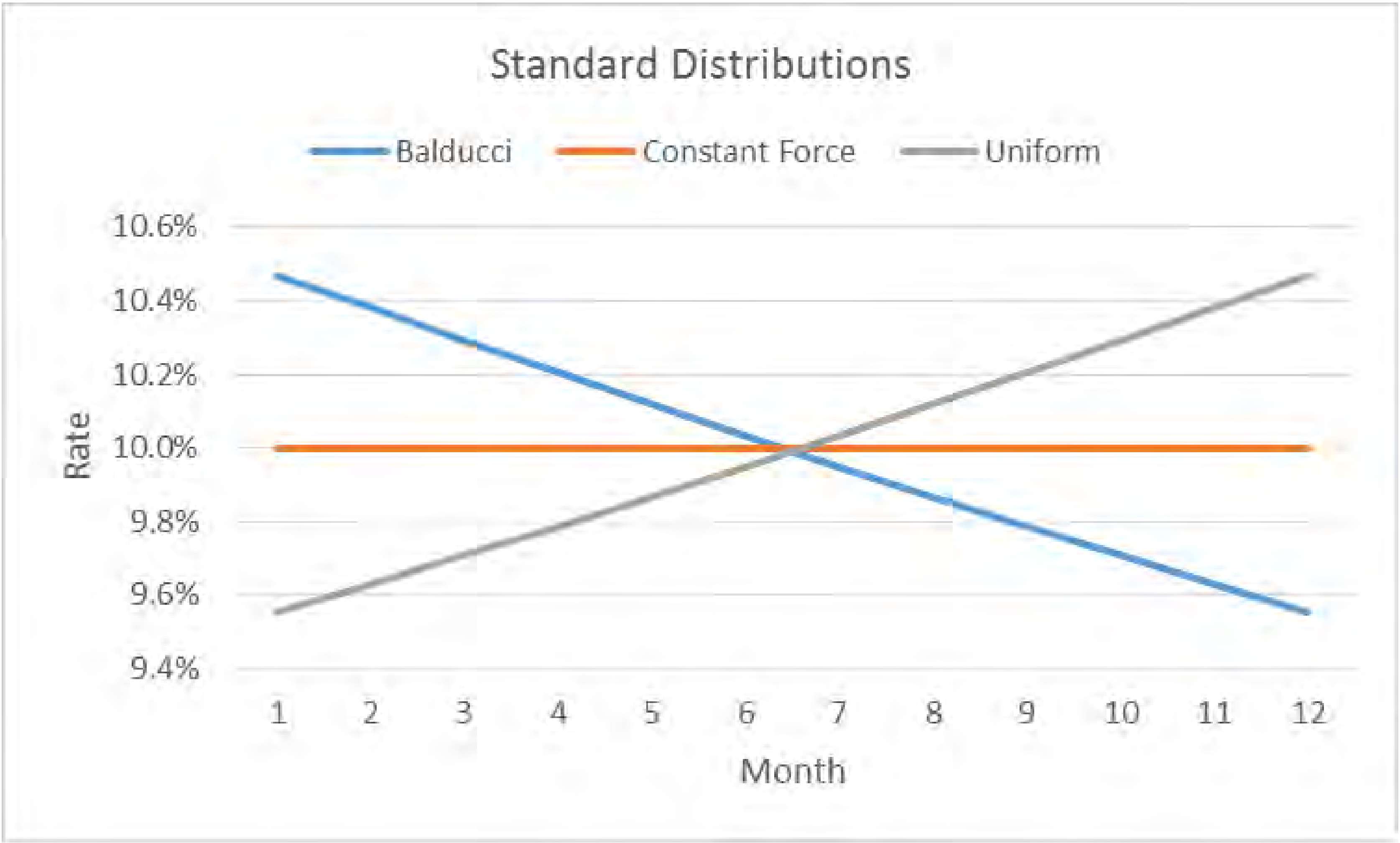
Annualized Rates (Centered)

Annualized Rates (Exact)

Main Study Methods

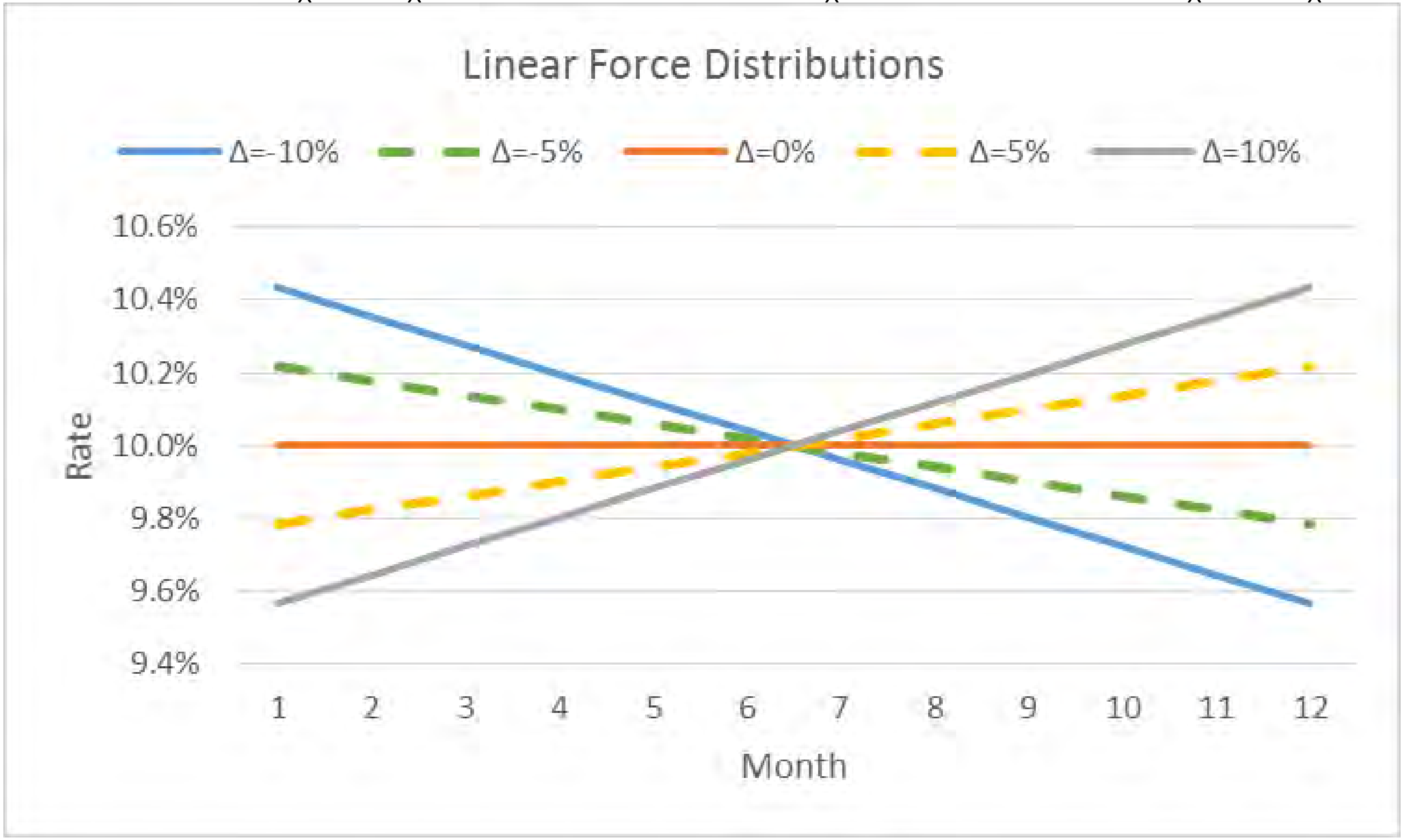
- For partial ages,
  - Traditional - Balducci:
    - the rate decreases over the year,
  - Daily – Constant Force:
    - the force is constant over the year, and
  - Distributed – Uniform Distribution of Deaths:
    - the rate increases over the year.
- These distributions can be estimated by the centered linear force distribution.
- 10% mortality rate example.
- Sample ages from VBT 2015 M NS ANB.

