GI ADV Model Solutions
Fall 2020

1. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

Estimate the experience rating loss cost, including ALAE, as a percentage of the subject premium.

Commentary on Question:

The solution for this question can be displayed in several different ways. The model solution in the Excel file is an example of one approach to display the solution.

Step 1: Select the proper trend period for each loss, in years, as the difference between the treaty year being priced (2021) and the year of the loss because all losses occurred at the midyear point.

Accident Date	Trend Period
7/1/2017	4
7/1/2017	4
7/1/2018	3
7/1/2018	3
7/1/2019	2
7/1/2019	2

1. Continued

Step 2: Calculate the trended loss in layer as the loss increased by the trend rate (5%) over the trend period adjusted by the layer.

Accident Date	Loss	Trended Loss	Trended Loss in Layer
7/1/2017	200,000	243,101	0
7/1/2017	350,000	425,427	175,427
7/1/2018	225,000	260,466	10,466
7/1/2018	900,000	1,041,863	750,000
7/1/2019	250,000	275,625	25,625
7/1/2019	800,000	882,000	632,000

Step 3: Calculate the covered ALAE as the ALAE increased by the trend rate (5%) over the trend period and allocated to the layer by using the ratio of the trended loss in layer to the trended loss.

Accident Date	ALAE	Trended ALAE	Covered ALAE
7/1/2017	150,000	182,326	-
7/1/2017	400,000	486,203	200,488
7/1/2018	-	-	-
7/1/2018	450,000	520,931	390,698
7/1/2019	50,000	55,125	5,125
7/1/2019	275,000	303,188	217,250

Step 4: Calculate the developed trended loss and ALAE for the layer by applying the appropriate development factor based on the age of the loss to the combined trended claim and ALAE for the layer.

Accident Date	Losses + ALAE	Development Factor	Developed Trended Layer Loss and ALAE
7/1/2017	-	1.10	-
7/1/2017	375,915	1.10	413,507
7/1/2018	10,466	1.40	14,652
7/1/2018	1,140,698	1.40	1,596,978
7/1/2019	30,750	2.40	73,800
7/1/2019	849,250	2.40	2,038,200
			4,137,137

Step 5: Calculate the rate as the total developed trended layer loss and ALAE divided by the total on level subject premium for the period 2017 to 2019.

$4,137,137 / (3 \times 10,000,000) = 13.8\%$

2. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

Sources:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the funds generating coefficient estimate, *k*.

Calculated as the sum of the "Percentage of the Insurer's Business" for each Group times the "Average Time Between Receipt of Premium and Payment of Losses and Expenses" for each Group.

$40\% \times 0.9 + 35\% \times 1.2 + 25\% \times 1.5 = 1.155$

(b) Calculate the underwriting beta.

Calculated as k, from part (a), times the liability beta times −1.

$1.155 \times -0.2 \times -1 = 0.231$

(c) Calculate the underwriting profit margin ignoring taxes.

The underwriting profit margin (UPM) is calculated as *k*, from part (a), times the risk-free rate times −1 plus the underwriting beta (β_u), from part (b), times the difference between the expected return on the market portfolio ($E(R_m)$)and the risk-free rate (R_f).

$UPM = (1.155 \times 2\% \times -1) + (0.231 \times (10\% - 2\%)) = -0.46\%$

2. **Continued**

(d) Calculate the underwriting profit margin using the version of CAPM that accounts for taxes.

First one needs to calculate the tax rate on investment income, TA, which is the sum of the Percentage of Total Assets for each asset type times the tax rate for each asset type.

Let T = tax rate on underwriting income and S/P = equity to premium ratio.

Then the underwriting profit margin UPM =
$-k \times R_f \times (1 - \text{TA})/(1 - \text{T}) + \beta_u \times [\text{E}(R_m) - R_f] + (\text{S/P}) \times R_f \times \text{TA}/(1 - \text{T})$.

