SOCIETY OF ACTUARIES

EXAM MLC Models for Life Contingencies

EXAM MLC SAMPLE QUESTIONS

The following questions or solutions have been modified since this document was prepared to use with the syllabus effective spring 2012

Prior to March 1, 2012:
Questions: 151, 181, 289, 300
Solutions: 2, 284, 289, 290, 295, 300

Changed on March 19, 2012: Questions: 20, 158, 199 (all are minor edits)

Changed on April 24, 2012: Solution: 292

Changed on August 20, 2012:Questions and Solutions 38, 54, 89, 180, 217 and 218 were restored from MLC-09-08 and reworded to conform to the current presentation. Question 288 was reworded to resolve an ambiguity. A typo in Question 122 was corrected. Questions and Solutions 301-309 are new questions

Changed on August 23, 2012:
Solution 47, initial formula corrected; Solution 72, minus signs added in the first integral

Changed on December 11, 2012: Question 300 deleted

The questions in this study note were previously presented in study note MLC-09-08. The questions in this study note have been edited for use under the 2012 learning objectives. Some questions were not applicable to the 2012 learning objectives and these questions were removed. Some of the questions in this study note are taken from past SOA examinations. No questions from published exams after 2005 are included. The November 2006 Exam M and May 2007 and May 2012 Exam MLC are available at http://www.soa.org/education/exam-req/syllabus-study-materials/edu-multiple-choice-exam.aspx. The weight of topics in these sample questions is not representative of the weight of topics on the exam. The syllabus indicates the exam weights by topic.

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1. For two independent lives now age 30 and 34, you are given:

x	q_x
30	0.1
31	0.2
32	0.3
33	0.4
34	0.5
35	0.6
36	0.7
37	0.8

Calculate the probability that the last death of these two lives will occur during the 3rd year from now (i.e. ${}_{2|}q_{\overline{30:34}}$).

(A) 0.01

(B) 0.03

(C) 0.14

(D) 0.18

(E) 0.24

2. For a whole life insurance of 1000 on (x) with benefits payable at the moment of death:

(i) The force of interest at time t, $\delta_t = \begin{cases} 0.04, & 0 < t \le 10 \\ 0.05, & 10 < t \end{cases}$

(ii) $\mu_{x+t} = \begin{cases} 0.06, & 0 < t \le 10 \\ 0.07, & 10 < t \end{cases}$

Calculate the single benefit premium for this insurance.

(A) 379

(B) 411

(C) 444

(D) 519

(E) 594

3. For a special whole life insurance on (x), payable at the moment of death:

(i) $\mu_{x+t} = 0.05$, $t > 0$

(ii) $\delta = 0.08$

(iii) The death benefit at time t is $b_t = e^{0.06t}$, $t > 0$.

(iv) Z is the present value random variable for this insurance at issue.

Calculate $\mathrm{Var}(Z)$.

(A) 0.038

(B) 0.041

(C) 0.043

(D) 0.045

(E) 0.048

4. For a group of individuals all age x, you are given:

(i) 25% are smokers (s); 75% are nonsmokers (ns).

(ii)

k	q_{x+k}^{s}	q_{x+k}^{ns}
0	0.10	0.05
1	0.20	0.10
2	0.30	0.15

$i = 0.02$

Calculate $10{,}000 A_{x:\overline{2}|}^{1}$ for an individual chosen at random from this group.

(A) 1690

(B) 1710

(C) 1730

(D) 1750

(E) 1770

5. A whole life policy provides that upon accidental death as a passenger on an airplane a benefit of 1,000,000 will be paid. If death occurs from other accidental causes, a death benefit of 500,000 will be paid. If death occurs from a cause other than an accident, a death benefit of 250,000 will be paid.

You are given:

(i) Death benefits are payable at the moment of death.

(ii) $\mu^{(1)} = 1/2,000,000$ where (1) indicates accidental death as a passenger on an airplane.

(iii) $\mu^{(2)} = 1/250,000$ where (2) indicates death from other accidental causes.

(iv) $\mu^{(3)} = 1/10,000$ where (3) indicates non-accidental death.

(v) $\delta = 0.06$

Calculate the single benefit premium for this insurance.

(A) 450

(B) 460

(C) 470

(D) 480

(E) 490

6. For a special fully discrete whole life insurance of 1000 on (40):

(i) The level benefit premium for each of the first 20 years is π.

(ii) The benefit premium payable thereafter at age x is $1000\,v\,q_x$, $x = 60, 61, 62, \ldots$

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate π.

(A) 4.79

(B) 5.11

(C) 5.34

(D) 5.75

(E) 6.07

7. For an annuity payable semiannually, you are given:

(i) Deaths are uniformly distributed over each year of age.

(ii) $q_{69} = 0.03$

(iii) $i = 0.06$

(iv) $1000\bar{A}_{70} = 530$

Calculate $\ddot{a}_{69}^{(2)}$.

(A) 8.35

(B) 8.47

(C) 8.59

(D) 8.72

(E) 8.85

8. Removed

9. Removed

10. For a fully discrete whole life insurance of 1000 on (40), the gross premium is the level annual benefit premium based on the mortality assumption at issue. At time 10, the actuary decides to increase the mortality rates for ages 50 and higher.

You are given:

(i) $d = 0.05$

(ii) Mortality assumptions:

At issue	${}_{k\|}q_{40} = 0.02,\ \ k = 0,1,2,...,49$
Revised prospectively at time 10	${}_{k\|}q_{50} = 0.04,\ \ k = 0,1,2,...,24$

(iii) ${}_{10}L$ is the prospective loss random variable at time 10 using the gross premium.

(iv) K_{40} is the curtate future lifetime of (40) random variable.

Calculate $\mathrm{E}[{}_{10}L \mid K_{40} \geq 10]$ using the revised mortality assumption.

(A) Less than 225

(B) At least 225, but less than 250

(C) At least 250, but less than 275

(D) At least 275, but less than 300

(E) At least 300

11. For a group of individuals all age x, of which 30% are smokers and 70% are non-smokers, you are given:

(i) $\delta = 0.10$

(ii) $\bar{A}_x^{\text{smoker}} = 0.444$

(iii) $\bar{A}_x^{\text{non-smoker}} = 0.286$

(iv) T is the future lifetime of (x).

(v) $\operatorname{Var}\left[\bar{a}_{\overline{T}|}^{\text{smoker}}\right] = 8.818$

(vi) $\operatorname{Var}\left[\bar{a}_{\overline{T}|}^{\text{non-smoker}}\right] = 8.503$

Calculate $\operatorname{Var}\left[\bar{a}_{\overline{T}|}\right]$ for an individual chosen at random from this group.

(A) 8.5

(B) 8.6

(C) 8.8

(D) 9.0

(E) 9.1

12. Removed

13. A population has 30% who are smokers with a constant force of mortality 0.2 and 70% who are non-smokers with a constant force of mortality 0.1.

Calculate the 75^{th} percentile of the distribution of the future lifetime of an individual selected at random from this population.

(A) 10.7

(B) 11.0

(C) 11.2

(D) 11.6

(E) 11.8

14. For a fully continuous whole life insurance of 1 on (x), you are given:

(i) The forces of mortality and interest are constant.

(ii) ${}^2\bar{A}_x = 0.20$

(iii) The benefit premium is 0.03.

(iv) ${}_0L$ is the loss-at-issue random variable based on the benefit premium.

Calculate $\mathrm{Var}\left({}_0L\right)$.

(A) 0.20

(B) 0.21

(C) 0.22

(D) 0.23

(E) 0.24

15. Removed

16. For a special fully discrete whole life insurance on (40):

(i) The death benefit is 1000 for the first 20 years; 5000 for the next 5 years; 1000 thereafter.

(ii) The annual benefit premium is $1000P_{40}$ for the first 20 years; $5000P_{40}$ for the next 5 years; π thereafter.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate ${}_{21}V$, the benefit reserve at the end of year 21 for this insurance.

(A) 255

(B) 259

(C) 263

(D) 267

(E) 271

17. For a whole life insurance of 1 on (41) with death benefit payable at the end of year of death, you are given:

(i) $i = 0.05$

(ii) $p_{40} = 0.9972$

(iii) $A_{41} - A_{40} = 0.00822$

(iv) ${}^2A_{41} - {}^2A_{40} = 0.00433$

(v) Z is the present-value random variable for this insurance.

Calculate Var(Z).

(A) 0.023

(B) 0.024

(C) 0.025

(D) 0.026

(E) 0.027

18. Removed

19. Removed

20. For a double decrement table, you are given:

(i) $\mu_{x+t}^{(1)} = 0.2\,\mu_{x+t}^{(\tau)}, \quad t > 0$

(ii) $\mu_{x+t}^{(\tau)} = kt^2, \quad t > 0$

(iii) $q_x^{\prime(1)} = 0.04$

Calculate ${}_2q_x^{(2)}$.

(A) 0.45

(B) 0.53

(C) 0.58

(D) 0.64

(E) 0.73

21. For (x):

(i) K is the curtate future lifetime random variable.

(ii) $q_{x+k} = 0.1(k+1), \quad k = 0, 1, 2, \ldots, 9$

(iii) $X = \min(K, 3)$

Calculate $\mathrm{Var}(X)$.

(A) 1.1

(B) 1.2

(C) 1.3

(D) 1.4

(E) 1.5

22. For a population which contains equal numbers of males and females at birth:

(i) For males, $\mu_x^m = 0.10, \quad x \geq 0$

(ii) For females, $\mu_x^f = 0.08, \quad x \geq 0$

Calculate q_{60} for this population.

(A) 0.076

(B) 0.081

(C) 0.086

(D) 0.091

(E) 0.096

23. Michel, age 45, is expected to experience higher than standard mortality only at age 64. For a special fully discrete whole life insurance of 1 on Michel, you are given:

(i) The benefit premiums are not level.

(ii) The benefit premium for year 20, π_{19}, exceeds P_{45} for a standard risk by 0.010.

(iii) Benefit reserves on his insurance are the same as benefit reserves for a fully discrete whole life insurance of 1 on (45) with standard mortality and level benefit premiums.

(iv) $i = 0.03$

(v) The benefit reserve at the end of year 20 for a fully discrete whole life insurance of 1 on (45), using standard mortality and interest, is 0.427.

Calculate the excess of q_{64} for Michel over the standard q_{64}.

(A) 0.012

(B) 0.014

(C) 0.016

(D) 0.018

(E) 0.020

24. For a block of fully discrete whole life insurances of 1 on independent lives age x, you are given:

(i) $i = 0.06$

(ii) $A_x = 0.24905$

(iii) ${}^2A_x = 0.09476$

(iv) $\pi = 0.025$, where π is the gross premium for each policy.

(v) Losses are based on the gross premium.

Using the normal approximation, calculate the minimum number of policies the insurer must issue so that the probability of a positive total loss on the policies issued is less than or equal to 0.05.

(A) 25

(B) 27

(C) 29

(D) 31

(E) 33

25. Your company currently offers a whole life annuity product that pays the annuitant 12,000 at the beginning of each year. A member of your product development team suggests enhancing the product by adding a death benefit that will be paid at the end of the year of death.

Using a discount rate, d, of 8%, calculate the death benefit that minimizes the variance of the present value random variable of the new product.

(A) 0

(B) 50,000

(C) 100,000

(D) 150,000

(E) 200,000

26. For a special fully continuous last survivor insurance of 1 on (x) and (y), you are given:

(i) T_x and T_y are independent.

(ii) For (x), $\mu_{x+t} = 0.08, \quad t > 0$

(iii) For (y), $\mu_{y+t} = 0.04, \quad t > 0$

(iv) $\delta = 0.06$

(v) π is the annual benefit premium payable until the first of (x) and (y) dies.

Calculate π.

(A) 0.055

(B) 0.080

(C) 0.105

(D) 0.120

(E) 0.150

27. For a special fully discrete whole life insurance of 1000 on (42):

(i) The gross premium for the first 4 years is equal to the level benefit premium for a fully discrete whole life insurance of 1000 on (40).

(ii) The gross premium after the fourth year is equal to the level benefit premium for a fully discrete whole life insurance of 1000 on (42).

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) ${}_3L$ is the prospective loss random variable at time 3, based on the gross premium.

(vi) K_{42} is the curtate future lifetime of (42).

Calculate $\mathrm{E}\left[{}_3L \middle| K_{42} \geq 3\right]$.

(A) 27

(B) 31

(C) 44

(D) 48

(E) 52

28. For T, the future lifetime random variable for (0):

(i) $\omega > 70$

(ii) ${}_{40}p_0 = 0.6$

(iii) E(T) = 62

(iv) $\mathrm{E}\left[\min\left(T, t\right)\right] = t - 0.005t^2, \quad 0 < t < 60$

Calculate the complete expectation of life at 40.

(A) 30

(B) 35

(C) 40

(D) 45

(E) 50

29. Two actuaries use the same mortality table to price a fully discrete 2-year endowment insurance of 1000 on (x).

(i) Kevin calculates non-level benefit premiums of 608 for the first year and 350 for the second year.

(ii) Kira calculates level annual benefit premiums of π.

(iii) $d = 0.05$

Calculate π.

(A) 482

(B) 489

(C) 497

(D) 508

(E) 517

30. For a fully discrete 10-payment whole life insurance of 100,000 on (x), you are given:

(i) $i = 0.05$

(ii) $q_{x+9} = 0.011$

(iii) $q_{x+10} = 0.012$

(iv) $q_{x+11} = 0.014$

(v) The level annual benefit premium is 2078.

(vi) The benefit reserve at the end of year 9 is 32,535.

Calculate $100,000 A_{x+11}$.

(A) 34,100

(B) 34,300

(C) 35,500

(D) 36,500

(E) 36,700

31. You are given:

(i) $l_x = 10(105 - x),\ 0 \le x \le 105$.

(ii) (45) and (65) have independent future lifetimes.

Calculate $\mathring{e}_{\overline{45:65}}$.

(A) 33

(B) 34

(C) 35

(D) 36

(E) 37

32. Given: The survival function $S_0(t)$, where

$$S_0(t) = 1, \qquad 0 \le t < 1$$

$$S_0(t) = 1 - \left\{\left(e^x\right)/100\right\}, \qquad 1 \le t < 4.5$$

$$S_0(t) = 0, \qquad 4.5 \le x$$

Calculate μ_4.

(A) 0.45

(B) 0.55

(C) 0.80

(D) 1.00

(E) 1.20

33. For a triple decrement table, you are given:

(i) $\mu_{x+t}^{(1)} = 0.3, \quad t > 0$

(ii) $\mu_{x+t}^{(2)} = 0.5, \quad t > 0$

(iii) $\mu_{x+t}^{(3)} = 0.7, \quad t > 0$

Calculate $q_x^{(2)}$.

(A) 0.26

(B) 0.30

(C) 0.33

(D) 0.36

(E) 0.39

34. You are given:

(i) the following select-and-ultimate mortality table with 3-year select period:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

(ii) $i = 0.03$

Calculate ${}_{2|2}A_{[60]}$, the actuarial present value of a 2-year deferred 2-year term insurance on $[60]$.