Standard Distributions

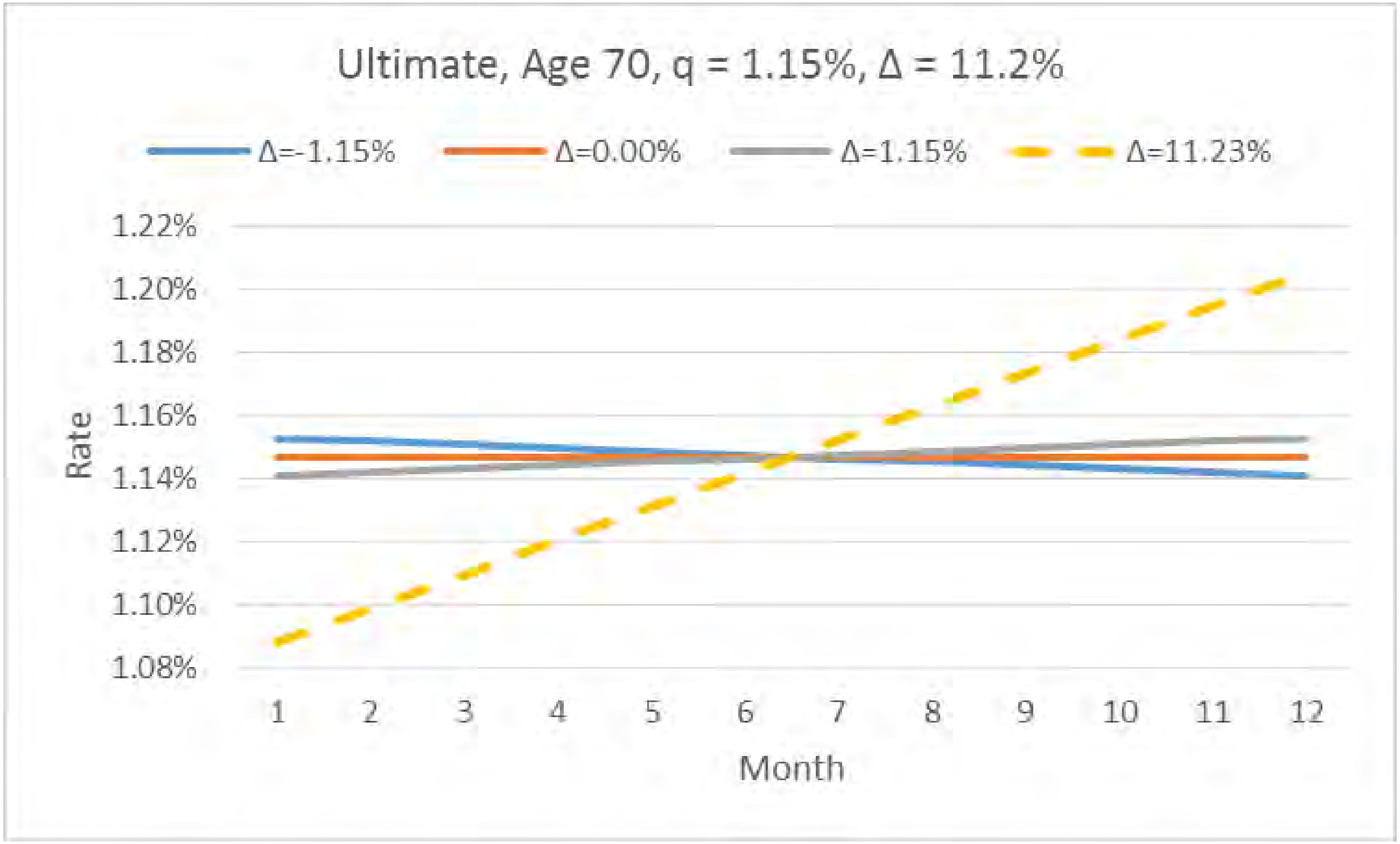


Centered Linear Force Distribution

- Balducci: $\Delta_x \approx -q_x$; Constant Force: $\Delta_x = 0$; Uniform: $\Delta_x \approx +q_x$