TA = 20% × 0% + 30% × 10% + 50% × 30% = 18%
UPM = −1.155 × 2% × (1 − 0.18) / (1 − 0.3) + 0.231 × (10% – 2%) +
(1/2) × 2% × 0.18 / (1 − 0.3)

UPM = −0.60%

3. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) through (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (b) through (d) are for explanatory purposes only. In Excel, the candidate was provided with data from the question in a tabular format. This table also included four columns with missing entries. These columns were labeled and were to be completed for the responses to parts (b) through (d).

Solution:

(a) Explain why this should not be a cause for concern.

Commentary on Question:

There were two reasons cited by Clark. Only one of these reasons was required for full credit. The model solution provides both reasons.

- The scale factor is generally small compared to the mean, so little precision is lost.
- The use of a discrete distribution allows for a mass point at zero, representing the cases in which no change in loss is seen in a given development increment.

(b) Calculate the value of ℓ at its maximum.

Commentary on Question:

The candidate was to complete the column in the Excel table labeled ℓ (cells I9 to I18). The sum of this column is the maximum value.

3. Continued

In order to complete this column, first complete the values in the column labeled "Expected increment", x (cells H9 to H18).

For each row in the table, calculate the expected increment *x* as onlevel premium times the ELR times the difference between G at the beginning of the interval and G at the end of the interval, where G is the CDF of the loglogistic distribution.

Each row of the ℓ column is the increment (in column E) times the natural logarithm of the expected increment *x* (calculated in column H) minus the expected increment *x.*

The sum of column I in the table, labeled ℓ, is equal to 169,574.4397. This the value of ℓ at its maximum.

(c) Estimate the scale factor, σ^2.

Commentary on Question:
The candidate was to complete the column in the Excel table labeled σ^2 (cells J9 to J18). The sum of this column divided by 7 is the scale factor.

In order to complete this column, first complete the values in the column for expected increment x (cells H9 to H18). This was completed for the response to part (b).

For each row in the table, calculate the amount in column J as the square of the difference between the increment (column E) and the expected increment (column H), divided by the expected increment.

The sum of column J, labeled σ^2, divided by 7 is equal to 43.5880387. The value of 7 is the number of rows (10) less the number of estimated parameters (3). This is the value of the scale factor σ^2.

(d) Create a scatter plot in which the *x* values are the expected incremental losses and the *y* values are the normalized residuals.

Commentary on Question:
The scatter plot was already set up to plot x, the expected increment, against y, the normalized residual. The expected increment column was completed for part (b). For this part, the candidate needed to complete the column for y, the normalized residual (column K). The scatter plot would be automatically created.

3. Continued

For each row in the table, calculate the normalized residual *y* as (the increment in column E minus the expected increment in column H) divided by the square root of the scale factor, σ^2, times the expected increment.

The scatter plot was automatically created (below row 40) using the values in columns H and K in the table.

(e) Interpret the scatter plot in part (d) with regard to determining if the model assumptions are correct.

Commentary on Question:
The model solution is an example of a full credit solution. It assumes that the x and y values were calculated correctly.

The residuals should be random about the zero line. That appears to be the case, providing evidence to support the assumptions.

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) through (e). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (b) through (e) are for explanatory purposes only. In Excel, the candidate was provided with data from the question with response cells color coded as indicated in the Excel version of the question, parts (b) through (e).

Solution:

(a) State the three statistical assumptions underlying the chain ladder model.

- The conditional expected accumulated total claims amount at a given development year is the accumulated total claims amount at the previous development year times a development factor that does not vary by accident year.
- The accumulated total claims amounts of different accident years are independent.
- The conditional variance of the accumulated total claims amount at a given development year is the accumulated total claims amount at the previous development year times a proportionality constant that does not vary by accident year.

(b) Complete the triangle of age-to-age factors.

Commentary on Question:

The color-coded cells from C32 to H37 were indicated as the response cells for part (b). Cells C32 to C37 were already prefilled with factors.

4. Continued

The factors are calculated for each accident year as the paid claims at development year x+1 divided by the paid claims at development year x. The values are as follows:

Age-to-Age Factors

	1	1.650	1.319	1.082	1.147	1.195	1.113
	2	40.425	1.259	1.977	1.292	1.132	
	3	2.637	1.543	1.163	1.161		
(b)	4	2.043	1.364	1.349			
	5	8.759	1.656				
	6	4.260					

(c) Calculate the remaining values of f_k and α_k^2.