(A) 0.156

(B) 0.160

(C) 0.186

(D) 0.190

(E) 0.195

35. You are given:

(i) $\mu_{x+t} = 0.01, \quad 0 \le t < 5$

(ii) $\mu_{x+t} = 0.02, \quad 5 \le t$

(iii) $\delta = 0.06$

Calculate $\bar{a}_x$.

(A) 12.5

(B) 13.0

(C) 13.4

(D) 13.9

(E) 14.3

36. For a double decrement table, you are given:

(i) $q'^{(1)}_x = 0.2$

(ii) $q'^{(2)}_x = 0.3$

(iii) Each decrement is uniformly distributed over each year of age in the double decrement table.

Calculate ${}_{0.3}q^{(1)}_{x+0.1}$.

(A) 0.020

(B) 0.031

(C) 0.042

(D) 0.053

(E) 0.064

37. For a fully continuous whole life insurance of 1 on (x), you are given:

(i) $\delta = 0.04$

(ii) $\bar{a}_x = 12$

(iii) $Var\left(v^T\right) = 0.10$

(iv) Expenses are
  (a) 0.02 initial expense
  (b) 0.003 per year, payable continuously

(v) The gross premium is the benefit premium plus 0.0066.

(vi) ${}_0L$ is the loss variable at issue.

Calculate $Var\left({}_0L\right)$.

(A) 0.208

(B) 0.217

(C) 0.308

(D) 0.434

(E) 0.472

38. For a discrete-time Markov model for an insured population:

(i) Annual transition probabilities between health states of individuals are as follows:

	Healthy	Sick	Terminated
Healthy	0.7	0.1	0.2
Sick	0.3	0.6	0.1
Terminated	0.0	0.0	1.0

(ii) The mean annual healthcare cost each year for each health state is:

	Mean
Healthy	500
Sick	3000
Terminated	0

(iii) Transitions occur at the end of the year.

(iv) $i = 0$

A gross premium of 800 is paid each year by an insured not in the terminated state.

Calculate the expected value of gross premiums less healthcare costs over the first 3 years for a new healthy insured.

(A) –390

(B) –200

(C) –20

(D) 160

(E) 340

39. Removed:

40. For a fully discrete whole life insurance of 1000 on (60), the annual benefit premium was calculated using the following:

(i) $i = 0.06$

(ii) $q_{60} = 0.01376$

(iii) $1000A_{60} = 369.33$

(iv) $1000A_{61} = 383.00$

A particular insured is expected to experience a first-year mortality rate ten times the rate used to calculate the annual benefit premium. The expected mortality rates for all other years are the ones originally used.

Calculate the expected loss at issue for this insured, based on the original benefit premium.

(A) 72

(B) 86

(C) 100

(D) 114

(E) 128

41. For a fully discrete whole life insurance of 1000 on (40), you are given:

(i) $i = 0.06$

(ii) Mortality follows the Illustrative Life Table.

(iii) $\ddot{a}_{40:\overline{10}|} = 7.70$

(iv) $\ddot{a}_{50:\overline{10}|} = 7.57$

(v) $1000 A^{1}_{40:\overline{20}|} = 60.00$

At the end of the tenth year, the insured elects an option to retain the coverage of 1000 for life, but pay premiums for the next ten years only.

Calculate the revised annual benefit premium for the next 10 years.

(A) 11

(B) 15

(C) 17

(D) 19

(E) 21

42. For a double-decrement table where cause 1 is death and cause 2 is withdrawal, you are given:

(i) Deaths are uniformly distributed over each year of age in the single-decrement table.

(ii) Withdrawals occur only at the end of each year of age.

(iii) $l_x^{(\tau)} = 1000$

(iv) $q_x^{(2)} = 0.40$

(v) $d_x^{(1)} = 0.45\ d_x^{(2)}$

Calculate $p_x'^{(2)}$.

(A) 0.51

(B) 0.53

(C) 0.55

(D) 0.57

(E) 0.59

43. You intend to hire 200 employees for a new management-training program. To predict the number who will complete the program, you build a multiple decrement table. You decide that the following associated single decrement assumptions are appropriate:

(i) Of 40 hires, the number who fail to make adequate progress in each of the first three years is 10, 6, and 8, respectively.

(ii) Of 30 hires, the number who resign from the company in each of the first three years is 6, 8, and 2, respectively.

(iii) Of 20 hires, the number who leave the program for other reasons in each of the first three years is 2, 2, and 4, respectively.

(iv) You use the uniform distribution of decrements assumption in each year in the multiple decrement table.

Calculate the expected number who fail to make adequate progress in the third year.

(A) 4

(B) 8

(C) 12

(D) 14

(E) 17

44. Removed

45. Your company is competing to sell a life annuity-due with an actuarial present value of 500,000 to a 50-year old individual.

Based on your company's experience, typical 50-year old annuitants have a complete life expectancy of 25 years. However, this individual is not as healthy as your company's typical annuitant, and your medical experts estimate that his complete life expectancy is only 15 years.

You decide to price the benefit using the issue age that produces a complete life expectancy of 15 years. You also assume:

(i) For typical annuitants of all ages, $l_x = 100(\omega - x)$, $0 \le x \le \omega$.

(ii) $i = 0.06$

Calculate the annual benefit that your company can offer to this individual.

(A) 38,000

(B) 41,000

(C) 46,000

(D) 49,000

(E) 52,000

46. For a temporary life annuity-immediate on independent lives (30) and (40):

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

Calculate $a_{30:40:\overline{10|}}$.

(A) 6.64

(B) 7.17

(C) 7.88

(D) 8.74

(E) 9.86

47. For a special whole life insurance on (35), you are given:

(i) The annual benefit premium is payable at the beginning of each year.

(ii) The death benefit is equal to 1000 plus the return of all benefit premiums paid in the past without interest.

(iii) The death benefit is paid at the end of the year of death.

(iv) $A_{35} = 0.42898$

(v) $(IA)_{35} = 6.16761$

(vi) $i = 0.05$

Calculate the annual benefit premium for this insurance.

(A) 73.66

(B) 75.28

(C) 77.42

(D) 78.95

(E) 81.66

48. Removed

49. For a special fully continuous whole life insurance of 1 on the last-survivor of (*x*) and (*y*), you are given:

(i) T_x and T_y are independent.

(ii) $\mu_{x+t} = \mu_{y+t} = 0.07, \quad t > 0$

(iii) $\delta = 0.05$

(iv) Premiums are payable until the first death.

Calculate the level annual benefit premium for this insurance.

(A) 0.04

(B) 0.07

(C) 0.08

(D) 0.10

(E) 0.14

50. For a fully discrete whole life insurance of 1000 on (20), you are given:

(i) $1000\,P_{20} = 10$

(ii) The following benefit reserves for this insurance

(a) ${}_{20}V = 490$
(b) ${}_{21}V = 545$
(c) ${}_{22}V = 605$

(iii) $q_{40} = 0.022$

Calculate q_{41}.

(A) 0.024

(B) 0.025

(C) 0.026

(D) 0.027

(E) 0.028

51. For a fully discrete whole life insurance of 1000 on (60), you are given:

(i) $i = 0.06$

(ii) Mortality follows the Illustrative Life Table, except that there are extra mortality risks at age 60 such that $q_{60} = 0.015$.

Calculate the annual benefit premium for this insurance.

(A) 31.5

(B) 32.0

(C) 32.1

(D) 33.1

(E) 33.2

52. Removed

53. The mortality of (x) and (y) follows a common shock model with states:

State 0 – both alive
State 1 – only (x) alive
State 2 – only (y) alive
State 3 – both dead

You are given:

(i) $\mu_{x+t} = \mu^{02}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{13}_{x+t:y+t} = g$, a constant, $0 \le t \le 5$

(ii) $\mu_{y+t} = \mu^{01}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{23}_{x+t:y+t} = h$, a constant, $0 \le t \le 5$

(iii) $p_{x+t} = 0.96,\ 0 \le t \le 4$

(iv) $p_{y+t} = 0.97,\ 0 \le t \le 4$

(v) $\mu^{03}_{x+t:y+t} = 0.01,\ 0 \le t \le 5$

Calculate the probability that (x) and (y) both survive 5 years.

(A) 0.65

(B) 0.67

(C) 0.70

(D) 0.72

(E) 0.74

54. Nancy reviews the interest rates each year for a 30-year fixed mortgage issued on July 1. She models interest rate behavior by a discrete-time Markov model assuming:

(i) Interest rates always change between years.

(ii) The change in any given year is dependent on the change in prior years as follows:

from year $t-3$ to year $t-2$	from year $t-2$ to year $t-1$	Probability that year t will increase from year $t-1$
Increase	Increase	0.10
Decrease	Decrease	0.20
Increase	Decrease	0.40
Decrease	Increase	0.25

She notes that interest rates decreased from year 2000 to 2001 and from year 2001 to 2002.

Calculate the probability that interest rates will decrease from year 2003 to 2004.

(A) 0.76

(B) 0.79

(C) 0.82

(D) 0.84

(E) 0.87

55. For a 20-year deferred whole life annuity-due of 1 per year on (45), you are given:

(i) $l_x = 10(105 - x),\ 0 \le x \le 105$

(ii) $i = 0$

Calculate the probability that the sum of the annuity payments actually made will exceed the actuarial present value at issue of the annuity.

(A) 0.425

(B) 0.450

(C) 0.475

(D) 0.500

(E) 0.525

56. For a continuously increasing whole life insurance on (x), you are given:

(i) The force of mortality is constant.

(ii) $\delta = 0.06$

(iii) ${}^2\overline{A}_x = 0.25$

Calculate $\left(\overline{IA}\right)_x$.

(A) 2.889

(B) 3.125

(C) 4.000

(D) 4.667

(E) 5.500

57. XYZ Co. has just purchased two new tools with independent future lifetimes.

You are given:

(i) Tools are considered age 0 at purchase.

(ii) For Tool 1, $S_0(t) = 1 - \frac{t}{10}, 0 \le t \le 10$.

(iii) For Tool 2, $S_0(t) = 1 - \frac{t}{7}, 0 \le t \le 7$,

Calculate the expected time until both tools have failed.

(A) 5.0

(B) 5.2

(C) 5.4

(D) 5.6

(E) 5.8

58. XYZ Paper Mill purchases a 5-year special insurance paying a benefit in the event its machine breaks down. If the cause is "minor" (1), only a repair is needed. If the cause is "major" (2), the machine must be replaced.

Given:

(i) The benefit for cause (1) is 2000 payable at the moment of breakdown.

(ii) The benefit for cause (2) is 500,000 payable at the moment of breakdown.

(iii) Once a benefit is paid, the insurance is terminated.

(iv) $\mu_t^{(1)} = 0.100$ and $\mu_t^{(2)} = 0.004$, for $t > 0$

(v) $\delta = 0.04$

Calculate the expected present value of this insurance.

(A) 7840

(B) 7880

(C) 7920

(D) 7960

(E) 8000

59. You are given:

(i) μ_{x+t} is the force of mortality

(ii) $R = 1 - e^{-\int_0^1 \mu_{x+t}\, dt}$

(iii) $S = 1 - e^{-\int_0^1 (\mu_{x+t} + k)\, dt}$

(iv) k is a constant such that $S = 0.75R$

Determine an expression for k.

(A) $1\text{n}\left((1-q_x)/(1-0.75q_x)\right)$

(B) $1\text{n}\left((1-0.75q_x)/(1-p_x)\right)$

(C) $1\text{n}\left((1-0.75p_x)/(1-p_x)\right)$

(D) $1\text{n}\left((1-p_x)/(1-0.75q_x)\right)$

(E) $1\text{n}\left((1-0.75q_x)/(1-q_x)\right)$

60. For a fully discrete whole life insurance of 100,000 on each of 10,000 lives age 60, you are given:

(i) The future lifetimes are independent.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$.

(iv) π is the premium for each insurance of 100,000.

Using the normal approximation, calculate π, such that the probability of a positive total loss is 1%.

(A) 3340

(B) 3360

(C) 3380

(D) 3390

(E) 3400

61. For a special fully discrete 3-year endowment insurance on (75), you are given:

(i) The maturity value is 1000.

(ii) The death benefit is 1000 plus the benefit reserve at the end of the year of death. For year 3, this benefit reserve is the benefit reserve just before the maturity benefit is paid.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.05$

Calculate the level benefit premium for this insurance.

(A) 321

(B) 339

(C) 356

(D) 364

(E) 373

62. A large machine in the ABC Paper Mill is 25 years old when ABC purchases a 5-year term insurance paying a benefit in the event the machine breaks down.

Given:

(i) Annual benefit premiums of 6643 are payable at the beginning of the year.

(ii) A benefit of 500,000 is payable at the moment of breakdown.

(iii) Once a benefit is paid, the insurance is terminated.

(iv) Machine breakdowns follow $l_x = 100 - x$.

(v) $i = 0.06$

Calculate the benefit reserve for this insurance at the end of the third year.

(A) –91

(B) 0

(C) 163

(D) 287

(E) 422

63. For a whole life insurance of 1 on (x), you are given:

(i) The force of mortality is μ_{x+t}.

(ii) The benefits are payable at the moment of death.

(iii) $\delta = 0.06$

(iv) $\bar{A}_x = 0.60$

Calculate the revised expected present value of this insurance assuming μ_{x+t} is increased by 0.03 for all t and δ is decreased by 0.03.

(A) 0.5

(B) 0.6

(C) 0.7

(D) 0.8

(E) 0.9

64. A maintenance contract on a hotel promises to replace burned out light bulbs at the end of each year for three years. The hotel has 10,000 light bulbs. The light bulbs are all new. If a replacement bulb burns out, it too will be replaced with a new bulb.

You are given:

(i) For new light bulbs, $q_0 = 0.10$
$q_1 = 0.30$
$q_2 = 0.50$

(ii) Each light bulb costs 1.

(iii) $i = 0.05$

Calculate the expected present value of this contract.

(A) 6700

(B) 7000

(C) 7300

(D) 7600

(E) 8000

65. You are given:

$$\mu_x = \begin{cases} 0.04, & 0 < x < 40 \\ 0.05, & x \geq 40 \end{cases}$$

Calculate $\mathring{e}_{25:\overline{25}|}$.

(A) 14.0

(B) 14.4

(C) 14.8

(D) 15.2

(E) 15.6

66. For a select-and-ultimate mortality table with a 3-year select period:

(i)

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

(ii) White was a newly selected life on 01/01/2000.

(iii) White's age on 01/01/2001 is 61.

(iv) P is the probability on 01/01/2001 that White will be alive on 01/01/2006.

Calculate P.

(A) $0 \le P < 0.43$

(B) $0.43 \le P < 0.45$

(C) $0.45 \le P < 0.47$

(D) $0.47 \le P < 0.49$

(E) $0.49 \le P \le 1.00$

67. For a continuous whole life annuity of 1 on (x):

(i) T_x is the future lifetime random variable for (x).

(ii) The force of interest and force of mortality are constant and equal.

(iii) $\bar{a}_x = 12.50$

Calculate the standard deviation of $\bar{a}_{\overline{T_x|}}$.