Method and Actual Distributions



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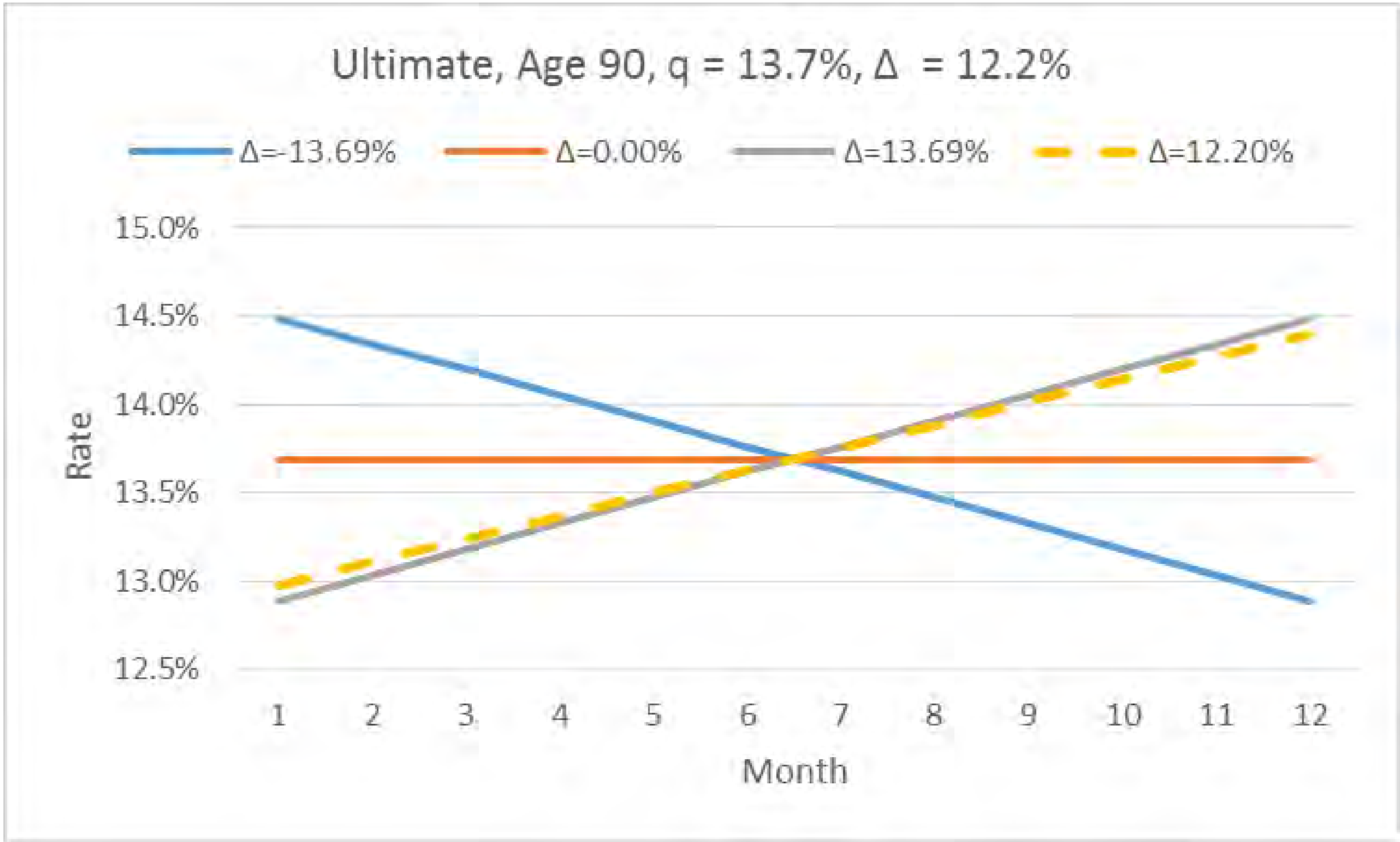