Commentary on Question:
The color-coded cells from C39 to H40 were indicated as the response cells for part (c). Cells C39 and C40 were already prefilled with values.

The f_k values are calculated as follows:

- For k = 2, it equals the sum of paid claims at development year 3 divided by the sum of paid claims at development year 2, for accident years 1 through 5.
- For k = 3, it equals the sum of paid claims at development year 4 divided by the sum of paid claims at development year 3, for accident years 1 through 4.
- The pattern continues for k = 4 through 6.

The α_k^2 values are calculated as follows for k = 2 to 5:

$$\alpha_k^2 = \frac{1}{7-k-1}\sum_{j=1}^{7-k} c_{j,k}\left(\frac{c_{j,k+1}}{c_{j,k}} - f_k\right)^2$$

where the $c_{j,k}$ are the paid losses in which j is the accident year and k is the development year.

For k = 6, $\alpha_6^2 = \frac{\left(\alpha_5^2\right)^2}{\alpha_4^2}$.

The values are as follows:

(c)	f_k	2.925	1.448	1.303	1.193	1.163	1.113
	α_k^2	40,350	216	1,094	73	27	10

4. Continued

(d) Square the development triangle by completing the remaining shaded cells, where one calculated value is provided.

Commentary on Question:
The color-coded cells from D25 to I30 were indicated as the response cells for part (d). Cell D30 was already prefilled with the value for $c_{7,2}$.

The remaining c values to square the development triangle are calculated as follows:

$$c_{j,k} = c_{j,k-1} \times f_{k-1}$$

The completed squared triangle is as follows:

	AY	1	2	3	4	5	6	7
	1	5,012	8,269	10,907	11,805	13,539	16,181	18,009
	2	106	4,285	5,396	10,666	13,782	15,599	17,361
	3	3,410	8,992	13,873	16,141	18,735	21,793	24,255
(d)	4	5,655	11,555	15,766	21,266	25,366	29,506	32,839
	5	1,092	9,565	15,836	20,640	24,619	28,637	31,872
	6	1,513	6,445	9,332	12,163	14,508	16,875	18,782
	7	557	1,629	2,359	3,074	3,667	4,265	4,747

(e) Calculate the remaining standard errors of the reserve estimators for the individual accident years.

Commentary on Question:
The color-coded cells from J24 to J30 were indicated as the response cells for part (e). Cells J24, J25, J28, J29 and J30 were already prefilled with the standard error (SE) values. The candidate was required to fill in cells J26 (SE for AY 3) and J27 (SE for AY 4).

$$SE_{AY3} = \sqrt{c_{3,7}^2 \times \left(\frac{\alpha_5^2}{f_5^2} \times \left(\frac{1}{c_{3,5}} + \frac{1}{c_{1,5}+c_{2,5}} \right) + \frac{\alpha_6^2}{f_6^2} \times \left(\frac{1}{c_{3,6}} + \frac{1}{c_{1,6}} \right) \right)} = 1,262$$

$$SE_{AY4} = \sqrt{c_{4,7}^2 \times \left(\frac{\alpha_4^2}{f_4^2} \times \left(\frac{1}{c_{4,4}} + \frac{1}{c_{1,4}+c_{2,4}+c_{3,4}} \right) + \frac{\alpha_5^2}{f_5^2} \times \left(\frac{1}{c_{4,5}} + \frac{1}{c_{1,5}+c_{2,5}} \right) + \frac{\alpha_6^2}{f_6^2} \times \left(\frac{1}{c_{4,6}} + \frac{1}{c_{1,6}} \right) \right)} = 2,562$$

4. Continued

(f) Describe how expected future emergence differs between the two models.

The chain ladder model assumes that expected future emergence for an accident year is proportional to losses emerged to date. The parameterized BF model assumes that expected future emergence for an accident year is proportional to expected ultimate losses.

5. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:

As Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the renewal risk load for each account using the Marginal Variance method.

- The risk load for each account is the account marginal variance times λ.
- Account marginal variance is the total variance less the variance of the other account.
  - $Var(X+Y) = \Sigma_i Var(X_i+Y_i) = \Sigma_i (X_i+Y_i)^2 \times p(i) \times (1-p(i)) = 14{,}419{,}500$
  - $Var(X) = \Sigma_i Var(X_i) = \Sigma_i (X_i)^2 \times p(i) \times (1-p(i)) = 6{,}880{,}000$
  - $Var(Y) = \Sigma_i Var(Y_i) = \Sigma_i (Y_i)^2 \times p(i) \times (1-p(i)) = 1{,}655{,}500$
  - X marginal variance = 14,419,500 – 1,655,500 = 12,764,000
  - Y marginal variance = 14,419,500 – 6,880,000 = 7,539,500

- X renewal risk load = 12,764,000 × λ = 306.34
- Y renewal risk load = 7,539,500 × λ = 180.95

(b) Demonstrate that the Marginal Variance method is not renewal additive.

- Risk load for both accounts combined = 14,419,500 × λ = 346.06
- The risk load from account X plus the risk load from account Y = 487.28
- The risk load for both accounts combined does not equal the sum of the risk loads for each account.

5. Continued

(c) Calculate the risk load for each account using the Covariance Share method.

For each event (i) calculate:

- Covariance to share as $\text{Cov}(i) = \text{Var}(X_i+Y_i) - \text{Var}(X_i) - \text{Var}(Y_i)$
- X-share of $\text{Cov}(i) = \text{Cov}(i) \times X_i / (X_i + Y_i)$
- Y-share of $\text{Cov}(i) = \text{Cov}(i) \times Y_i / (X_i + Y_i)$

X risk load = $\lambda \times \Sigma_i$ [Var(X_i) + X-share of Cov(i)] = 258.12
Y risk load = $\lambda \times \Sigma_i$ [Var(Y_i) + Y-share of Cov(i)] = 87.95

6. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question required the candidate to respond in Excel for parts (c) and (d). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (c) and (d) are for explanatory purposes only.

Solution:

(a) There are two sources of systemic risk: internal risk and external risk.

Define each source.

Internal risk is the extent to which the adopted actuarial valuation approach is an imperfect representation of the real-life process. External risk is all risks that are outside the modeling process.

(b) There are two sources of independent risk: parameter risk and process risk.

Define each source.

Parameter risk is the extent to which randomness prevents accurate selection of parameters. Process risk is the pure randomness associated with the insurance process.

(c) Calculate the coefficient of variation for each risk source for both lines combined.

The solution below uses the following abbreviations:
M = line of business Motor
H = line of business Home
CoV = coefficient of variation
p_x = percentage of liabilities for line of business x
IND_x = independent risk CoV for line of business x
ISR_x = internal systemic risk CoV for line of business x
ESR_x = external systemic risk CoV for line of business x
$\rho_{IND(MH)}$ = correlation between motor and home liabilities for IND

6. **Continued**

$\rho_{ISR(MH)}$ = correlation between motor and home liabilities for ISR
$\rho_{ESR(MH)}$ = correlation between motor and home liabilities for ESR

IND CoV
$= [p_M^2 \times IND_M^2 + p_H^2 \times IND_H^2 + 2 \times p_M \times p_H \times IND_M \times IND_H \times \rho_{IND(MH)}]^{0.5}$
= 5.200%

ISR CoV =
$= [p_M^2 \times ISR_M^2 + p_H^2 \times ISR_H^2 + 2 \times p_M \times p_H \times ISR_M \times ISR_H \times \rho_{ISR(MH)}]^{0.5}$
= 4.508%

ESR CoV =
$= [p_M^2 \times ESR_M^2 + p_H^2 \times ESR_H^2 + 2 \times p_M \times p_H \times ESR_M \times ESR_H \times \rho_{ESR(MH)}]^{0.5}$
= 2.988%

(d) Calculate the consolidated coefficient of variation from the three sources of uncertainty. Assume independence between each of the sources of uncertainty.