(A) 1.67

(B) 2.50

(C) 2.89

(D) 6.25

(E) 7.22

68. For a special fully discrete whole life insurance on (x):

(i) The death benefit is 0 in the first year and 5000 thereafter.

(ii) Level benefit premiums are payable for life.

(iii) $q_x = 0.05$

(iv) $v = 0.90$

(v) $\ddot{a}_x = 5.00$

(vi) The benefit reserve at the end of year 10 for a fully discrete whole life insurance of 1 on (x) is 0.20.

(vii) ${}_{10}V$ is the benefit reserve at the end of year 10 for this special insurance.

Calculate ${}_{10}V$.

(A) 795

(B) 1000

(C) 1090

(D) 1180

(E) 1225

69. For a fully discrete 2-year term insurance of 1 on (x):

(i) 0.95 is the lowest premium such that there is a 0% chance of loss in year 1.

(ii) $p_x = 0.75$

(iii) $p_{x+1} = 0.80$

(iv) Z is the random variable for the present value at issue of future benefits.

Calculate $\mathrm{Var}(Z)$.

(A) 0.15

(B) 0.17

(C) 0.19

(D) 0.21

(E) 0.23

70. For a special fully discrete 3-year term insurance on (55), whose mortality follows a double decrement model:

(i) Decrement 1 is accidental death; decrement 2 is all other causes of death.

(ii)

x	$q_x^{(1)}$	$q_x^{(2)}$
55	0.002	0.020
56	0.005	0.040
57	0.008	0.060

(iii) $i = 0.06$

(iv) The death benefit is 2000 for accidental deaths and 1000 for deaths from all other causes.

(v) The level annual gross premium is 50.

(vi) ${}_1L$ is the prospective loss random variable at time 1, based on the gross premium.

(vii) K_{55} is the curtate future lifetime of (55).

Calculate $\mathrm{E}\left[{}_1L \middle| K_{55} \geq 1\right]$.

(A) 5

(B) 9

(C) 13

(D) 17

(E) 20

71. Removed

72. Each of 100 independent lives purchase a single premium 5-year deferred whole life insurance of 10 payable at the moment of death. You are given:

(i) $\mu = 0.04$

(ii) $\delta = 0.06$

(iii) F is the aggregate amount the insurer receives from the 100 lives.

Using the normal approximation, calculate F such that the probability the insurer has sufficient funds to pay all claims is 0.95.

(A) 280

(B) 390

(C) 500

(D) 610

(E) 720

73. For a select-and-ultimate table with a 2-year select period:

x	$p_{[x]}$	$p_{[x]+1}$	p_{x+2}	$x+2$
48	0.9865	0.9841	0.9713	50
49	0.9858	0.9831	0.9698	51
50	0.9849	0.9819	0.9682	52
51	0.9838	0.9803	0.9664	53

Keith and Clive are independent lives, both age 50. Keith was selected at age 45 and Clive was selected at age 50.

Calculate the probability that exactly one will be alive at the end of three years.

(A) Less than 0.115

(B) At least 0.115, but less than 0.125

(C) At least 0.125, but less than 0.135

(D) At least 0.135, but less than 0.145

(E) At least 0.145

74. Removed

75. Removed

76. A fund is established by collecting an amount P from each of 100 independent lives age 70. The fund will pay the following benefits:

- 10, payable at the end of the year of death, for those who die before age 72, or
- P, payable at age 72, to those who survive.

You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.08$

Calculate P, using the equivalence principle.

(A) 2.33

(B) 2.38

(C) 3.02

(D) 3.07

(E) 3.55

77. You are given:

(i) $P_x = 0.090$

(ii) The benefit reserve at the end of year n for a fully discrete whole life insurance of 1 on (x) is 0.563.

(iii) $P_{x:\overline{n|}}^{\ \ 1} = 0.00864$

Calculate $P_{x:\overline{n|}}^{1}$.

(A) 0.008

(B) 0.024

(C) 0.040

(D) 0.065

(E) 0.085

78. For a fully continuous whole life insurance of 1 on (40), you are given:

(i) Mortality follows $l_x = 10(100 - x),\ 0 \le x \le 100$.

(ii) $i = 0.05$

(iii) The following annuity-certain values:

$$\bar{a}_{\overline{40}|} = 17.58$$
$$\bar{a}_{\overline{50}|} = 18.71$$
$$\bar{a}_{\overline{60}|} = 19.40$$

Calculate the benefit reserve at the end of year 10 for this insurance.

(A) 0.075

(B) 0.077

(C) 0.079

(D) 0.081

(E) 0.083

79. For a group of individuals all age x, you are given:

(i) 30% are smokers and 70% are non-smokers.

(ii) The constant force of mortality for smokers is 0.06 at all ages.

(iii) The constant force of mortality for non-smokers is 0.03 at all ages.

(iv) $\delta = 0.08$

Calculate $\mathrm{Var}\left(\bar{a}_{\overline{T_x}|}\right)$ for an individual chosen at random from this group.

(A) 13.0

(B) 13.3

(C) 13.8

(D) 14.1

(E) 14.6

80. For (80) and (84), whose future lifetimes are independent:

x	p_x
80	0.50
81	0.40
82	0.60
83	0.25
84	0.20
85	0.15
86	0.10

Calculate the change in the value ${}_{2|}q_{\overline{80:84}}$ if p_{82} is decreased from 0.60 to 0.30.

(A) 0.03

(B) 0.06

(C) 0.10

(D) 0.16

(E) 0.19

81. Removed

82. Don, age 50, is an actuarial science professor. His career is subject to two decrements:

(i) Decrement 1 is mortality. The associated single decrement table follows $l_x = 100 - x,\ 0 \le x \le 100$.

(ii) Decrement 2 is leaving academic employment, with

$$\mu_{50+t}^{(2)} = 0.05, \quad t \ge 0$$

Calculate the probability that Don remains an actuarial science professor for at least five but less than ten years.

(A) 0.22

(B) 0.25

(C) 0.28

(D) 0.31

(E) 0.34

83. For a double decrement model:

(i) In the single decrement table associated with cause (1), $q_{40}'^{(1)} = 0.100$ and decrements are uniformly distributed over the year.

(ii) In the single decrement table associated with cause (2), $q_{40}'^{(2)} = 0.125$ and all decrements occur at time 0.7.

Calculate $q_{40}^{(2)}$.

(A) 0.114

(B) 0.115

(C) 0.116

(D) 0.117

(E) 0.118

84. For a special 2-payment whole life insurance on (80):

(i) Premiums of π are paid at the beginning of years 1 and 3.

(ii) The death benefit is paid at the end of the year of death.

(iii) There is a partial refund of premium feature:

If (80) dies in either year 1 or year 3, the death benefit is $1000+\frac{\pi}{2}$.

Otherwise, the death benefit is 1000.

(iv) Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

Calculate π, using the equivalence principle.

(A) 369

(B) 381

(C) 397

(D) 409

(E) 425

85. For a special fully continuous whole life insurance on (65):

(i) The death benefit at time t is $b_t = 1000\,e^{0.04t}$, $t \geq 0$.

(ii) Level benefit premiums are payable for life.

(iii) $\mu_{65+t} = 0.02$, $t \geq 0$

(iv) $\delta = 0.04$

Calculate the benefit reserve at the end of year 2.

(A) 0

(B) 29

(C) 37

(D) 61

(E) 83

86. You are given:

(i) $A_x = 0.28$

(ii) $A_{x+20} = 0.40$

(iii) $A_{x:\overline{20}|}^{\;1} = 0.25$

(iv) $i = 0.05$

Calculate $a_{x:\overline{20}|}$.

(A) 11.0

(B) 11.2

(C) 11.7

(D) 12.0

(E) 12.3

87. Removed

88. At interest rate i:

(i) $\ddot{a}_x = 5.6$

(ii) The expected present value of a 2-year certain and life annuity-due of 1 on (x) is $\ddot{a}_{\overline{x:\overline{2|}}} = 5.6459$.

(iii) $e_x = 8.83$

(iv) $e_{x+1} = 8.29$

Calculate i.

(A) 0.077

(B) 0.079

(C) 0.081

(D) 0.083

(E) 0.084

89. A machine is in one of four states (F, G, H, I) and migrates annually among them according to a discrete-time Markov process with transition probability matrix:

	F	G	H	I
F	0.20	0.80	0.00	0.00
G	0.50	0.00	0.50	0.00
H	0.75	0.00	0.00	0.25
I	1.00	0.00	0.00	0.00

At time 0, the machine is in State F. A salvage company will pay 500 at the end of 3 years if the machine is in State F.

Assuming $v = 0.90$, calculate the actuarial present value at time 0 of this payment.

(A) 150

(B) 155

(C) 160

(D) 165

(E) 170

90. Removed

91. You are given:

(i) The survival function for males is $S_0(t) = 1 - \frac{t}{75}, \quad 0 \le t \le 75$.

(ii) Female mortality follows $S_0(t) = 1 - \frac{t}{\omega}, 0 \le t \le \omega$.

(iii) At age 60, the female force of mortality is 60% of the male force of mortality.

For two independent lives, a male age 65 and a female age 60, calculate the expected time until the second death.

(A) 4.33

(B) 5.63

(C) 7.23

(D) 11.88

(E) 13.17

92. For a fully continuous whole life insurance of 1:

(i) $\mu_x = 0.04,\ x > 0$

(ii) $\delta = 0.08$

(iii) L is the loss-at-issue random variable based on the benefit premium.

Calculate Var (L).

(A) $\frac{1}{10}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

93. For a deferred whole life annuity-due on (25) with annual payment of 1 commencing at age 60, you are given:

(i) Level benefit premiums are payable at the beginning of each year during the deferral period.

(ii) During the deferral period, a death benefit equal to the benefit reserve is payable at the end of the year of death.

Which of the following is a correct expression for the benefit reserve at the end of the 20th year?

(A) $\left(\ddot{a}_{60} / \ddot{s}_{\overline{35}|}\right)\ddot{s}_{\overline{20}|}$

(B) $\left(\ddot{a}_{60} / \ddot{s}_{\overline{20}|}\right)\ddot{s}_{\overline{35}|}$

(C) $\left(\ddot{s}_{\overline{20}|} / \ddot{a}_{60}\right)\ddot{s}_{\overline{35}|}$

(D) $\left(\ddot{s}_{\overline{35}|} / \ddot{a}_{60}\right)\ddot{s}_{\overline{20}|}$

(E) $\left(\ddot{a}_{60} / \ddot{s}_{\overline{35}|}\right)$

94. You are given:

(i) The future lifetimes of (50) and (50) are independent.

(ii) Mortality follows the Illustrative Life Table.

(iii) Deaths are uniformly distributed over each year of age.

Calculate the force of failure at duration 10.5 for the last survivor status of (50) and (50).

(A) 0.001

(B) 0.002

(C) 0.003

(D) 0.004

(E) 0.005

95. For a special whole life insurance:

(i) The benefit for accidental death is 50,000 in all years.

(ii) The benefit for non-accidental death during the first 2 years is return of the single benefit premium without interest.

(iii) The benefit for non-accidental death after the first 2 years is 50,000.

(iv) Benefits are payable at the moment of death.

(v) Force of mortality for accidental death: $\mu_x^{(1)} = 0.01, \quad x \geq 0$

(vi) Force of mortality for non-accidental death: $\mu_x^{(2)} = 2.29, \quad x \geq 0$

(vii) $\delta = 0.10$

Calculate the single benefit premium for this insurance.

(A) 1,000

(B) 4,000

(C) 7,000

(D) 11,000

(E) 15,000

96. For a special 3-year deferred whole life annuity-due on (x):

(i) $i = 0.04$

(ii) The first annual payment is 1000.

(iii) Payments in the following years increase by 4% per year.

(iv) There is no death benefit during the three year deferral period.

(v) Level benefit premiums are payable at the beginning of each of the first three years.

(vi) $e_x = 11.05$ is the curtate expectation of life for (x).

(vii)

k	1	2	3
${}_k p_x$	0.99	0.98	0.97

Calculate the annual benefit premium.

(A) 2625

(B) 2825

(C) 3025

(D) 3225

(E) 3425

97. For a special fully discrete 10-payment whole life insurance on (30) with level annual benefit premium π:

(i) The death benefit is equal to 1000 plus the refund, without interest, of the benefit premiums paid.

(ii) $A_{30} = 0.102$

(iii) ${}_{10|}A_{30} = 0.088$

(iv) $(IA)^{1}_{30:\overline{10|}} = 0.078$

(v) $\ddot{a}_{30:\overline{10|}} = 7.747$

Calculate π.

(A) 14.9

(B) 15.0

(C) 15.1

(D) 15.2

(E) 15.3

98. For a life age 30, it is estimated that an impact of a medical breakthrough will be an increase of 4 years in $\mathring{e}_{30}$, the complete expectation of life.

Prior to the medical breakthrough, $S_0(t) = 1 - \frac{t}{100}$, $0 \le t \le 100$.

After the medical breakthrough, $S_0(t) = 1 - \frac{t}{\omega}$, $0 \le t \le \omega$.

Calculate ω.

(A) 104

(B) 105

(C) 106

(D) 107

(E) 108

99. On January 1, 2002, Pat, age 40, purchases a 5-payment, 10-year term insurance of 100,000:

(i) Death benefits are payable at the moment of death.

(ii) Gross premiums of 4000 are payable annually at the beginning of each year for 5 years.

(iii) $i = 0.05$

(iv) L is the loss random variable at time of issue.

Calculate the value of L if Pat dies on June 30, 2004.

(A) 77,100

(B) 80,700

(C) 82,700

(D) 85,900

(E) 88,000

100. A special whole life insurance on (x) pays 10 times salary if the cause of death is an accident and 500,000 for all other causes of death.

You are given:

(i) $\mu_{x+t}^{(\tau)} = 0.01, \quad t \geq 0$

(ii) $\mu_{x+t}^{(\text{accident})} = 0.001, \quad t \geq 0$

(iii) Benefits are payable at the moment of death.

(iv) $\delta = 0.05$

(v) Salary of (x) at time t is $50{,}000 e^{0.04t}, \quad t \geq 0$.

Calculate the expected present value of the benefits at issue.

(A) 78,000

(B) 83,000

(C) 92,000

(D) 100,000

(E) 108,000

101. Removed

102. For a fully discrete 20-payment whole life insurance of 1000 on (x), you are given:

(i) $i = 0.06$

(ii) $q_{x+19} = 0.01254$

(iii) The level annual benefit premium is 13.72.

(iv) The benefit reserve at the end of year 19 is 342.03.

Calculate 1000 P_{x+20}, the level annual benefit premium for a fully discrete whole life insurance of 1000 on (x+20).

(A) 27

(B) 29

(C) 31

(D) 33

(E) 35

103. For a multiple decrement model on (60):

(i) $\mu_{x+t}^{(1)}, \quad t \geq 0,$ follows the Illustrative Life Table.

(ii) $\mu_{60+t}^{(\tau)} = 2\mu_{60+t}^{(1)}, \quad t \geq 0$

Calculate ${}_{10|}q_{60}^{(\tau)}$, the probability that decrement occurs during the 11th year.