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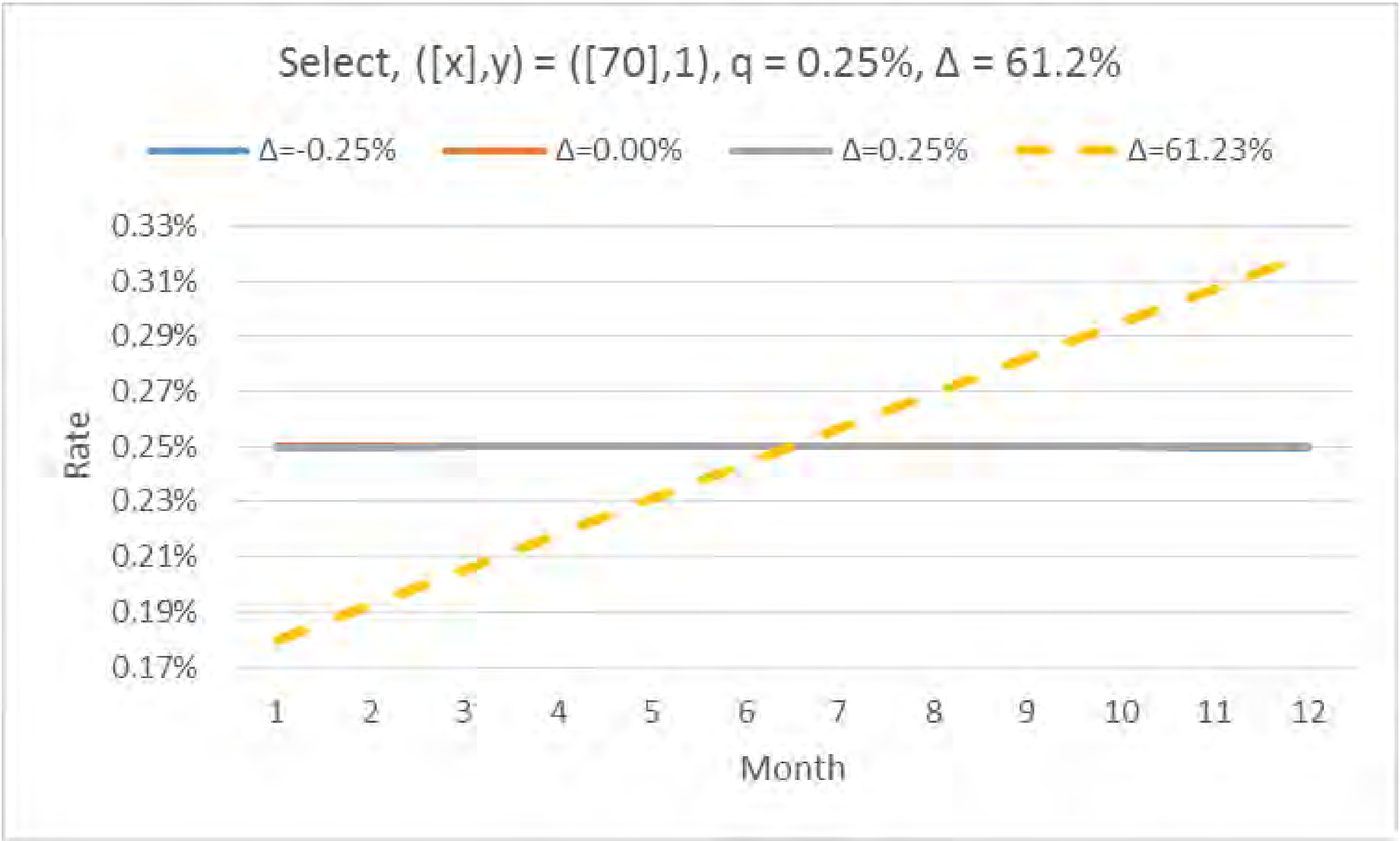
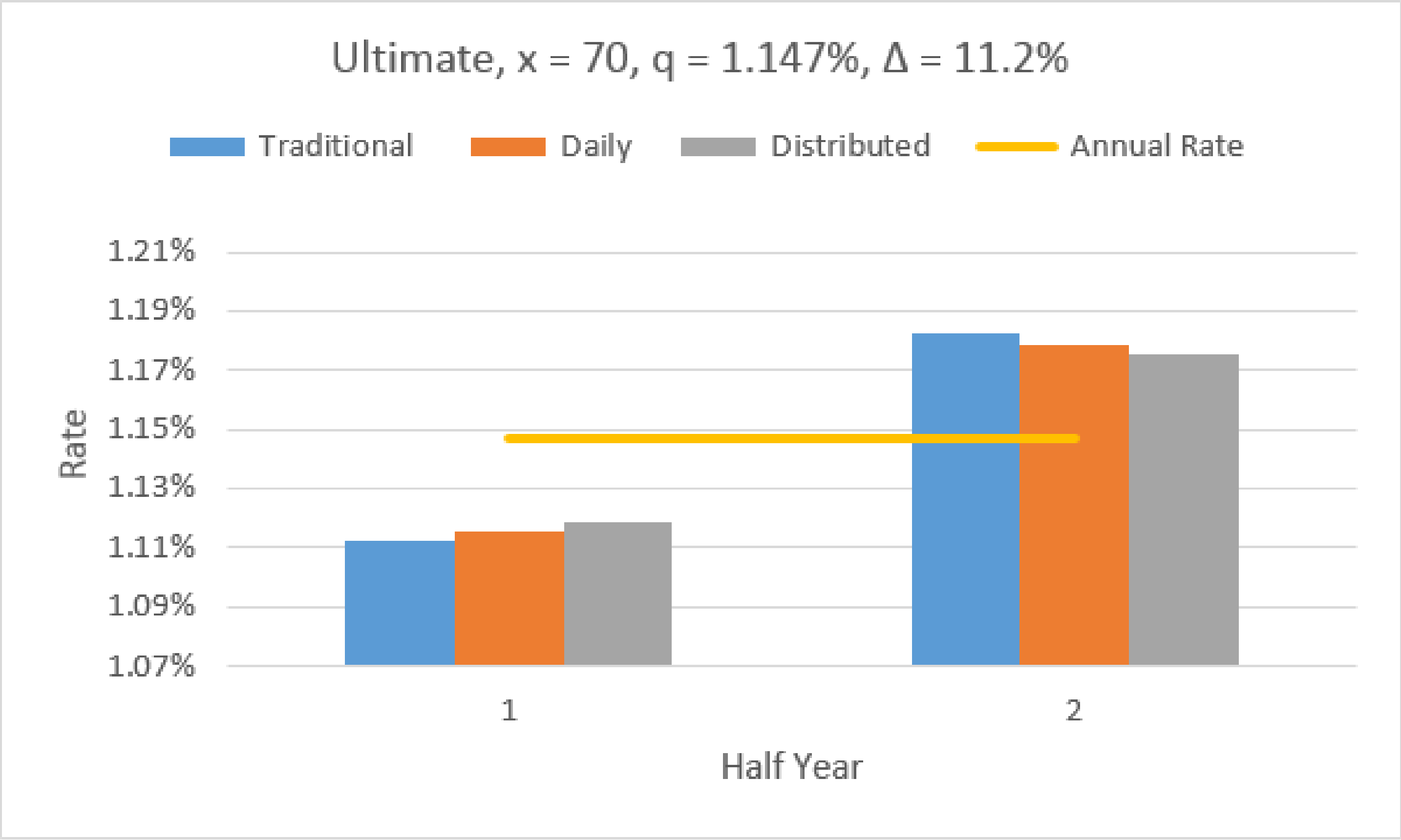
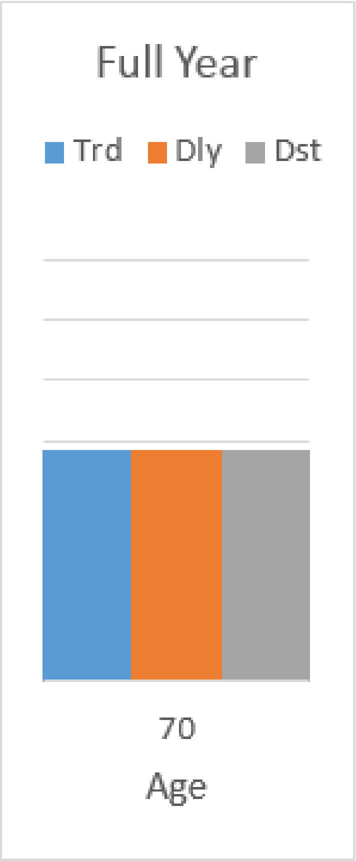


Errors for Partial Ages

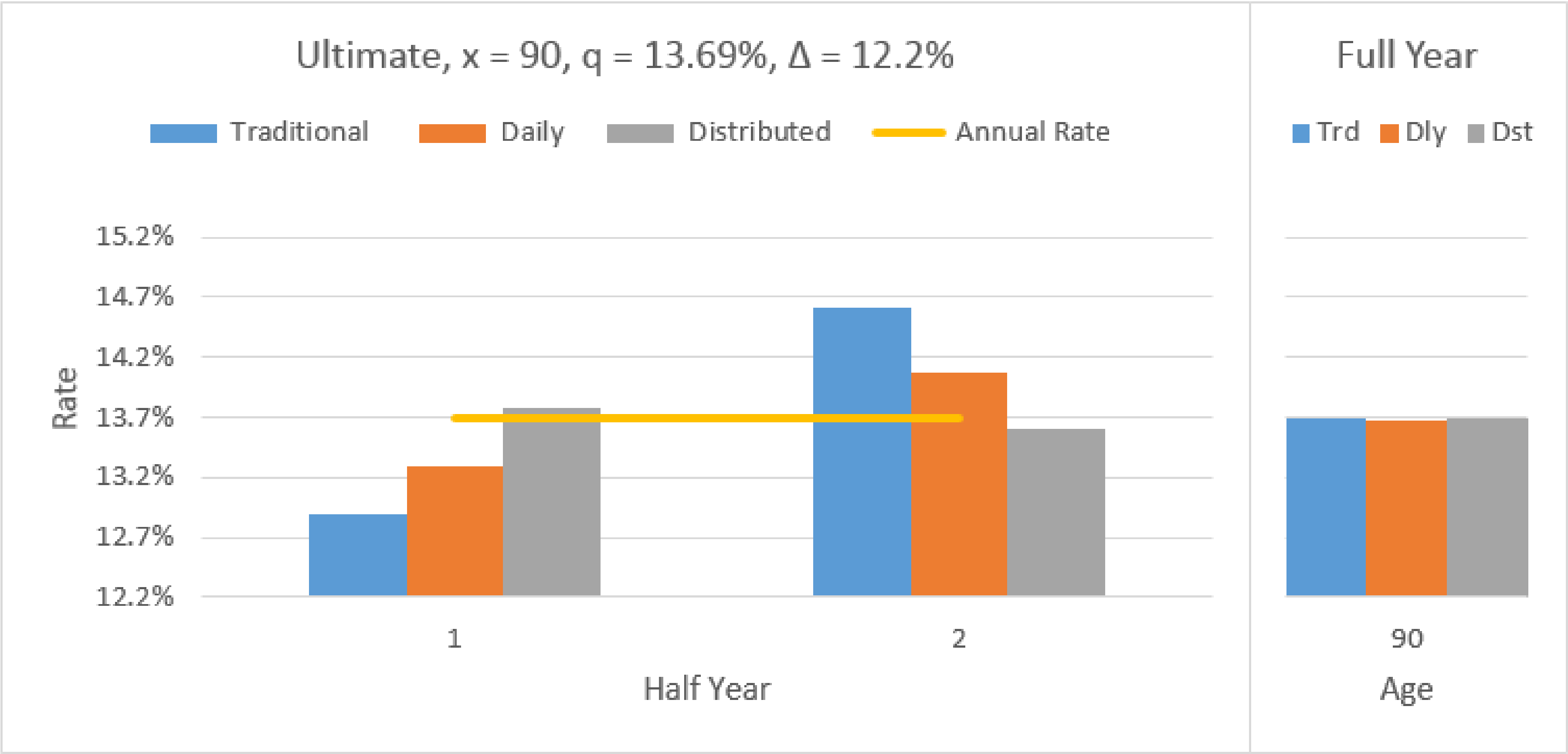
- For sample ages, lives are projected using the linear force distribution, with the exposure and rates calculated for partial ages. The rates for partial ages are compared to annual rate for the full year of age.
- If the age anniversaries are uniformly distributed over the year, the rates for partial ages that arise in a study can be estimated using half-year ages.
- Sample ages from VBT 2015 M NS ANB