Consolidated CoV = $[\text{IND CoV}^2 + \text{ISR CoV}^2 + \text{ESR CoV}^2]^{0.5}$ = 7.503%

7. Learning Objectives:

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3e) Explain Table M and Table L construction in graphical terms.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Commentary on Question:

This question required the candidate to respond in Excel for parts (b) and (c). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (b) and (c) are for explanatory purposes only.

Solution:

(a) Determine the values of r_H and r_G.

From the first equation, $r_G - r_H = 0.80$.
From the second equation, $\phi(r_H) - \phi(r_G) = 0.48$.
Using Table M then yields $r_H = 0.40$ and $r_G = 1.20$.

(b) Calculate the net insurance charge.

Net insurance charge = $I = E \times [\phi(1.2) - \psi(0.4)] = 6$

(c) Calculate the basic premium.

Basic premium = $b = e - (C - 1) \times E + C \times I = 7.5$

8. **Learning Objectives:**

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (a) and (b). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The model solutions in this file for parts (a) and (b) are for explanatory purposes only.

Solution:

(a) Complete the following aggregate loss probability table:

Aggregate Losses (millions)	Probability
0	
1	0.1673
2	0.1966
3	0.1496
4	
5	
6	0.0411
7	
8	
9	
10	
11	
12	
13	
14	0.0001
15	0.0000

Commentary on Question:

Note that the calculation of the probability of 0 for aggregate losses is from the Poisson distribution and is equal to $e^{-\lambda}$. The probability of 0 for aggregate losses could have been estimated as 1 minus the sum of the probabilities for aggregate losses from 1 to 15 million. This is not exact because aggregate losses could exceed 15 million. There was a minor deduction for using this approximation.

8. Continued

Let p_y = probability of loss size of y million.

The annual number of losses is Poisson with mean 1.5. That is, $\lambda = 1.5$.

For aggregate losses of 0, the probability is $e^{-\lambda} = 0.2231$.

For aggregate losses of x, in millions, the probability is given by the formula:

$$(\lambda/x) \times [(x-1) \times p_1 + 2 \times (x-2) \times p_2 + 3 \times (x-3) \times p_3]$$

The table of values is then given by:

Aggregate Losses (million)	Probability
0	0.2231
1	0.1673
2	0.1966
3	0.1496
4	0.1059
5	0.0695
6	0.0411
7	0.0231
8	0.0122
9	0.0062
10	0.0030
11	0.0014
12	0.0006
13	0.0003
14	0.0001
15	0.0000

(b) Verify the following underwriting results for Specialist:

(i) A profit of 0.3 million if aggregate losses are 2 million.

(ii) A loss of 1.125 million if aggregate losses are 5 million.

8. Continued

Profit is premium less the losses and margin.

- For (i) this is 0.25
- For (ii) this is −2.75

When profit is greater than zero, there is a profit commission of 80% of the profit.

- For (i) this is 0.2
- For (ii) this is 0

When profit is less than zero (i.e., a loss), there is additional premium of 50% of (losses plus the margin minus the annual premium).

- For (i) this is 0
- For (ii) this is 1.375

The underwriting result is premium plus additional premium less losses less profit commission.

- For (i) this is 2.5 + 0 – 2 – 0.2 = 0.3
- For (ii) this is 2.5 + 1.375 – 5 – 0 = −1.125

(c) State the two conditions that a finite reinsurance arrangement must fulfill for a ceding company to consider it insurance.

- The reinsurer must assume significant insurance risk.
- It must be reasonably possible that the reinsurer will realize a significant loss.

(d) Explain whether the finite reinsurance can be considered insurance by Ceding Insurance Company.

Commentary on Question:
There is no single correct answer to this question. However, an explanation for this should consider loss sizes relative to the premium and their probabilities. The model solution is an example of a full credit solution.

We may consider that a significant loss is one that is at least 25% of the premium. This level of loss has a probability of over 25% which can be considered reasonably possible. As such, it can be considered as insurance.