(A) 0.03

(B) 0.04

(C) 0.05

(D) 0.06

(E) 0.07

104. (x) and (y) are two lives with identical expected mortality.
You are given:

$P_x = P_y = 0.1$

$P_{\overline{xy}} = 0.06$, where $P_{\overline{xy}}$ is the annual benefit premium for a fully discrete whole life insurance of 1 on $(\overline{xy})$.

$d = 0.06$

Calculate the premium P_{xy}, the annual benefit premium for a fully discrete whole life insurance of 1 on (xy).

(A) 0.14

(B) 0.16

(C) 0.18

(D) 0.20

(E) 0.22

105. For students entering a college, you are given the following from a multiple decrement model:

(i) 1000 students enter the college at $t = 0$.

(ii) Students leave the college for failure (1) or all other reasons (2).

(iii) $\mu_{x+t}^{(1)} = \mu \qquad 0 \le t \le 4$

$\mu_{x+t}^{(2)} = 0.04 \qquad 0 \le t < 4$

(iv) 48 students are expected to leave the college during their first year due to all causes.

Calculate the expected number of students who will leave because of failure during their fourth year.

(A) 8

(B) 10

(C) 24

(D) 34

(E) 41

106. The following graph is related to current human mortality:

Which of the following functions of age does the graph most likely show?

(A) μ_x

(B) $l_x \mu_x$

(C) $l_x p_x$

(D) l_x

(E) l_x^2

107. Z is the present value random variable for a 15-year pure endowment of 1 on (x):

(i) The force of mortality is constant over the 15-year period.

(ii) $v = 0.9$

(iii) $\mathrm{Var}(Z) = 0.065\,\mathrm{E}[Z]$

Calculate q_x.

(A) 0.020

(B) 0.025

(C) 0.030

(D) 0.035

(E) 0.040

108. You are given:

(i) ${}_kV^A$ is the benefit reserve at the end of year k for type A insurance, which is a fully discrete 10-payment whole life insurance of 1000 on (x).

(ii) ${}_kV^B$ is the benefit reserve at the end of year k for type B insurance, which is a fully discrete whole life insurance of 1000 on (x).

(iii) $q_{x+10} = 0.004$

(iv) The annual benefit premium for type B is 8.36.

(v) ${}_{10}V^A - {}_{10}V^B = 101.35$

(vi) $i = 0.06$

Calculate ${}_{11}V^A - {}_{11}V^B$.

(A) 91

(B) 93

(C) 95

(D) 97

(E) 99

109. For a special 3-year term insurance on (x), you are given:

(i) Z is the present-value random variable for the death benefits.

(ii) $q_{x+k} = 0.02(k+1)$ $k = 0, 1, 2$

(iii) The following death benefits, payable at the end of the year of death:

k	b_{k+1}
0	300,000
1	350,000
2	400,000

(iv) $i = 0.06$

Calculate $\mathrm{E}(Z)$.

(A) 36,800

(B) 39,100

(C) 41,400

(D) 43,700

(E) 46,000

110. For a special fully discrete 20-year endowment insurance on (55):

(i) Death benefits in year k are given by $b_k = (21 - k)$, $k = 1, 2, \ldots, 20$.

(ii) The maturity benefit is 1.

(iii) Annual benefit premiums are level.

(iv) ${}_kV$ denotes the benefit reserve at the end of year k, $k = 1, 2, \ldots, 20$.

(v) ${}_{10}V = 5.0$

(vi) ${}_{19}V = 0.6$

(vii) $q_{65} = 0.10$

(viii) $i = 0.08$

Calculate ${}_{11}V$.

(A) 4.5

(B) 4.6

(C) 4.8

(D) 5.1

(E) 5.3

111. For a special fully discrete 3-year term insurance on (x):

(i) The death benefit payable at the end of year k+1 is

$$b_{k+1} = \begin{cases} 0 & \text{for } k = 0 \\ 1{,}000(11-k) & \text{for } k = 1, 2 \end{cases}$$

(ii)

k	q_{x+k}
0	0.200
1	0.100
2	0.097

(iii) $i = 0.06$

Calculate the level annual benefit premium for this insurance.

(A) 518

(B) 549

(C) 638

(D) 732

(E) 799

112. A continuous two-life annuity pays:

> 100 while both (30) and (40) are alive;
> 70 while (30) is alive but (40) is dead; and
> 50 while (40) is alive but (30) is dead.

The expected present value of this annuity is 1180. Continuous single life annuities paying 100 per year are available for (30) and (40) with actuarial present values of 1200 and 1000, respectively.

Calculate the expected present value of a two-life continuous annuity that pays 100 while at least one of them is alive.

(A) 1400

(B) 1500

(C) 1600

(D) 1700

(E) 1800

113. For a disability insurance claim:

(i) The claimant will receive payments at the rate of 20,000 per year, payable continuously as long as she remains disabled.

(ii) The length of the payment period in years is a random variable with the gamma distribution with parameters $\alpha = 2$ and $\theta = 1$. That is,

$f(t) = te^{-t}, \quad t > 0$

(iii) Payments begin immediately.

(iv) $\delta = 0.05$

Calculate the actuarial present value of the disability payments at the time of disability.

(A) 36,400

(B) 37,200

(C) 38,100

(D) 39,200

(E) 40,000

114. For a special 3-year temporary life annuity-due on (x), you are given:

(i)

t	Annuity Payment	p_{x+t}
0	15	0.95
1	20	0.90
2	25	0.85

(ii) $i = 0.06$

Calculate the variance of the present value random variable for this annuity.

(A) 91

(B) 102

(C) 114

(D) 127

(E) 139

115. For a fully discrete 3-year endowment insurance of 1000 on (x), you are given:

(i) ${}_{k}L$ is the prospective loss random variable at time k.

(ii) $i = 0.10$

(iii) $\ddot{a}_{x:\overline{3}|} = 2.70182$

(iv) Premiums are determined by the equivalence principle.

Calculate ${}_{1}L$, given that (x) dies in the second year from issue.

(A) 540

(B) 630

(C) 655

(D) 720

(E) 910

116. For a population of individuals, you are given:

(i) Each individual has a constant force of mortality.

(ii) The forces of mortality are uniformly distributed over the interval (0,2).

Calculate the probability that an individual drawn at random from this population dies within one year.

(A) 0.37

(B) 0.43

(C) 0.50

(D) 0.57

(E) 0.63

117. For a double-decrement model:

(i) $\quad {}_t p'^{(1)}_{40} = 1 - \frac{t}{60}, \qquad 0 \le t \le 60$

(ii) $\quad {}_t p'^{(2)}_{40} = 1 - \frac{t}{40}, \qquad 0 \le t \le 40$

Calculate $\mu^{(\tau)}_{40+20}$.

(A) 0.025

(B) 0.038

(C) 0.050

(D) 0.063

(E) 0.07

118. For a special fully discrete 3-year term insurance on (x):

(i) Level benefit premiums are paid at the beginning of each year.

(ii)

k	Death benefit b_{k+1}	q_{x+k}
0	200,000	0.03
1	150,000	0.06
2	100,000	0.09

(iii) $i = 0.06$

Calculate the initial benefit reserve for year 2.

(A) 6,500

(B) 7,500

(C) 8,100

(D) 9,400

(E) 10,300

119. For a special fully continuous whole life insurance on (x):

(i) The level premium is determined using the equivalence principle.

(ii) Death benefits are given by $b_t = (1+i)^t$ where i is the interest rate.

(iii) L is the loss random variable at $t = 0$ for the insurance.

(iv) T is the future lifetime random variable of (x).

Which of the following expressions is equal to L?

(A) $\dfrac{\left(v^T - \bar{A}_x\right)}{\left(1-\bar{A}_x\right)}$

(B) $\left(v^T - \bar{A}_x\right)\left(1+\bar{A}_x\right)$

(C) $\dfrac{\left(v^T - \bar{A}_x\right)}{\left(1+\bar{A}_x\right)}$

(D) $\left(v^T - \bar{A}_x\right)\left(1-\bar{A}_x\right)$

(E) $\dfrac{\left(v^T + \bar{A}_x\right)}{\left(1+\bar{A}_x\right)}$

120. For a 4-year college, you are given the following probabilities for dropout from all causes:

$$q_0 = 0.15$$
$$q_1 = 0.10$$
$$q_2 = 0.05$$
$$q_3 = 0.01$$

Dropouts are uniformly distributed over each year.

Compute the temporary 1.5-year complete expected college lifetime of a student entering the second year, $\overset{\circ}{e}_{1:\overline{1.5}|}$.

(A) 1.25

(B) 1.30

(C) 1.35

(D) 1.40

(E) 1.45

121. Lee, age 63, considers the purchase of a single premium whole life insurance of 10,000 with death benefit payable at the end of the year of death.

The company calculates benefit premiums using:

(i) mortality based on the Illustrative Life Table,

(ii) $i = 0.05$

The company calculates gross premiums as 112% of benefit premiums.

The single gross premium at age 63 is 5233.

Lee decides to delay the purchase for two years and invests the 5233.

Calculate the minimum annual rate of return that the investment must earn to accumulate to an amount equal to the single gross premium at age 65.

(A) 0.030

(B) 0.035

(C) 0.040

(D) 0.045

(E) 0.050

122A-C. Note to candidates – in reformatting the prior question 122 to match the new syllabus it has been split into three parts. While this problem uses a constant force for the common shock (which was the only version presented in the prior syllabus), it should be noted that in the multi-state context, that assumption is not necessary. 122C represents the former problem 122.

Use the following information for problems 122A-122C.

You want to impress your supervisor by calculating the expected present value of a last-survivor whole life insurance of 1 on (x) and (y) using multi-state methodology. You defined states as

State 0 = both alive
State 1 = only (x) alive
State 2 = only (y) alive
State 3 = neither alive

You assume:

(i) Death benefits are payable at the moment of death.
(ii) The future lifetimes of (x) and (y) are independent.
(iii) $\mu^{01}_{x+t:y+t} = \mu^{02}_{x+t:y+t} = \mu^{13}_{x+t:y+t} = \mu^{23}_{x+t:y+t} = 0.06, t \geq 0$
(iv) $\mu^{03}_{x+t:y+t} = 0, t \geq 0$
(v) $\delta = 0.05$

Your supervisor points out that the particular lives in question do not have independent future lifetimes. While your model correctly projects the survival function of (x) and (y), a common shock model should be used for their joint future lifetime. Based on her input, you realize you should be using

$$\mu^{03}_{x+t:y+t} = 0.02, t \geq 0 .$$

122A. To ensure that you get off to a good start, your supervisor suggests that you calculate the expected present value of a whole life insurance of 1 payable at the first death of (x) and (y). You make the necessary changes to your model to incorporate the common shock.

Calculate the expected present value for the first-to-die benefit.

(A) 0.55

(B) 0.61

(C) 0.67

(D) 0.73

(E) 0.79

122B. Having checked your work and ensured it is correct, she now asks you to calculate the probability that both have died by the end of year 3.

Calculate that probability.

(A) 0.03

(B) 0.04

(C) 0.05

(D) 0.06

(E) 0.07

122C. You are now ready to calculate the expected present value of the last-to-die insurance, payable at the moment of the second death.

Calculate the expected present value for the last-to-die benefit.

(A) 0.39

(B) 0.40

(C) 0.41

(D) 0.42

(E) 0.43

123. For independent lives (35) and (45):

(i) ${}_5p_{35} = 0.90$

(ii) ${}_5p_{45} = 0.80$

(iii) $q_{40} = 0.03$

(iv) $q_{50} = 0.05$

Calculate the probability that the last death of (35) and (45) occurs in the 6^{th} year.

(A) 0.0095

(B) 0.0105

(C) 0.0115

(D) 0.0125

(E) 0.0135

124. Removed

125. Removed

126. A government creates a fund to pay this year's lottery winners.

You are given:

(i) There are 100 winners each age 40.

(ii) Each winner receives payments of 10 per year for life, payable annually, beginning immediately.

(iii) Mortality follows the Illustrative Life Table.

(iv) The lifetimes are independent.

(v) $i = 0.06$

(vi) The amount of the fund is determined, using the normal approximation, such that the probability that the fund is sufficient to make all payments is 95%.

Calculate the initial amount of the fund.

(A) 14,800

(B) 14,900

(C) 15,050

(D) 15,150

(E) 15,250

127. For a special fully discrete 35-payment whole life insurance on (30):

(i) The death benefit is 1 for the first 20 years and is 5 thereafter.

(ii) The initial benefit premium paid during the each of the first 20 years is one fifth of the benefit premium paid during each of the 15 subsequent years.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) $A_{30:\overline{20}|} = 0.32307$

(vi) $\ddot{a}_{30:\overline{35}|} = 14.835$

Calculate the initial annual benefit premium.

(A) 0.010

(B) 0.015

(C) 0.020

(D) 0.025

(E) 0.030

128. For independent lives (x) and (y):

(i) $q_x = 0.05$

(ii) $q_y = 0.10$

(iii) Deaths are uniformly distributed over each year of age.

Calculate ${}_{0.75}q_{xy}$.

(A) 0.1088

(B) 0.1097

(C) 0.1106

(D) 0.1116

(E) 0.1125

129. For a fully discrete whole life insurance of 100,000 on (35) you are given:

(i) Percent of premium expenses are 10% per year.

(ii) Per policy expenses are 25 per year.

(iii) Per thousand expenses are 2.50 per year.

(iv) All expenses are paid at the beginning of the year.

(v) $1000P_{35} = 8.36$

Calculate the level annual premium using the equivalence principle.

(A) 930

(B) 1041

(C) 1142

(D) 1234

(E) 1352

130. A person age 40 wins 10,000 in the actuarial lottery. Rather than receiving the money at once, the winner is offered the actuarially equivalent option of receiving an annual payment of K (at the beginning of each year) guaranteed for 10 years and continuing thereafter for life.

You are given:

(i) $i = 0.04$

(ii) $A_{40} = 0.30$

(iii) $A_{50} = 0.35$

(iv) $A^{1}_{40:\overline{10|}} = 0.09$

Calculate K.

(A) 538

(B) 541

(C) 545

(D) 548

(E) 551

131. Mortality for Audra, age 25, follows $l_x = 50(100 - x),\ 0 \le x \le 100$.

If she takes up hot air ballooning for the coming year, her assumed mortality will be adjusted so that for the coming year only, she will have a constant force of mortality of 0.1.

Calculate the decrease in the 11-year temporary complete life expectancy for Audra if she takes up hot air ballooning.

(A) 0.10

(B) 0.35

(C) 0.60

(D) 0.80

(E) 1.00

132. For a 5-year fully continuous term insurance on (x):

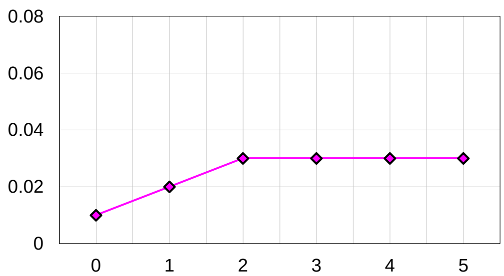
(i) $\delta = 0.10$

(ii) All the graphs below are to the same scale.