Errors for Half-Year Ages

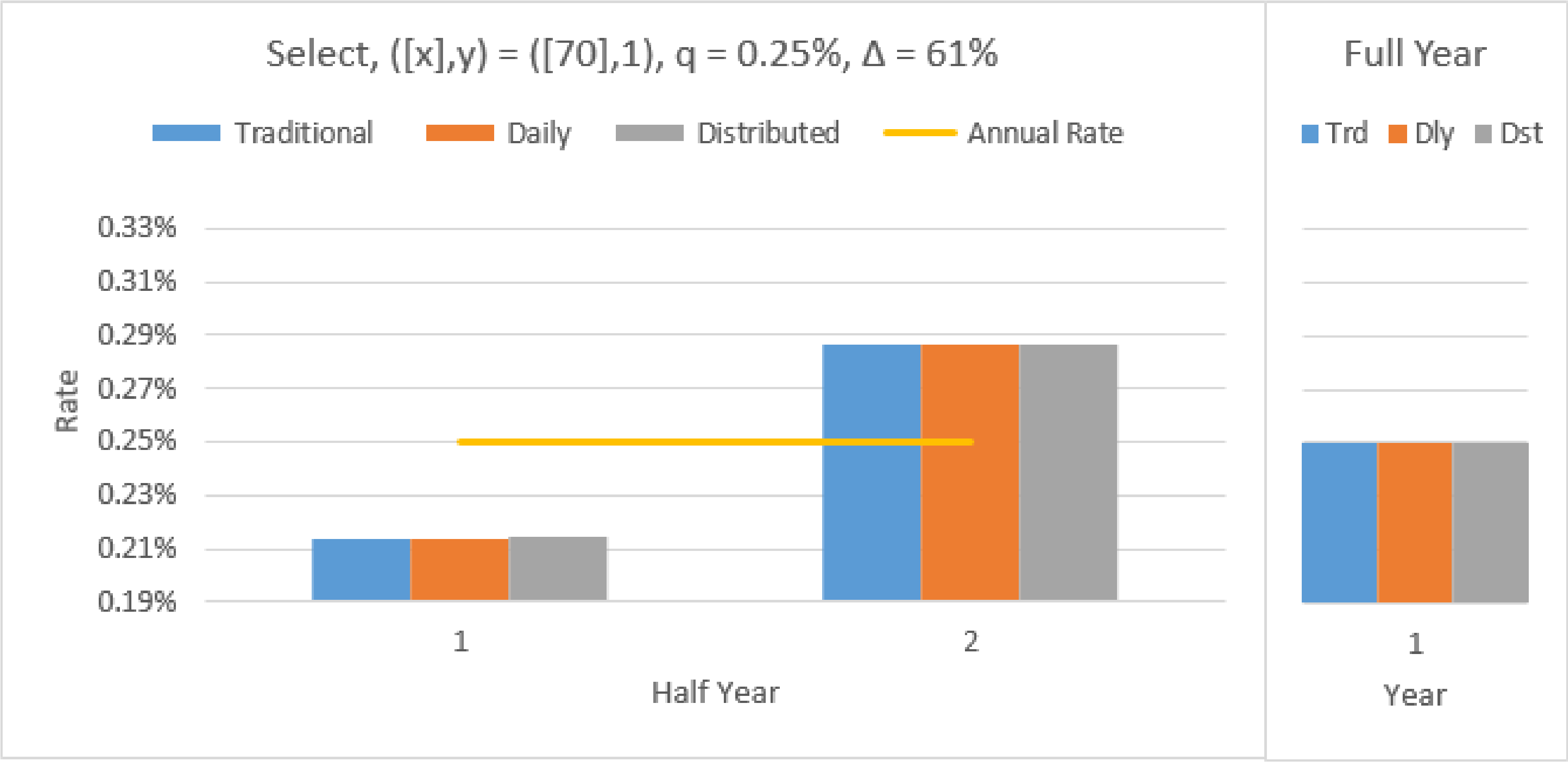




Errors for Half-Year Ages



Errors for Half-Year Ages



Error Formula

Error From the Annual Rate given Centered Linear Force

= Time at Mid Point from Mid Year

* (Gradient * Rate + Flag * Rate Squared)

$= T(\Delta_x q_x + F q_x^2)$

where, for half years, $H = 1,2$,

- Time $T = (H - 1.5)/2$,
- Method Flag F = Traditional 1: $T(\Delta_x q_x + q_x^2)$,
  Daily 0: $T(\Delta_x q_x)$,
  Distributed -1: $T(\Delta_x q_x - q_x^2)$.

Sample Age Error Estimates

- Ultimate, x = 70, q = 1.15%, Δ = 11.2%

Half Year	Time	Traditional	Daily	Distributed
1	-0.25	-0.035%	-0.032%	-0.029%
2	0.25	0.035%	0.032%	0.029%

- Ultimate, x = 90, q = 13.7%, Δ = 12.2%

Half Year	Time	Traditional	Daily	Distributed
1	-0.25	-0.89%	-0.42%	0.05%
2	0.25	0.89%	0.42%	-0.05%

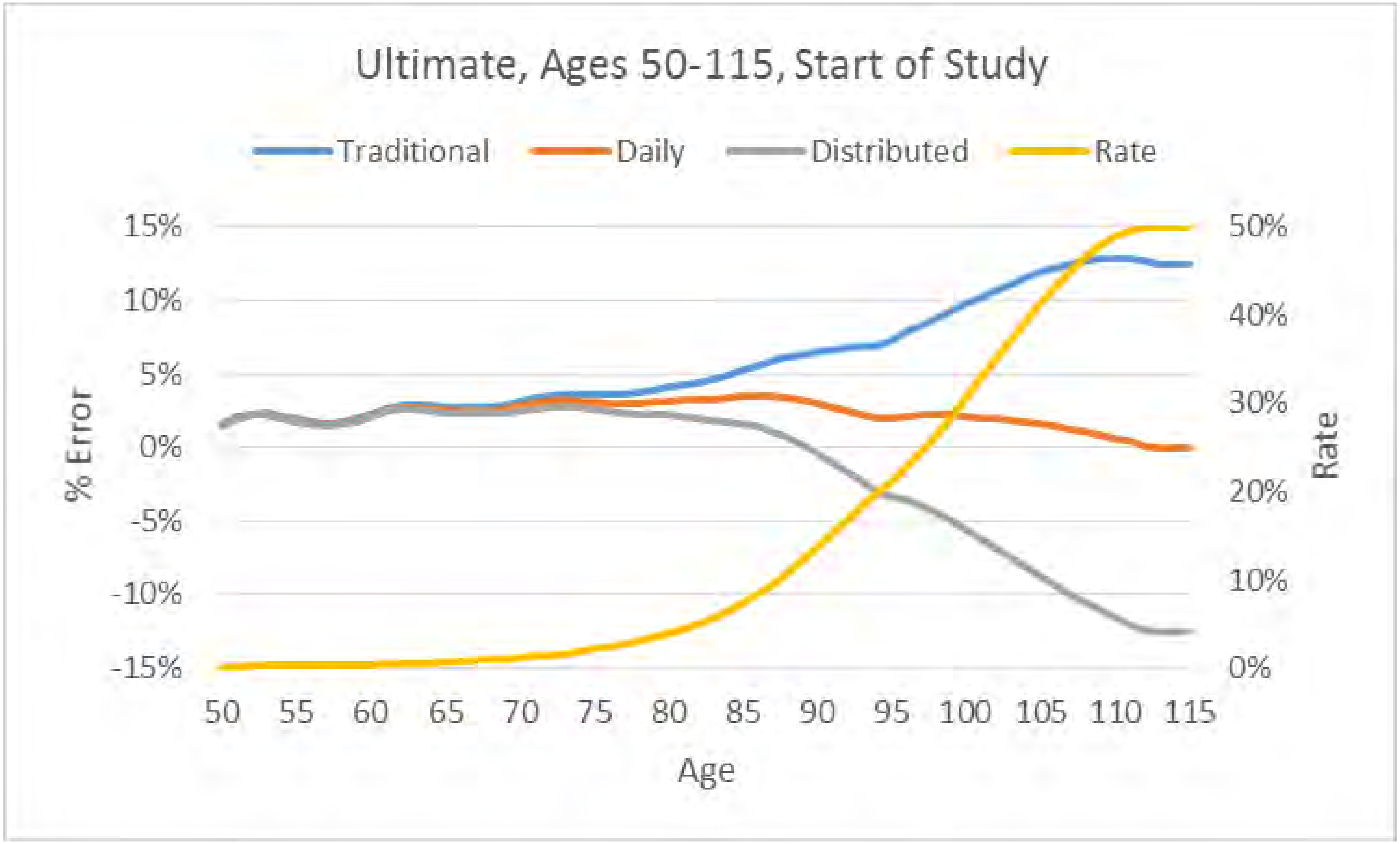
- Select, ([x],y) = ([70],1), q = 0.25%, Δ = 61%

Half Year	Time	Traditional	Daily	Distributed
1	-0.25	-0.0384%	-0.0383%	-0.0381%
2	0.25	0.0384%	0.0383%	0.0381%