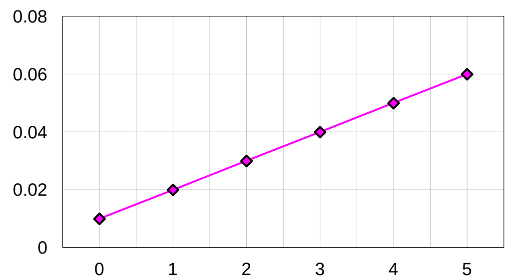
(iii) All the graphs show μ_{x+t} on the vertical axis and t on the horizontal axis.

Which of the following mortality assumptions would produce the highest benefit reserve at the end of year 2?

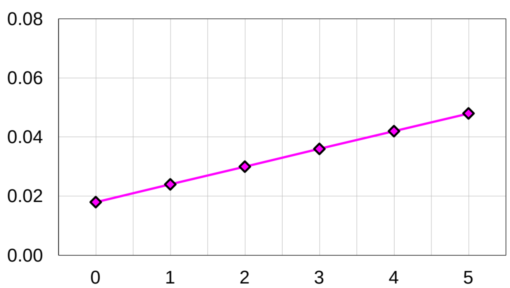
(A)



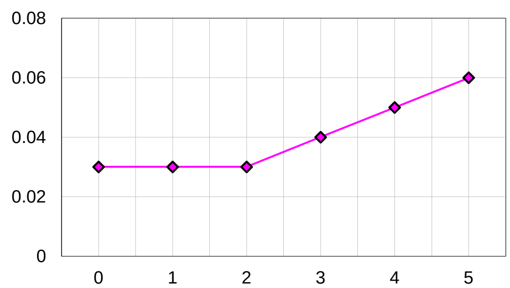
(B)



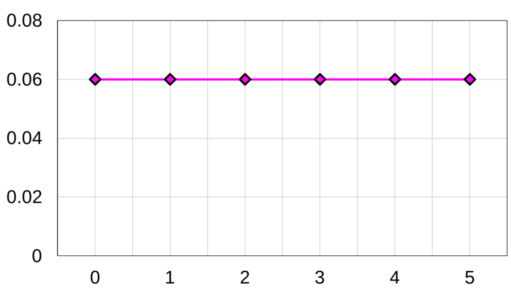
(C)



(D)



(E)



133. For a multiple decrement table, you are given:

(i) Decrement (1) is death, decrement (2) is disability, and decrement (3) is withdrawal.

(ii) $q'^{(1)}_{60} = 0.010$

(iii) $q'^{(2)}_{60} = 0.050$

(iv) $q'^{(3)}_{60} = 0.100$

(v) Withdrawals occur only at the end of the year.

(vi) Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

Calculate $q^{(3)}_{60}$.

(A) 0.088

(B) 0.091

(C) 0.094

(D) 0.097

(E) 0.100

134. Removed

135. For a special whole life insurance of 100,000 on (x), you are given:

(i) $\delta = 0.06$

(ii) The death benefit is payable at the moment of death.

(iii) If death occurs by accident during the first 30 years, the death benefit is doubled.

(iv) $\mu_{x+t}^{(\tau)} = 0.008,\ t \ge 0$

(v) $\mu_{x+t}^{(1)} = 0.001, \quad t \ge 0,$ is the force of decrement due to death by accident.

Calculate the single benefit premium for this insurance.

(A) 11,765

(B) 12,195

(C) 12,622

(D) 13,044

(E) 13,235

136. You are given the following extract from a select-and-ultimate mortality table with a 2-year select period:

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
60	80,625	79,954	78,839	62
61	79,137	78,402	77,252	63
62	77,575	76,770	75,578	64

Assume that deaths are uniformly distributed between integral ages.

Calculate ${}_{0.9}q_{[60]+0.6}$.

(A) 0.0102

(B) 0.0103

(C) 0.0104

(D) 0.0105

(E) 0.0106

137. Removed

138. For a double decrement table with $l_{40}^{(\tau)} = 2000$:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x'^{(1)}$	$q_x'^{(2)}$
40	0.24	0.10	0.25	y
41	--	--	0.20	$2y$

Calculate $l_{42}^{(\tau)}$.

(A) 800

(B) 820

(C) 840

(D) 860

(E) 880

139. For a fully discrete whole life insurance of 10,000 on (30):

(i) π denotes the annual premium and $L(\pi)$ denotes the loss-at-issue random variable for this insurance.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

Calculate the lowest premium, π', such that the probability is less than 0.5 that the loss L (π') is positive.

(A) 34.6

(B) 36.6

(C) 36.8

(D) 39.0

(E) 39.1

140. Y is the present-value random variable for a special 3-year temporary life annuity-due on (x). You are given:

(i) ${}_t p_x = 0.9^t, \quad t \geq 0$

(ii) K_x is the curtate-future-lifetime random variable for (x).

(iii) $$Y = \begin{cases} 1.00, & K_x = 0 \\ 1.87, & K_x = 1 \\ 2.72, & K_x = 2, 3, \ldots \end{cases}$$

Calculate Var(Y).

(A) 0.19

(B) 0.30

(C) 0.37

(D) 0.46

(E) 0.55

141. Z is the present-value random variable for a whole life insurance of b payable at the moment of death of (x).

You are given:

(i) $\delta = 0.04$

(ii) $\mu_{x+t} = 0.02, \quad t \geq 0$

(iii) The single benefit premium for this insurance is equal to Var(Z).

Calculate b.

(A) 2.75

(B) 3.00

(C) 3.25

(D) 3.50

(E) 3.75

142. For a fully continuous whole life insurance of 1 on (x):

(i) π is the benefit premium.

(ii) L is the loss-at-issue random variable with the premium equal to π.

(iii) L^* is the loss-at-issue random variable with the premium equal to 1.25 π.

(iv) $\bar{a}_x = 5.0$

(v) $\delta = 0.08$

(vi) $\mathrm{Var}(L) = 0.5625$

Calculate the sum of the expected value and the standard deviation of L^*.

(A) 0.59

(B) 0.71

(C) 0.86

(D) 0.89

(E) 1.01

143. Removed

144. For students entering a three-year law school, you are given:

(i) The following double decrement table:

Academic Year	For a student at the beginning of that academic year, probability of		
	Academic Failure	Withdrawal for All Other Reasons	Survival Through Academic Year
1	0.40	0.20	--
2	--	0.30	--
3	--	--	0.60

(ii) Ten times as many students survive year 2 as fail during year 3.

(iii) The number of students who fail during year 2 is 40% of the number of students who survive year 2.

Calculate the probability that a student entering the school will withdraw for reasons other than academic failure before graduation.

(A) Less than 0.35

(B) At least 0.35, but less than 0.40

(C) At least 0.40, but less than 0.45

(D) At least 0.45, but less than 0.50

(E) At least 0.50

145. Given:

(i) Superscripts *M* and *N* identify two forces of mortality and the curtate expectations of life calculated from them.

(ii) $$\mu_{25+t}^{N} = \begin{cases} \mu_{25+t}^{M} + 0.1*(1-t), & 0 \le t \le 1 \\ \mu_{25+t}^{M}, & t>1 \end{cases}$$

(iii) $e_{25}^{M} = 10.0$

Calculate e_{25}^{N}.

(A) 9.2

(B) 9.3

(C) 9.4

(D) 9.5

(E) 9.6

146. A fund is established to pay annuities to 100 independent lives age *x*. Each annuitant will receive 10,000 per year continuously until death. You are given:

(i) $\delta = 0.06$

(ii) $\bar{A}_x = 0.40$

(iii) ${}^2\bar{A}_x = 0.25$

Calculate the amount (in millions) needed in the fund so that the probability, using the normal approximation, is 0.90 that the fund will be sufficient to provide the payments.

(A) 9.74

(B) 9.96

(C) 10.30

(D) 10.64

(E) 11.10

147. For a special 3-year term insurance on (30), you are given:

(i) Premiums are payable semiannually.

(ii) Premiums are payable only in the first year.

(iii) Benefits, payable at the end of the year of death, are:

k	b_{k+1}
0	1000
1	500
2	250

(iv) Mortality follows the Illustrative Life Table.

(v) Deaths are uniformly distributed within each year of age.

(vi) $i = 0.06$

Calculate the amount of each semiannual benefit premium for this insurance.

(A) 1.3

(B) 1.4

(C) 1.5

(D) 1.6

(E) 1.7

148. A decreasing term life insurance on (80) pays (20-k) at the end of the year of death if (80) dies in year k+1, for k = 0,1,2,…,19.

You are given:

(i) $i = 0.06$

(ii) For a certain mortality table with $q_{80} = 0.2$, the single benefit premium for this insurance is 13.

(iii) For this same mortality table, except that $q_{80} = 0.1$, the single benefit premium for this insurance is P.

Calculate P.

(A) 11.1

(B) 11.4

(C) 11.7

(D) 12.0

(E) 12.3

149. Removed

150. For independent lives (50) and (60):

$$\mu_x = \frac{1}{100-x}, \qquad 0 \le x < 100$$

Calculate $\overset{\circ}{e}_{\overline{50:60}}$.

(A) 30

(B) 31

(C) 32

(D) 33

(E) 34

151. For a multi-state model with three states, Healthy (0), Disabled (1), and Dead (2):

(i) For k = 0, 1:

$$p_{x+k}^{00} = 0.70$$
$$p_{x+k}^{01} = 0.20$$
$$p_{x+k}^{10} = 0.10$$
$$p_{x+k}^{12} = 0.25$$

(ii) There are 100 lives at the start, all Healthy. Their future states are independent.

Calculate the variance of the number of the original 100 lives who die within the first two years.

(A) 11

(B) 14

(C) 17

(D) 20

(E) 23

152. An insurance company issues a special 3-year insurance to a high risk individual (x). You are given the following multi-state model:

(i) State 1: active
State 2: disabled
State 3: withdrawn
State 4: dead

Annual transition probabilities for k =0, 1, 2:

i	p_{x+k}^{i1}	p_{x+k}^{i2}	p_{x+k}^{i3}	p_{x+k}^{i4}
1	0.4	0.2	0.3	0.1
2	0.2	0.5	0.0	0.3
3	0.0	0.0	1,0	0.0
4	0.0	0.0	0.0	1.0

(ii) The death benefit is 1000, payable at the end of the year of death.

(iii) i = 0.05

(iv) The insured is disabled (in State 2) at the beginning of year 2.

Calculate the expected present value of the prospective death benefits at the beginning of year 2.

(A) 440

(B) 528

(C) 634

(D) 712

(E) 803

153. Removed

154. For a special 30-year deferred annual whole life annuity-due of 1 on (35):

(i) If death occurs during the deferral period, the single benefit premium is refunded without interest at the end of the year of death.

(ii) $\ddot{a}_{65} = 9.90$

(iii) $A_{35:\overline{30|}} = 0.21$

(iv) $A^{\;\;1}_{35:\overline{30|}} = 0.07$

Calculate the single benefit premium for this special deferred annuity.

(A) 1.3

(B) 1.4

(C) 1.5

(D) 1.6

(E) 1.7

155. Given:

(i) $\mu_x = F + e^{2x}, \qquad x \geq 0$

(ii) ${}_{0.4}p_0 = 0.50$

Calculate F.

(A) -0.20

(B) -0.09

(C) 0.00

(D) 0.09

(E) 0.20

156. For a fully discrete whole life insurance of b on (x), you are given:

(i) $q_{x+9} = 0.02904$

(ii) $i = 0.03$

(iii) The benefit reserve at the start of year 10, after the premium is paid is 343.

(iv) The net amount at risk for year 10 is 872.

(v) $\ddot{a}_x = 14.65976$

Calculate the benefit reserve at the end of year 9.

(A) 280

(B) 288

(C) 296

(D) 304

(E) 312

157. For a special fully discrete 2-year endowment insurance of 1000 on (x), you are given:

(i) The first year benefit premium is 668.

(ii) The second year benefit premium is 258.

(iii) $d = 0.06$

Calculate the level annual premium using the equivalence principle.

(A) 469

(B) 479

(C) 489

(D) 499

(E) 509

158. For an increasing 10-year term insurance, you are given:

(i) The benefit for death during year $k + 1$ is $b_{k+1} = 100,000(k+1)$, $k = 0, 1, \ldots, 9$

(ii) Benefits are payable at the end of the year of death.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) The single benefit premium for this insurance on (41) is 16,736.

Calculate the single benefit premium for this insurance on (40).

(A) 12,700

(B) 13,600

(C) 14,500

(D) 15,500

(E) 16,300

159. For a fully discrete whole life insurance of 1000 on (x):

(i) Death is the only decrement.

(ii) The annual benefit premium is 80.

(iii) The annual gross premium is 100.

(iv) Expenses in year 1, payable at the start of the year, are 40% of gross premiums.

(v) Mortality and interest are the same for asset shares and benefit reserves.

(vi) $i = 0.10$

(vii) The benefit reserve at the end of year 1 is 40.

(viii) The asset share at time 0 is 0.

Calculate the asset share at the end of the first year.

(A) 17

(B) 18

(C) 19

(D) 20

(E) 21

160. A fully discrete 3-year term insurance of 10,000 on (40) is based on a double-decrement model, death and withdrawal:

(i) Decrement 1 is death.

(ii) $\mu_{40+t}^{(1)} = 0.02, \quad t \geq 0$

(iii) Decrement 2 is withdrawal, which occurs at the end of the year.

(iv) $q'^{(2)}_{40+k} = 0.04, \qquad k = 0, 1, 2$

(v) $v = 0.95$

Calculate the actuarial present value of the death benefits for this insurance.

(A) 487

(B) 497

(C) 507

(D) 517

(E) 527

161. You are given:

(i) $\overset{\circ}{e}_{30:\overline{40}|} = 27.692$

(ii) $S_0(t) = 1 - \dfrac{t}{\omega}, \qquad 0 \le t \le \omega$

(iii) T_x is the future lifetime random variable for (x).

Calculate $\text{Var}\left(T_{30}\right)$.

(A) 332

(B) 352

(C) 372

(D) 392

(E) 412

162. For a fully discrete 5-payment 10-year decreasing term insurance on (60), you are given:

(i) The death benefit during year k + 1 is $b_{k+1} = 1000\left(10 - k\right)$, k = 0, 1, 2,…, 9

(ii) Level benefit premiums are payable for five years and equal 218.15 each.

(iii) $q_{60+k} = 0.02 + 0.001\,k$, k = 0, 1, 2,…, 9

(iv) i = 0.06

Calculate ${}_2V$, the benefit reserve at the end of year 2.

(A) 70

(B) 72

(C) 74

(D) 76

(E) 78

163. You are given:

(i) T_x and T_y are not independent.

(ii) $q_{x+k} = q_{y+k} = 0.05$, $k = 0, 1, 2, \ldots$

(iii) ${}_k p_{xy} = 1.02\ {}_k p_x\ {}_k p_y$, $k = 1, 2, 3\ldots$

Into which of the following ranges does $e_{\overline{x:y}}$, the curtate expectation of life of the last survivor status, fall?