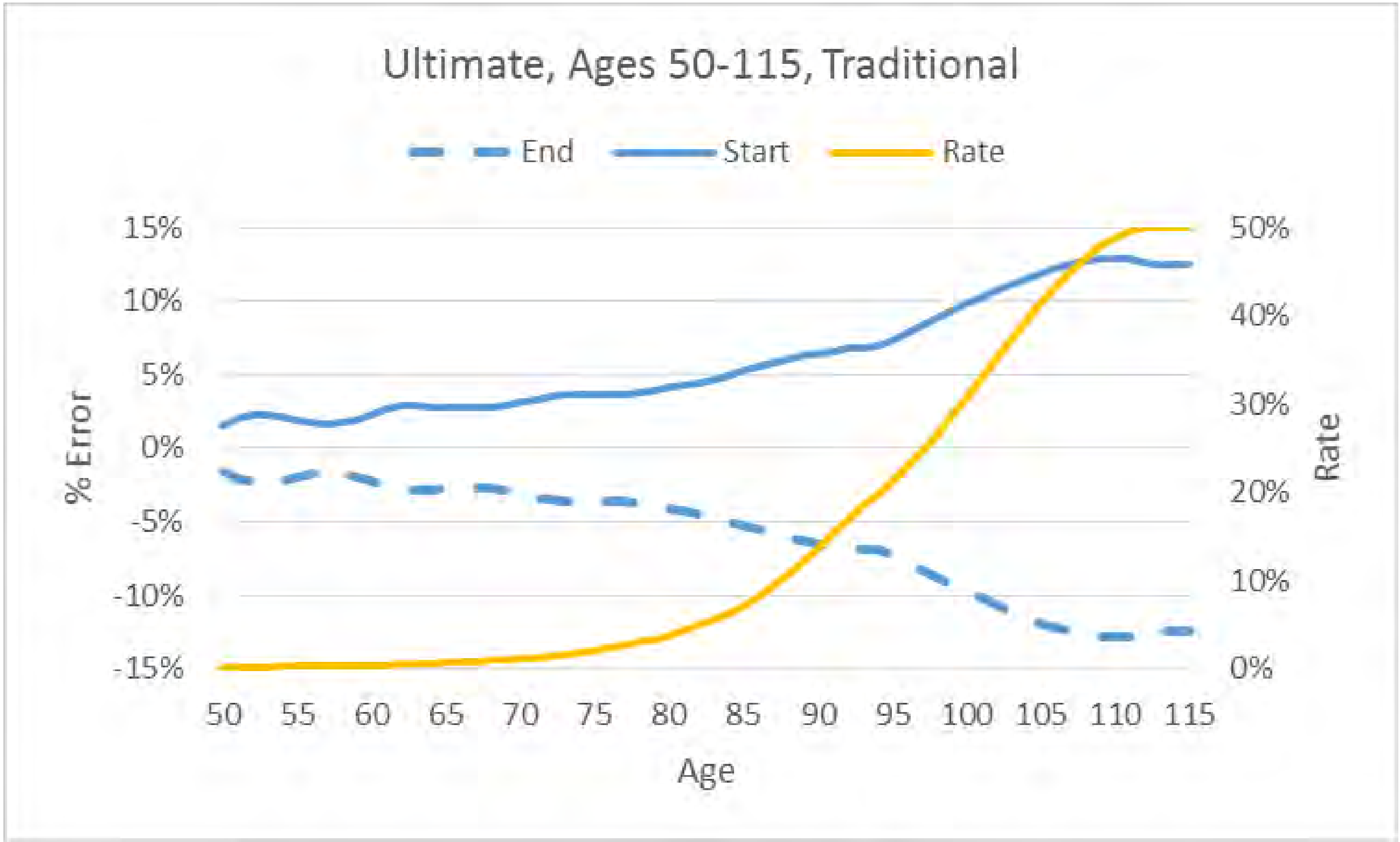
Errors at Start and End of Study

- Errors given uniform anniversaries.
  - Partial Age at Start of Study
    - $e_{x,Start} = +¼(\Delta_x q_x + Fq_x^2) = +\varepsilon_x.$
  - Partial Age at End of Study
    - $e_{x,End} = -¼(\Delta_x q_x + Fq_x^2) = -\varepsilon_x.$
- VBT 2015 M NS ANB

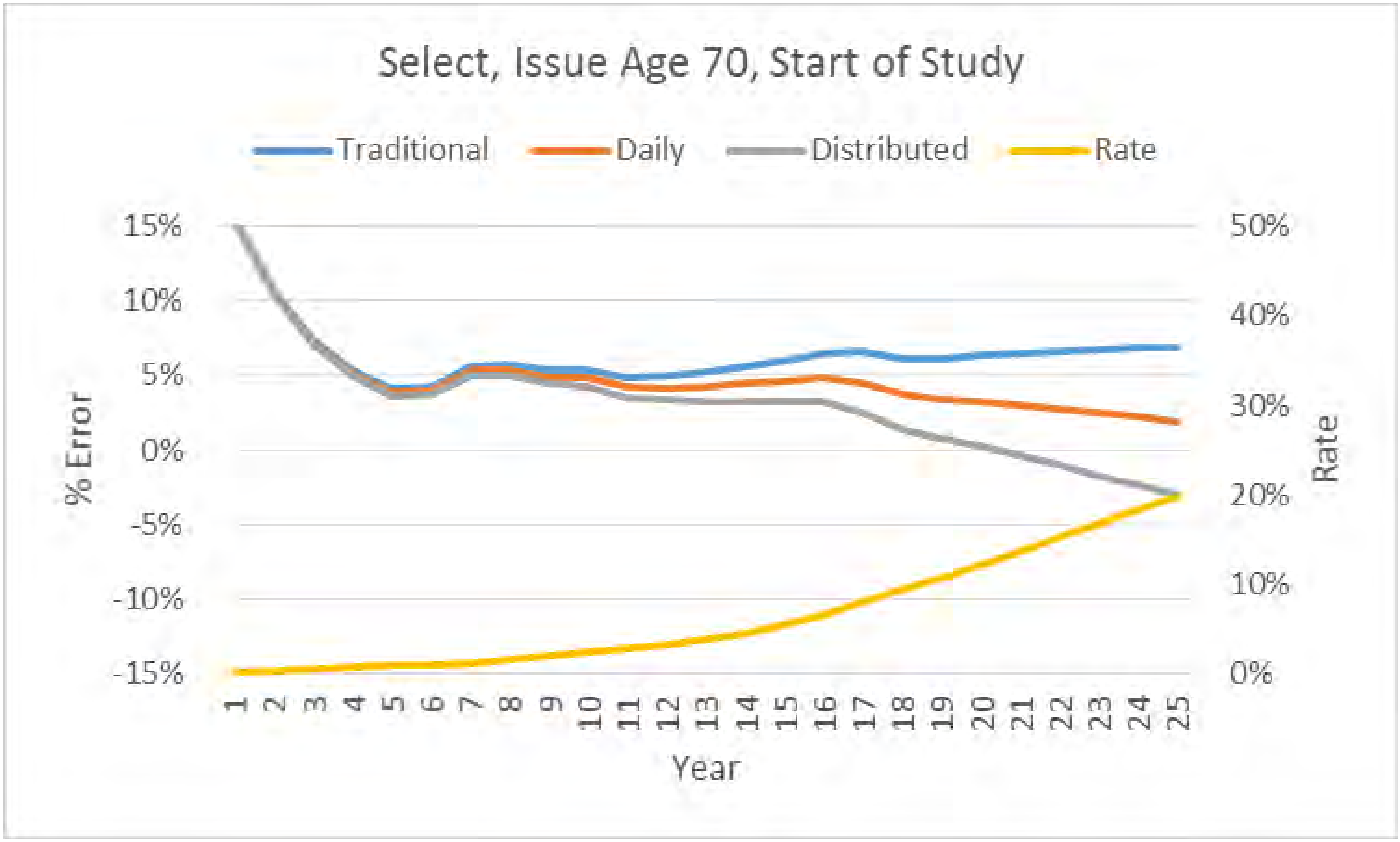
Errors at Start of Study



Traditional Errors at Start and End



Errors at Start of Study



Traditional Errors at Start and End

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Study Errors – Single Cohort

- A full year of age spans across two calendar years. The rates for the partial ages in each calendar year will contain errors that are equal in size (given uniform anniversaries) and opposite in sign.
- For the ages at the start and end of a calendar year study, only one partial age will fall into the study period.
- These “method” errors occur for a single cohort of lives born in the same year, that contribute to the same ages at the same time in the study. For example, in a three year study, 2012-2014, the lives born in 1942 will contribute ages 70 to 73.

Study Errors – Single Cohort

- Study Period, 2012-14, Lives Born 1942.
- Exact age range, errors and errors by study year.
- Traditional Method.