(A) $e_{\overline{x:y}} \le 25.7$

(B) $25.7 < e_{\overline{x:y}} \le 26.7$

(C) $26.7 < e_{\overline{x:y}} \le 27.7$

(D) $27.7 < e_{\overline{x:y}} \le 28.7$

(E) $28.7 < e_{\overline{x:y}}$

164. Removed

165. Removed

166. You are given:

(i) $\mu_{x+t} = 0.03$, $t \ge 0$

(ii) $\delta = 0.05$

(iii) T_x is the future lifetime random variable.

(iv) g is the standard deviation of $\bar{a}_{\overline{T_x}|}$.

Calculate $\Pr\left(\bar{a}_{\overline{T_x}|} > \bar{a}_x - g\right)$.

(A) 0.53

(B) 0.56

(C) 0.63

(D) 0.68

(E) 0.79

167. (50) is an employee of XYZ Corporation. Future employment with XYZ follows a double decrement model:

(i) Decrement 1 is retirement.

(ii) $\mu_{50+t}^{(1)} = \begin{cases} 0.00 & 0 \le t < 5 \\ 0.02 & 5 \le t \end{cases}$

(iii) Decrement 2 is leaving employment with XYZ for all other causes.

(iv) $\mu_{50+t}^{(2)} = \begin{cases} 0.05 & 0 \le t < 5 \\ 0.03 & 5 \le t \end{cases}$

(v) If (50) leaves employment with XYZ, he will never rejoin XYZ.

Calculate the probability that (50) will retire from XYZ before age 60.

(A) 0.069

(B) 0.074

(C) 0.079

(D) 0.084

(E) 0.089

168. For a life table with a one-year select period, you are given:

(i)

x	$l_{[x]}$	$d_{[x]}$	l_{x+1}	$\mathring{e}_{[x]}$
80	1000	90	–	8.5
81	920	90	–	–

(ii) Deaths are uniformly distributed over each year of age.

Calculate $\mathring{e}_{[81]}$.

(A) 8.0

(B) 8.1

(C) 8.2

(D) 8.3

(E) 8.4

169. For a fully discrete 3-year endowment insurance of 1000 on (x):

(i) $i = 0.05$

(ii) $p_x = p_{x+1} = 0.7$

Calculate the benefit reserve at the end of year 2.

(A) 526

(B) 632

(C) 739

(D) 845

(E) 952

170. For a fully discrete whole life insurance of 1000 on (50), you are given:

(i) The annual per policy expense is 1.

(ii) There is an additional first year expense of 15.

(iii) The claim settlement expense of 50 is payable when the claim is paid.

(iv) All expenses, except the claim settlement expense, are paid at the beginning of the year.

(v) $l_x = 20(100 - x),\ 0 \le x \le 100$.

(vi) $i = 0.05$

Calculate the level gross premium using the equivalence principle.

(A) 27

(B) 28

(C) 29

(D) 30

(E) 31

171. You are given:

$$\mu_x = \begin{cases} 0.05 & 50 \le x < 60 \\ 0.04 & 60 \le x < 70 \end{cases}$$

Calculate ${}_{4|14}q_{50}$.

(A) 0.38

(B) 0.39

(C) 0.41

(D) 0.43

(E) 0.44

172. For a special fully discrete 5-year deferred whole life insurance of 100,000 on (40), you are given:

(i) The death benefit during the 5-year deferral period is return of benefit premiums paid without interest.

(ii) Annual benefit premiums are payable only during the deferral period.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) $(IA)^{1}_{40:\overline{5|}} = 0.04042$

Calculate the annual benefit premium.

(A) 3300

(B) 3320

(C) 3340

(D) 3360

(E) 3380

173. You are pricing a special 3-year annuity-due on two independent lives, both age 80. The annuity pays 30,000 if both persons are alive and 20,000 if only one person is alive.

You are given:

(i)

k	${}_k p_{80}$
1	0.91
2	0.82
3	0.72

(ii) $i = 0.05$

Calculate the actuarial present value of this annuity.

(A) 78,300

(B) 80,400

(C) 82,500

(D) 84,700

(E) 86,800

174. Company ABC sets the gross premium for a continuous life annuity of 1 per year on (x) equal to the single benefit premium calculated using:

(i) $\delta = 0.03$

(ii) $\mu_{x+t} = 0.02, \quad t \ge 0$

However, a revised mortality assumption reflects future mortality improvement and is given by

$$\mu_{x+t} = \begin{cases} 0.02 & \text{for } t \le 10 \\ 0.01 & \text{for } t > 10 \end{cases}$$

Calculate the expected loss at issue for ABC (using the revised mortality assumption) as a percentage of the gross premium.

(A) 2%

(B) 8%

(C) 15%

(D) 20%

(E) 23%

175. A group of 1000 lives each age 30 sets up a fund to pay 1000 at the end of the first year for each member who dies in the first year, and 500 at the end of the second year for each member who dies in the second year. Each member pays into the fund an amount equal to the single benefit premium for a special 2-year term insurance, with:

(i) Benefits:

k	b_{k+1}
0	1000
1	500

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

The actual experience of the fund is as follows:

k	Interest Rate Earned	Number of Deaths
0	0.070	1
1	0.069	1

Calculate the difference, at the end of the second year, between the expected size of the fund as projected at time 0 and the actual fund.

(A) 840

(B) 870

(C) 900

(D) 930

(E) 960

176. For a special whole life insurance on (x), you are given:

(i) Z is the present value random variable for this insurance.

(ii) Death benefits are paid at the moment of death.

(iii) $\mu_{x+t} = 0.02, \quad t \geq 0$

(iv) $\delta = 0.08$

(v) The death benefit at time t is $b_t = e^{0.03t}, \quad t \geq 0$

Calculate $\mathrm{Var}(Z)$.

(A) 0.075

(B) 0.080

(C) 0.085

(D) 0.090

(E) 0.095

177. For a whole life insurance of 1 on (x), you are given:

(i) Benefits are payable at the moment of death.

(ii) Level premiums are payable at the beginning of each year.

(iii) Deaths are uniformly distributed over each year of age.

(iv) $i = 0.10$

(v) $\ddot{a}_x = 8$

(vi) $\ddot{a}_{x+10} = 6$

Calculate the benefit reserve at the end of year 10 for this insurance.

(A) 0.18

(B) 0.25

(C) 0.26

(D) 0.27

(E) 0.30

178. A special whole life insurance of 100,000 payable at the moment of death of (x) includes a double indemnity provision. This provision pays during the first ten years an additional benefit of 100,000 at the moment of death for death by accidental means.

You are given:

(i) $\mu_{x+t}^{(\tau)} = 0.001, \quad t \geq 0$

(ii) $\mu_{x+t}^{(1)} = 0.0002, \quad t \geq 0$, is the force of decrement due to death by accidental means.

(iii) $\delta = 0.06$

Calculate the single benefit premium for this insurance.

(A) 1640

(B) 1710

(C) 1790

(D) 1870

(E) 1970

179. Kevin and Kira are modeling the future lifetime of (60).

(i) Kevin uses a double decrement model:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
60	1000	120	80
61	800	160	80
62	560	–	–

(ii) Kira uses a multi-state model:

(a) The states are 0 (alive), 1 (death due to cause 1), 2 (death due to cause 2).

(b) Her calculations include the annual transition probabilities.

(iii) The two models produce equal probabilities of decrement.

Calculate $p_{61}^{00} + p_{61}^{01} + p_{61}^{10} + p_{61}^{11}$.

(A) 1.64

(B) 1.88

(C) 1.90

(D) 1.92

(E) 2.12

180. A certain species of flower has three states: sustainable, endangered and extinct. Transitions between states are modeled as a non-homogeneous discrete-time Markov chain with transition probability matrices Q_i as follows, where Q_i denotes the matrix from time *i* to *i*+1.

$$Q_0 = \begin{array}{l} \\ \text{Sustainable} \\ \text{Endangered} \\ \text{Extinct} \end{array} \begin{pmatrix} \text{Sustainable} & \text{Endangered} & \text{Extinct} \\ 0.85 & 0.15 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.7 & 0.2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_i = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad i = 3, 4, \ldots$$

Calculate the probability that a species endangered at time 0 will ever become extinct.

(A) 0.45

(B) 0.47

(C) 0.49

(D) 0.51

(E) 0.53

181. For a multi-state model of a special 3-year term insurance on (x):

(i) Insureds may be in one of three states at the beginning of each year: active (State 0), disabled (State 1), or dead (State 2). The annual transition probabilities are as follows for k = 0, 1, 2:

State i	p_{x+k}^{i0}	p_{x+k}^{i1}	p_{x+k}^{i2}
Active (0)	0.8	0.1	0.1
Disabled (1)	0.1	0.7	0.2
Dead (2)	0.0	0.0	1.0

(ii) A 100,000 benefit is payable at the end of the year of death whether the insured was active or disabled.

(iii) Premiums are paid at the beginning of each year when active. Insureds do not pay any annual premiums when they are disabled at the start of the year.

(iv) All insureds are active (State 0) at issue.

(v) $d = 0.10$

Calculate the level annual benefit premium for this insurance.

(A) 9,000

(B) 10,700

(C) 11,800

(D) 13,200

(E) 20,800

182. Removed

183. Removed

184. For a special fully discrete 30-payment whole life insurance on (45), you are given:

(i) The death benefit of 1000 is payable at the end of the year of death.

(ii) The benefit premium for this insurance is equal to $1000P_{45}$ for the first 15 years followed by an increased level annual premium of π for the remaining 15 years.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate π.

(A) 16.8

(B) 17.3

(C) 17.8

(D) 18.3

(E) 18.8

185. For a special fully discrete 2-year endowment insurance on (x):

(i) The pure endowment is 2000.

(ii) The death benefit for year k is $(1000k)$ plus the benefit reserve at the end of year k, $k=1,2$. For k = 2, this benefit reserve is the benefit reserve just before the maturity benefit is paid.

(iii) π is the level annual benefit premium.

(iv) i = 0.08

(v) $p_{x+k-1}=0.9, \quad k=1,2$

Calculate π.

(A) 1027

(B) 1047

(C) 1067

(D) 1087

(E) 1107

186. For a group of 250 individuals age x, you are given:

(i) The future lifetimes are independent.

(ii) Each individual is paid 500 at the beginning of each year, if living.

(iii) $A_x = 0.369131$

(iv) ${}^2A_x = 0.1774113$

(v) $i = 0.06$

Using the normal approximation, calculate the size of the fund needed at inception in order to be 90% certain of having enough money to pay the life annuities.

(A) 1.43 million

(B) 1.53 million

(C) 1.63 million

(D) 1.73 million

(E) 1.83 million

187. For a double decrement table, you are given:

Age	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	1000	60	55
41	–	–	70
42	750	–	–

Each decrement is uniformly distributed over each year of age in the double decrement table.

Calculate $q_{41}'^{(1)}$.

(A) 0.077

(B) 0.078

(C) 0.079

(D) 0.080

(E) 0.081

188. The actuarial department for the SharpPoint Corporation models the lifetime of pencil sharpeners from purchase using $S_0(t) = \left(1 - \frac{t}{\omega}\right)^{\alpha}$, for $\alpha > 0$ and $0 \le t \le \omega$.

A senior actuary examining mortality tables for pencil sharpeners has determined that the original value of α must change. You are given:

(i) The new complete expectation of life at purchase is half what it was previously.

(ii) The new force of mortality for pencil sharpeners is 2.25 times the previous force of mortality for all durations.

(iii) ω remains the same.

Calculate the original value of α.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

189. You are given:

(i) T is the future lifetime random variable.

(ii) $\mu_x = \mu$, $\quad x \geq 0$

(iii) $\mathrm{Var}[T] = 100$.

(iv) $X = \min(T, 10)$

Calculate $\mathrm{E}[X]$.

(A) 2.6

(B) 5.4

(C) 6.3

(D) 9.5

(E) 10.0

190. For a fully discrete 15-payment whole life insurance of 100,000 on (x), you are given:

(i) The level gross annual premium using the equivalence principle is 4669.95.

(ii) $100,000 A_x = 51,481.97$

(iii) $\ddot{a}_{x:\overline{15|}} = 11.35$

(iv) $d = 0.02913$

(v) Expenses are incurred at the beginning of the year.

(vi) Percent of premium expenses are 10% in the first year and 2% thereafter.

(vii) Per policy expenses are K in the first year and 5 in each year thereafter until death.

Calculate K.

(A) 10.0

(B) 16.5

(C) 23.0

(D) 29.5

(E) 36.5

191. For the future lifetimes of (x) and (y):

(i) With probability 0.4, $T_x = T_y$ (i.e., deaths occur simultaneously).

(ii) With probability 0.6, the joint density function is

$$f_{T_x,T_y}(t,s) = 0.0005\,,\quad 0 < t < 40\,,\qquad 0 < s < 50$$

Calculate $\text{Prob}\left[T_x < T_y\right]$.

(A) 0.30

(B) 0.32

(C) 0.34

(D) 0.36

(E) 0.38

192. For a group of lives age x, you are given:

(i) Each member of the group has a constant force of mortality that is drawn from the uniform distribution on $\left[0.01, 0.02\right]$.

(ii) $\delta = 0.01$

For a member selected at random from this group, calculate the actuarial present value of a continuous lifetime annuity of 1 per year.

(A) 40.0

(B) 40.5

(C) 41.1

(D) 41.7

(E) 42.3

193. For a population whose mortality follows $l_x = 100(\omega - x)$, $0 \le x \le \omega$, you are given:

(i) $\mathring{e}_{\overline{40:40}} = 3\mathring{e}_{\overline{60:60}}$

(ii) $\mathring{e}_{\overline{20:20}} = k\,\mathring{e}_{\overline{60:60}}$

Calculate k.

(A) 3.0

(B) 3.5

(C) 4.0

(D) 4.5

(E) 5.0

194. For multi-state model of an insurance on (x) and (y):

(i) The death benefit of 10,000 is payable at the moment of the second death.

(ii) You use the states:
State 0 = both alive
State 1 = only (x) is alive
State 2 = only (y) is alive
State 3 = neither alive

(iii) $\mu^{01}_{x+t:y+t} = \mu^{02}_{x+t:y+t} = 0.06, t \geq 0$

(iv) $\mu^{03}_{x+t:y+t} = 0, t \geq 0$

(v) $\mu^{13}_{x+t:y+t} = \mu^{23}_{x+t:y+t} = 0.10, t \geq 0$

(vi) $\delta = 0.04$

Calculate the expected present value of this insurance on (x) and (y).

(A) 4500

(B) 5400

(C) 6000

(D) 7100

(E) 7500

195. Kevin and Kira are in a history competition:

(i) In each round, every child still in the contest faces one question. A child is out as soon as he or she misses one question. The contest will last at least 5 rounds.

(ii) For each question, Kevin's probability and Kira's probability of answering that question correctly are each 0.8; their answers are independent.