			Study Year				
Age	Exact Age	Error	2011	2012	2013	2014	2015
70	70+t,71	$+\varepsilon_{70}$	$-\varepsilon_{70}$	$+\varepsilon_{70}$			
71	71,72	0		$-\varepsilon_{71}$	$+\varepsilon_{71}$		
72	72,73	0			$-\varepsilon_{72}$	$+\varepsilon_{72}$	
73	73,74+t	$-\varepsilon_{73}$				$-\varepsilon_{73}$	$+\varepsilon_{73}$

Study Errors – Seven Cohorts

- Study Period, 2012-14, Lives Born 1939-45.
- Cohorts equal in size, homogeneous population.

	Cohort Errors							Total	
Age	1939	1940	1941	1942	1943	1944	1945	Error	Exposure
67							$+\varepsilon_{67}$	$+\varepsilon_{67}$	½
68						$+\varepsilon_{68}$	0	$+\frac{1}{3}\varepsilon_{68}$	1½
69					$+\varepsilon_{69}$	0	0	$+\frac{1}{5}\varepsilon_{69}$	2½
70				$+\varepsilon_{70}$	0	0	$-\varepsilon_{70}$	0	3
71			$+\varepsilon_{71}$	0	0	$-\varepsilon_{71}$		0	3
72		$+\varepsilon_{72}$	0	0	$-\varepsilon_{72}$			0	3
73	$+\varepsilon_{73}$	0	0	$-\varepsilon_{73}$				0	3
74	0	0	$-\varepsilon_{74}$					$-\frac{1}{5}\varepsilon_{74}$	2½
75	0	$-\varepsilon_{75}$						$-\frac{1}{3}\varepsilon_{75}$	1½
76	$-\varepsilon_{76}$							$-\varepsilon_{76}$	½

Study Errors – Seven Cohorts

- Study Period, 2012-14, Policies Issued in 2008-14.
- Cohorts equal in size, homogeneous population.

	Cohort Errors							Total	
Year	2008	2009	2010	2011	2012	2013	2014	Error	Exposure
1				$+\varepsilon_1$	0	0	$-\varepsilon_1$	0	3
2			$+\varepsilon_2$	0	0	$-\varepsilon_2$		0	3
3		$+\varepsilon_3$	0	0	$-\varepsilon_3$			0	3
4	$+\varepsilon_4$	0	0	$-\varepsilon_4$				0	3
5	0	0	$-\varepsilon_5$					$-\frac{1}{5}\varepsilon_5$	2½
6	0	$-\varepsilon_6$						$-\frac{1}{3}\varepsilon_6$	1½
7	$-\varepsilon_7$							$-\varepsilon_7$	½

Study Errors – Full Exposure Ages

- In an N year study, each full-exposure age has $N+1$ cohorts, $n = 0, N$.
- Study Error
  - $e_x = \left(E_{x,0}e_{x,0} + E_{x,N}e_{x,N}\right)/E_x$
- Equal Cohorts
  - $e_x = 0$
- Increasing Cohorts, i% per year, Ni% across age.
  - $\alpha_x = E_{x,SY}/\left(E_{x,SY} + E_{x,SY+1}\right)$ - exposure distribution
  - $e_x = e_{x,N}\,\alpha_x Ni/(N + ½(N+1)Ni + \alpha_x Ni)$
- Simplifying, $\alpha_x \approx ½$, $e_{x,N} \approx -\varepsilon_x$
  - $e_x = -½\varepsilon_x\, i/(1 + (½N + 1)i) < -½\varepsilon_x i$

Study Errors – Increasing Cohorts

- Ultimate rates, VBT 2015 M NS ANB

x	50	70	90
q_x	0.192%	1.147%	13.690%
Δ_x	6.0%	11.2%	12.2%
ε_x	0.003%	0.04%	0.89%
ε/q	2%	3%	6%
$q_{x,0}$	0.195%	1.18%	14.58%
$q_{x,N}$	0.189%	1.11%	12.80%
α_x	50.05%	50.24%	53.79%

Study Errors – Increasing Cohorts

- Ultimate rates, VBT 2015 M NS ANB

Increase		% Study Error		
Annual	Study	50	70	90
0%	0%	0.000%	0.000%	0.000%
1%	3%	-0.008%	-0.015%	-0.034%
5%	15%	-0.034%	-0.070%	-0.158%
10%	30%	-0.062%	-0.129%	-0.288%
50%	150%	-0.172%	-0.385%	-0.861%
100%	300%	-0.221%	-0.514%	-1.149%

Study Errors – Increasing Cohorts

- Select rates, VBT 2015 M NS ANB

$[x], y$	[50],1	[70],1	[90],1
$q_{[x],1}$	0.052%	0.250%	2.069%
$\Delta_{[x],1}$	41.9%	61.2%	125.0%
$\varepsilon_{[x],1}$	0.005%	0.04%	0.66%
ε/q	10%	15%	32%
$q_{[x],1,0}$	0.057%	0.29%	2.73%
$q_{[x],1,N}$	0.047%	0.21%	1.41%
$\alpha_{[x],1}$	50.02%	50.04%	50.03%

Study Errors – Increasing Cohorts

- Select rates, VBT 2015 M NS ANB

Increase		% Study Error		
Annual	Study	[50],1	[70],1	[90],1
0%	0%	0.000%	0.000%	0.000%
1%	3%	-0.051%	-0.075%	-0.156%
5%	15%	-0.233%	-0.348%	-0.720%
10%	30%	-0.420%	-0.637%	-1.316%
50%	150%	-1.165%	-1.905%	-3.932%
100%	300%	-1.498%	-2.544%	-5.248%

Eliminating Errors

- Adjust Study Period (not fractional exposure)
  - "Rate Year" Study Period – individual lives only contribute full ages, excluding partial ages at start and end.
- Adjust Age Range (cohorts roughly equal)
  - Only ages with full exposure, excluding ages with exposure less than study period.
- Adjust Study Rates
  - Estimate the annual rate using the partial age rate and an estimate of the study error.
    - $q_x^E = q_x - e_x.$
- Adjust Exposure (Daily Exposure Method)
  - "Weighted" Exposure Method - Identify the gradients in the study, and directly weight the daily exposure to reflect the gradients.

Weighted Daily Exposure

- Calculate exposure for force, E_x^F and estimate the gradients, Δ_x.
- Calculate weights for partial ages, $x + t$ to $x + t + f$:
  - ${}_f w_{x+t} = 1 + T\Delta_x$.
  - ${}_f E_{x+t}^W = {}_f w_{x+t} \cdot {}_f E_{x+t}^F$.
- Where
  - $\Sigma_1^P \left({}_f E_{x+t}^W \right) = E_x^F$.
- Calculate average force of mortality,
  - $\bar{\mu}_x \approx d_x / E_x^W$.
- Calculate the annual rate,
  - $q_x = 1 - e^{-\bar{\mu}_x}$.

Questions?