Calculate the conditional probability that both Kevin and Kira are out by the start of round five, given that at least one of them participates in round 3.

(A) 0.13

(B) 0.16

(C) 0.19

(D) 0.22

(E) 0.25

196. For a special increasing whole life annuity-due on (40), you are given:

(i) Y is the present-value random variable.

(ii) Payments are made once every 30 years, beginning immediately.

(iii) The payment in year 1 is 10, and payments increase by 10 every 30 years.

(iv) ${}_t p_0 = 1 - \dfrac{t}{110}, 0 \le t \le 110$

(v) $i = 0.04$

Calculate $\text{Var}(Y)$.

(A) 10.5

(B) 11.0

(C) 11.5

(D) 12.0

(E) 12.5

197. For a special 3-year term insurance on (x), you are given:

(iv) Z is the present-value random variable for this insurance.

(v) $q_{x+k} = 0.02(k+1)$, $k = 0, 1, 2$

(vi) The following benefits are payable at the end of the year of death:

k	b_{k+1}
0	300
1	350
2	400

(iv) $i = 0.06$

Calculate $\text{Var}(Z)$.

(A) 9,600

(B) 10,000

(C) 10,400

(D) 10,800

(E) 11,200

198. For a fully discrete whole life insurance of 1000 on (60), you are given:

(i) The expenses, payable at the beginning of the year, are:

Expense Type	First Year	Renewal Years
% of Premium	20%	6%
Per Policy	8	2

(ii) The level gross premium is 41.20.

(iii) $i = 0.05$

(iv) ${}_0L$ is the present value of the loss random variable at issue.

Calculate the value of ${}_0L$ if the insured dies in the third policy year.

(A) 770

(B) 790

(C) 810

(D) 830

(E) 850

199. For a fully discrete whole life insurance of 1000 on (45), you are given:

(i) ${}_kV$ denotes the benefit reserve at the end of year k, k = 1, 2, 3, …for an insurance of 1.

(ii)

k	$1000\,{}_kV$	q_{45+k}
22	235	0.015
23	255	0.020
24	272	0.025

Calculate $1000\,{}_{25}V$.

(A) 279

(B) 282

(C) 284

(D) 286

(E) 288

200. The graph of a piecewise linear survival function, $S_0(t)$, consists of 3 line segments with endpoints (0, 1), (25, 0.50), (75, 0.40), (100, 0).

Calculate $\dfrac{{}_{20|55}q_{15}}{{}_{55}q_{35}}$.

(A) 0.69

(B) 0.71

(C) 0.73

(D) 0.75

(E) 0.77

201. For a group of lives aged 30, containing an equal number of smokers and non-smokers, you are given:

(i) For non-smokers, $\mu_x^n = 0.08$, $x \geq 30$

(ii) For smokers, $\mu_x^s = 0.16$, $x \geq 30$

Calculate q_{80} for a life randomly selected from those surviving to age 80.

(A) 0.078

(B) 0.086

(C) 0.095

(D) 0.104

(E) 0.112

202. For a 3-year fully discrete term insurance of 1000 on (40), subject to a double decrement model:

(i)

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	2000	20	60
41	–	30	50
42	–	40	–

(ii) Decrement 1 is death. Decrement 2 is withdrawal.

(iii) There are no withdrawal benefits.

(iv) $i = 0.05$

Calculate the level annual benefit premium for this insurance.

(A) 14.3

(B) 14.7

(C) 15.1

(D) 15.5

(E) 15.7

203. For a fully continuous whole life insurance of 1 on (30), you are given:

(i) The force of mortality is 0.05 in the first 10 years and 0.08 thereafter.

(ii) $\delta = 0.08$

Calculate the benefit reserve at time 10 for this insurance.

(A) 0.144

(B) 0.155

(C) 0.166

(D) 0.177

(E) 0.188

204. For a 10-payment, 20-year term insurance of 100,000 on Pat:

(i) Death benefits are payable at the moment of death.

(ii) Gross premiums of 1600 are payable annually at the beginning of each year for 10 years.

(iii) $i = 0.05$

(iv) L is the loss random variable at the time of issue.

Calculate the minimum value of L as a function of the time of death of Pat.

(A) $-21{,}000$

(B) $-17{,}000$

(C) $-13{,}000$

(D) $-12{,}400$

(E) $-12{,}000$

205. Removed

206. Michael, age 45, is a professional motorcycle jumping stuntman who plans to retire in three years. He purchases a three-year term insurance policy. The policy pays 500,000 for death from a stunt accident and nothing for death from other causes. The benefit is paid at the end of the year of death.

You are given:

(i) $i = 0.08$

(ii)

x	$l_x^{(\tau)}$	$d_x^{(-s)}$	$d_x^{(s)}$
45	2500	10	4
46	2486	15	5
47	2466	20	6

where $d_x^{(s)}$ represents deaths from stunt accidents and $d_x^{(-s)}$ represents deaths from other causes.

(iii) Level annual benefit premiums are payable at the beginning of each year.

Calculate the annual benefit premium.

(A) 920

(B) 1030

(C) 1130

(D) 1240

(E) 1350

207. You are given the survival function

$$S_0(t) = 1 - \left(0.01t\right)^2, \qquad 0 \le t \le 100$$

Calculate $\mathring{e}_{30:\overline{50}|}$, the 50-year temporary complete expectation of life of (30).

(A) 27

(B) 30

(C) 34

(D) 37

(E) 41

208. For a fully discrete whole life insurance of 1000 on (50), you are given:

(i) $1000P_{50} = 25$

(ii) $1000A_{61} = 440$

(iii) $1000q_{60} = 20$

(iv) $i = 0.06$

Calculate the benefit reserve at the end of year 10.

(A) 170

(B) 172

(C) 174

(D) 176

(E) 178

209. For a pension plan portfolio, you are given:

(i) 80 individuals with mutually independent future lifetimes are each to receive a whole life annuity-due.

(ii) $i = 0.06$

(iii)

Age	Number of annuitants	Annual annuity payment	$\ddot{a}_x$	A_x	${}^2A_x$
65	50	2	9.8969	0.43980	0.23603
75	30	1	7.2170	0.59149	0.38681

(iv) X is the random variable for the present value of total payments to the 80 annuitants.

Using the normal approximation, calculate the 95th percentile of the distribution of X.

(A) 1220

(B) 1239

(C) 1258

(D) 1277

(E) 1296

210. Your company sells a product that pays the cost of nursing home care for the remaining lifetime of the insured.

(i) Insureds who enter a nursing home remain there until death.

(ii) The force of mortality, μ, for each insured who enters a nursing home is constant.

(iii) μ is uniformly distributed on the interval [0.5, 1].

(iv) The cost of nursing home care is 50,000 per year payable continuously.

(v) $\delta = 0.045$

Calculate the actuarial present value of this benefit for a randomly selected insured who has just entered a nursing home.

(A) 60,800

(B) 62,900

(C) 65,100

(D) 67,400

(E) 69,800

211. Removed

212. Removed

213. Removed.

214. For a fully discrete 20-year endowment insurance of 10,000 on (45) that has been in force for 15 years, you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) At issue, the premium was calculated using the equivalence principle.

(iv) When the insured decides to stop paying premiums after 15 years, the death benefit remains at 10,000 but the pure endowment value is reduced such that the expected prospective loss at age 60 is unchanged.

Calculate the reduced pure endowment value.

(A) 8120

(B) 8500

(C) 8880

(D) 9260

(E) 9640

215. For a whole life insurance of 1 on (x) with benefits payable at the moment of death, you are given:

(i) δ_t, the force of interest at time t is $\delta_t = \begin{cases} 0.02, & t < 12 \\ 0.03, & t \geq 12 \end{cases}$

(ii) $\mu_{x+t} = \begin{cases} 0.04, & t < 5 \\ 0.05, & t \geq 5 \end{cases}$

Calculate the actuarial present value of this insurance.

(A) 0.59

(B) 0.61

(C) 0.64

(D) 0.66

(E) 0.68

216. For a fully continuous whole life insurance on (x), you are given:

(i) The benefit is 2000 for death by accidental means (decrement 1).

(ii) The benefit is 1000 for death by other means (decrement 2).

(iii) The initial expense at issue is 50.

(iv) Termination expenses are 5% of the benefit, payable at the moment of death.

(v) Maintenance expenses are 3 per year, payable continuously.

(vi) The gross premium is 100 per year, payable continuously.

(vii) $\mu_{x+t}^{(1)} = 0.004, \quad t > 0$

(viii) $\mu_{x+t}^{(2)} = 0.040, \quad t > 0$

(ix) $\delta = 0.05$

(x) ${}_0L$ is the random variable for the present value at issue of the insurer's loss.

Calculate $\mathrm{E}({}_0L)$.

(A) – 446

(B) – 223

(C) 0

(D) 223

(E) 446

217. A homogeneous discrete-time Markov model has three states representing the status of the members of a population.

State 1 = healthy, no benefits
State 2 = disabled, receiving Home Health Care benefits
State 3 = disabled, receiving Nursing Home benefits

The annual transition probability matrix is given by:

$$\begin{pmatrix} 0.80 & 0.15 & 0.05 \\ 0.05 & 0.90 & 0.05 \\ 0.00 & 0.00 & 1.00 \end{pmatrix}$$

Transitions occur at the end of each year.
At the start of year 1, there are 50 members, all in state 1, healthy.

Calculate the variance of the number of those 50 members who will be receiving Nursing Home benefits during year 3.

(A) 2.3

(B) 2.7

(C) 4.4

(D) 4.5

(E) 4.6

218. A non-homogenous discrete-time Markov model has:

(i) Three states: 0, 1, and 2

(ii) Annual transition probability matrix Q_n from time n to time n+1 as follows:

$$Q_n = \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{for } n = 0 \text{ and } 1, \text{ and}$$

$$Q_n = \begin{pmatrix} 0 & 0.3 & 0.7 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{for } n = 2, 3, 4, \ldots$$

An individual starts out in state 0 and transitions occur mid-year.

An insurance is provided whereby:

(i) A premium of 1 is paid at the beginning of each year that an individual is in state 0 or 1.

(ii) A benefit of 4 is paid at the end of any year that the individual is in state 1 at the end of the year.

$i = 0.1$

Calculate the actuarial present value of premiums minus the actuarial present value of benefits at the start of this insurance.

(A) – 0.17

(B) 0.00

(C) 0.34

(D) 0.50

(E) 0.66

219. You are given the following information on participants entering a special 2-year program for treatment of a disease:

(i) Only 10% survive to the end of the second year.

(ii) The force of mortality is constant within each year.

(iii) The force of mortality for year 2 is three times the force of mortality for year 1.

Calculate the probability that a participant who survives to the end of month 3 dies by the end of month 21.

(A) 0.61

(B) 0.66

(C) 0.71

(D) 0.75

(E) 0.82

220. In a population, non-smokers have a force of mortality equal to one half that of smokers.

For non-smokers, $l_x = 500(110 - x), \quad 0 \le x \le 110$.

Calculate $\overset{\circ}{e}_{20:25}$ for a smoker (20) and a non-smoker (25) with independent future lifetimes.

(A) 18.3

(B) 20.4

(C) 22.1

(D) 24.5

(E) 26.8

221. For a special fully discrete 20-year term insurance on (30):

(i) The death benefit is 1000 during the first ten years and 2000 during the next ten years.

(ii) The benefit premium is π for each of the first ten years and 2π for each of the next ten years.

(iii) $\ddot{a}_{30:\overline{20}|} = 15.0364$

(iv)

x	$\ddot{a}_{x:\overline{10}\mid}$	$1000A^{1}_{x:\overline{10}\mid}$
30	8.7201	16.66
40	8.6602	32.61

Calculate π.

(A) 2.9

(B) 3.0

(C) 3.1

(D) 3.2

(E) 3.3

222. For a fully discrete whole life insurance of 25,000 on (25), you are given:

(i) $P_{25} = 0.01128$

(ii) $P_{25:\overline{15}|}^{\;\;1} = 0.05107$

(iii) $P_{25:\overline{15}|} = 0.05332$

Calculate the benefit reserve at the end of year 15.

(A) 4420

(B) 4460

(C) 4500

(D) 4540

(E) 4580

223. You are given 3 mortality assumptions:

(i) Illustrative Life Table (ILT),

(ii) Constant force model (CF), where $S_0(t) = e^{-\mu t},\ t \ge 0$

(iii) DeMoivre model (DM), where $S_0(t) = 1 - \dfrac{t}{\omega},\ 0 \le t \le \omega$, and $\omega \ge 72$.

For the constant force and DeMoivre models, ${}_2 p_{70}$ is the same as for the Illustrative Life Table.

Rank $e_{70:\overline{2}|}$ for these 3 models.

(A) ILT $<$ CF $<$ DM

(B) ILT $<$ DM $<$ CF

(C) CF $<$ DM $<$ ILT

(D) DM $<$ CF $<$ ILT

(E) DM $<$ ILT $<$ CF

224. A population of 1000 lives age 60 is subject to 3 decrements, death (1), disability (2), and retirement (3). You are given:

(i) The following independent rates of decrement:

x	$q_x'^{(1)}$	$q_x'^{(2)}$	$q_x'^{(3)}$
60	0.010	0.030	0.100
61	0.013	0.050	0.200

(ii) Decrements are uniformly distributed over each year of age in the multiple decrement table.

Calculate the expected number of people who will retire before age 62.

(A) 248

(B) 254

(C) 260

(D) 266

(E) 272

225. You are given:

(i) The future lifetimes of (40) and (50) are independent.

(ii) The survival function for (40) is based on a constant force of mortality, $\mu = 0.05$.

(iii) The survival function for (50) follows $l_x = 100(110 - x),\ 0 \le x \le 110$.

Calculate the probability that (50) dies within 10 years and dies before (40).

(A) 10%

(B) 13%

(C) 16%

(D) 19%

(E) 25%

226. Oil wells produce until they run dry. The survival function for a well is given by:

t (years)	$S_0(t)$
0	1.00
1	0.90
2	0.80
3	0.60
4	0.30
5	0.10
6	0.05
7	0.00

An oil company owns 10 wells age 3. It insures them for 1 million each against failure for two years where the loss is payable at the end of the year of failure.

You are given:

(i) R is the present-value random variable for the insurer's aggregate losses on the 10 wells.

(ii) The insurer actually experiences 3 failures in the first year and 5 in the second year.

(iii) $i = 0.10$

Calculate the ratio of the actual value of R to the expected value of R.

(A) 0.94

(B) 0.96

(C) 0.98

(D) 1.00

(E) 1.02

227. For a fully discrete 2-year term insurance of 1 on (x):

(i) $q_x = 0.1 \quad q_{x+1} = 0.2$

(ii) $v = 0.9$

(iii) K_x is the curtate future lifetime of (x).

(iv) ${}_1L$ is the prospective loss random variable at time 1 using the premium determined by the equivalence principle.

Calculate $\text{Var}\left({}_1L \middle| K_x > 0\right)$.

(A) 0.05

(B) 0.07

(C) 0.09

(D) 0.11

(E) 0.13

228. For a fully continuous whole life insurance of 1 on (x):

(i) $\bar{A}_x = 1/3$

(ii) $\delta = 0.10$

(iii) L is the loss at issue random variable using the premium based on the equivalence principle.

(iv) $\mathrm{Var}[L] = 1/5$

(v) L' is the loss at issue random variable using the premium π.

(vi) $\mathrm{Var}[L'] = 16/45$.

Calculate π.

(A) 0.05

(B) 0.08

(C) 0.10

(D) 0.12

(E) 0.15

229. You are given:

(i) Y is the present value random variable for a continuous whole life annuity of 1 per year on (40).

(ii) Mortality follows $S_0(t) = 1 - \frac{t}{120}, 0 \le t \le 120$.

(iii) $\delta = 0.05$

Calculate the 75th percentile of the distribution of Y.

(A) 12.6

(B) 14.0

(C) 15.3

(D) 17.7

(E) 19.0

230. For a special fully discrete 20-year endowment insurance on (40):

(i) The death benefit is 1000 for the first 10 years and 2000 thereafter. The pure endowment benefit is 2000.

(ii) The annual benefit premium is 40 for each of the first 10 years and 100 for each year thereafter.

(iii) $q_{40+k} = 0.001k + 0.001, \qquad k = 8, 9, \ldots 13$

(iv) $i = 0.05$

(v) $\ddot{a}_{51:\overline{9}|} = 7.1$

Calculate the benefit reserve at the end of year 10.

(A) 490

(B) 500

(C) 530

(D) 550

(E) 560

231. For a whole life insurance of 1000 on (80), with death benefits payable at the end of the year of death, you are given:

(i) Mortality follows a select and ultimate mortality table with a one-year select period.

(ii) $q_{[80]} = 0.5 q_{80}$

(iii) $i = 0.06$

(iv) $1000 A_{80} = 679.80$

(v) $1000 A_{81} = 689.52$

Calculate $1000 A_{[80]}$.

(A) 655

(B) 660

(C) 665

(D) 670

(E) 675

232. For a fully discrete 4-year term insurance on (40), who is subject to a double-decrement model:

(i) The benefit is 2000 for decrement 1 and 1000 for decrement 2.

(ii) The following is an extract from the double-decrement table for the last 3 years of this insurance:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
41	800	8	16
42	–	8	16
43	–	8	16

(iii) $v = 0.95$

(iv) The benefit premium is 34.

Calculate ${}_2V$, the benefit reserve at the end of year 2.

(A) 8

(B) 9

(C) 10

(D) 11

(E) 12

233. You are pricing a special 3-year temporary life annuity-due on two lives each age x, with independent future lifetimes, each following the same mortality table. The annuity pays 10,000 if both persons are alive and 2000 if exactly one person is alive.

You are given:

(i) $q_{xx} = 0.04$

(ii) $q_{x+1:x+1} = 0.01$

(iii) $i = 0.05$

Calculate the expected present value of this annuity.

(A) 27,800

(B) 27,900

(C) 28,000

(D) 28,100

(E) 28,200

234. For a triple decrement table, you are given:

(i) Each decrement is uniformly distributed over each year of age in its associated single decrement table.

(ii) $q_x'^{(1)} = 0.200$

(iii) $q_x'^{(2)} = 0.080$

(iv) $q_x'^{(3)} = 0.125$

Calculate $q_x^{(1)}$.

(A) 0.177

(B) 0.180

(C) 0.183

(D) 0.186

(E) 0.189

235. For a fully discrete whole life insurance of 1000 on (40), you are given:

(i) Death and withdrawal are the only decrements.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

(iv) The probabilities of withdrawal are:

$$q_{40+k}^{(w)} = \begin{cases} 0.2, & k = 0 \\ 0, & k > 0 \end{cases}$$

(v) Withdrawals occur at the end of the year.

(vi) The following expenses are payable at the beginning of the year:

	Percent of Premium	Per 1000 Insurance
All Years	10%	1.50

(vii) The cash value at the end of year 1 is 2.93.

(viii) The asset share at the end of year 2 is 24.

Calculate the gross premium, G.

(A) 15.4

(B) 15.8

(C) 16.3

(D) 16.7

(E) 17.2

236. For a fully discrete insurance of 1000 on (x), you are given:

(i) ${}_4AS = 396.63$ is the asset share at the end of year 4.

(ii) ${}_5AS = 694.50$ is the asset share at the end of year 5.

(iii) $G = 281.77$ is the gross premium.

(iv) ${}_5CV = 572.12$ is the cash value at the end of year 5.

(v) $c_4 = 0.05$ is the fraction of the gross premium paid at time 4 for expenses.

(vi) $e_4 = 7.0$ is the amount of per policy expenses paid at time 4.

(vii) $q_{x+4}^{(1)} = 0.09$ is the probability of decrement by death.

(viii) $q_{x+4}^{(2)} = 0.26$ is the probability of decrement by withdrawal.

Calculate i.

(A) 0.050

(B) 0.055

(C) 0.060

(D) 0.065

(E) 0.070

237. Removed

238. Removed

239. For a semicontinuous 20-year endowment insurance of 25,000 on (x), you are given:

(i) The following expenses are payable at the beginning of the year:

	Percent of Premium	Per 1000 Insurance	Per Policy
First Year	25%	2.00	15.00
Renewal	5%	0.50	3.00

(ii) Deaths are uniformly distributed over each year of age.

(iii) $\bar{A}_{x:\overline{20}|} = 0.4058$

(iv) $A_{x:\overline{20}|}^{\;1} = 0.3195$

(v) $\ddot{a}_{x:\overline{20}|} = 12.522$

(vi) $i = 0.05$

(vii) Premiums are determined using the equivalence principle.

Calculate the level annual premium.

(A) 884

(B) 888

(C) 893

(D) 909

(E) 913

240. For a 10-payment 20-year endowment insurance of 1000 on (40), you are given:

(i) The following expenses:

	First Year		Subsequent Years	
	Percent of Premium	Per Policy	Percent of Premium	Per Policy
Taxes	4%	0	4%	0
Sales Commission	25%	0	5%	0
Policy Maintenance	0	10	0	5

(ii) Expenses are paid at the beginning of each policy year.

(iii) Death benefits are payable at the moment of death.

(iv) The premium is determined using the equivalence principle.

Which of the following is a correct expression for the premium?

(A) $\left(1000\bar{A}_{40:\overline{20}|}+10+5a_{40:\overline{9}|}\right)/\left(0.96\ddot{a}_{40:\overline{10}|}-0.25-0.05\ddot{a}_{40:\overline{9}|}\right)$

(B) $\left(1000\bar{A}_{40:\overline{20}|}+10+5a_{40:\overline{9}|}\right)/\left(0.91\ddot{a}_{40:\overline{10}|}-0.2\right)$

(C) $\left(1000\bar{A}_{40:\overline{20}|}+10+5a_{40:\overline{19}|}\right)/\left(0.96\ddot{a}_{40:\overline{10}|}-0.25-0.05\ddot{a}_{40:\overline{9}|}\right)$

(D) $\left(1000\bar{A}_{40:\overline{20}|}+10+5a_{40:\overline{19}|}\right)/\left(0.91\ddot{a}_{40:\overline{10}|}-0.2\right)$

(E) $\left(1000\bar{A}_{40:\overline{20}|}+10+5a_{40:\overline{9}|}\right)/\left(0.95\ddot{a}_{40:\overline{10}|}-0.2-0.04\ddot{a}_{40:\overline{20}|}\right)$

241. Removed

242. For a fully discrete whole life insurance of 10,000 on (x), you are given:

(i) ${}_{10}AS = 1600$ is the asset share at the end of year 10.

(ii) $G = 200$ is the gross premium.

(iii) ${}_{11}CV = 1700$ is the cash value at the end of year 11.

(iv) $c_{10} = 0.04$ is the fraction of gross premium paid at time 10 for expenses.

(v) $e_{10} = 70$ is the amount of per policy expense paid at time 10.

(vi) Death and withdrawal are the only decrements.

(vii) $q_{x+10}^{(d)} = 0.02$

(viii) $q_{x+10}^{(w)} = 0.18$

(ix) $i = 0.05$

Calculate ${}_{11}AS$, the asset share at the end of year 11.

(A) 1302

(B) 1520

(C) 1628

(D) 1720

(E) 1878

243. For a fully discrete 10-year endowment insurance of 1000 on (35), you are given:

(i) Expenses are paid at the beginning of each year.

(ii) Annual per policy renewal expenses are 5.

(iii) Percent of premium renewal expenses are 10% of the gross premium.

(iv) There are expenses during year 1.

(v) $1000P_{35:\overline{10}|} = 76.87$

(vi) Gross premiums were calculated using the equivalence principle.

(vii) At the end of year 9, the excess of the benefit reserve over the gross premium reserve is 1.67.

Calculate the gross premium for this insurance.

(A) 80.20

(B) 83.54

(C) 86.27

(D) 89.11

(E) 92.82

244. For a fully discrete whole life insurance of 1000 on (x), you are given:

(i) $G = 30$ is the gross premium

(ii) $e_k = 5, \quad k = 1, 2, 3, \ldots$ is the per policy expense at the start of year k.

(iii) $c_k = 0.02, \quad k = 1, 2, 3, \ldots$ is the fraction of premium expense at the start of year k.

(iv) $i = 0.05$

(v) ${}_4CV = 75$ is the cash value payable upon withdrawal at the end of year 4.

(vi) $q_{x+3}^{(d)} = 0.013$

(vii) $q_{x+3}^{(w)} = 0.05$; withdrawals occur at the end of the year.

(viii) ${}_3AS = 25.22$ is the asset share at the end of year 3.

If the probability of withdrawal and all expenses for year 4 are each 120% of the values shown above, by how much does the asset share at the end of year 4 decrease?

(A) 1.59

(B) 1.64

(C) 1.67

(D) 1.93

(E) 2.03

245. For a fully discrete 5-payment 10-year deferred 20-year term insurance of 1000 on (30), you are given:

(i) The following expenses:

	Year 1		Years 2-10	
	Percent of Premium	Per Policy	Percent of Premium	Per Policy
Taxes	5%	0	5%	0
Sales commission	25%	0	10%	0
Policy maintenance	0	20	0	10

(ii) Expenses are paid at the beginning of each policy year.

(iii) The gross premium is determined using the equivalence principle.

Which of the following is a correct expression for the gross premium?

(A) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{19|}}\right) / \left(0.95\,\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\,\ddot{a}_{30:\overline{4|}}\right)$

(B) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{19|}}\right) / \left(0.85\,\ddot{a}_{30:\overline{5|}} - 0.15\right)$

(C) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{19|}}\right) / \left(0.95\,\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\,a_{30:\overline{4|}}\right)$

(D) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9|}}\right) / \left(0.95\,\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\,\ddot{a}_{30:\overline{4|}}\right)$

(E) $\left(1000\,{}_{10|20}A_{30} + 20 + 10\,a_{30:\overline{9|}}\right) / \left(0.85\,\ddot{a}_{30:\overline{5|}} - 0.15\right)$

246. For a special single premium 2-year endowment insurance on (x), you are given:

(i) Death benefits, payable at the end of the year of death, are:

$$b_1 = 3000$$
$$b_2 = 2000$$

(ii) The maturity benefit is 1000.

(iii) Expenses, payable at the beginning of the year:
- (a) Taxes are 2% of the gross premium.
- (b) Commissions are 3% of the gross premium.
- (c) Other expenses are 15 in the first year and 2 in the second year.

(iv) $i = 0.04$

(v) $p_x = 0.9$
$p_{x+1} = 0.8$

Calculate the single gross premium using the equivalence principle.

(A) 670

(B) 940

(C) 1000

(D) 1300

(E) 1370

247. For a fully discrete 2-payment, 3-year term insurance of 10,000 on (x), you are given:

(i) $i = 0.05$

(ii) $q_x = 0.10$
$q_{x+1} = 0.15$
$q_{x+2} = 0.20$

(iii) Death is the only decrement.

(iv) Expenses, paid at the beginning of the year, are:

Policy Year	Per policy	Per 1000 of insurance	Fraction of premium
1	25	4.50	0.20
2	10	1.50	0.10
3	10	1.50	–

(v) Settlement expenses, paid at the end of the year of death, are 20 per policy plus 1 per 1000 of insurance.

(vi) G is the gross annual premium for this insurance.

(vii) The single benefit premium for this insurance is 3499.

Calculate G, using the equivalence principle.

(A) 1597

(B) 2296

(C) 2303

(D) 2343

(E) 2575

248. For a fully discrete 20-year endowment insurance of 10,000 on (50), you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) The annual gross premium is 495.

(iv) Expenses are payable at the beginning of the year.

(v) The expenses are:

	Percent of Premium	Per Policy	Per 1000 of Insurance
First Year	35%	20	15.00
Renewal	5%	5	1.50

Calculate the expected present value of amounts available for profit and contingencies.

(A) 930

(B) 1080

(C) 1130

(D) 1180

(E) 1230

249. For (x) and (y) with independent future lifetimes, you are given:

(i) (x) is subject to a uniform distribution of deaths over each year of age.

(ii) (y) is subject to a constant force of mortality of 0.25.

(iii) $q_{\overset{1}{x}y} = 0.125$

Calculate q_x.

(A) 0.130

(B) 0.141

(C) 0.167

(D) 0.214

(E) 0.250

250. The CAS Insurance Company classifies its auto drivers as Preferred (State 1) or Standard (State 2) at time 0, which is the start of the first year the driver is insured. After issue, drivers are continuously reclassified.

For a driver, Anne, you are given:

(i) $[x]$ denotes Anne's age at time 0.

(ii) For $k = 0, 1, 2, \ldots$,

$$p_{[x]+k}^{11} = 0.7 + \frac{0.1}{k+1}$$
$$p_{[x]+k}^{12} = 0.3 - \frac{0.1}{k+1}$$
$$p_{[x]+k}^{21} = 0.4 - \frac{0.2}{k+1}$$
$$p_{[x]+k}^{22} = 0.6 + \frac{0.2}{k+1}$$

(iii) Anne is classified Preferred at the start of year 2.

Calculate the probability that Anne is classified Preferred at the start of year 4.

(A) 0.55

(B) 0.59

(C) 0.63

(D) 0.67

(E) 0.71

251-260. These questions have been removed.

261. You are given:

(i) Z is the present value random variable for an insurance on the lives of (x) and (y), where

$$Z = \begin{cases} v^{T_y}, & T_x \le T_y \\ 0, & T_x > T_y \end{cases}$$

(ii) (x) is subject to a constant force of mortality, 0.07.

(iii) (y) is subject to a constant force of mortality, 0.09

(iv) (x) and (y) are independent lives.

(v) $\delta = 0.06$

Calculate E[Z].

(A) 0.191

(B) 0.318

(C) 0.409

(D) 0.600

(E) 0.727

262. You are given:

(i) T_x and T_y are independent.

(ii) The survival function for (x) follows $l_x = 100(95 - x), 0 \le x \le 95$.

(iii) The survival function for (y) is based on a constant force of mortality, $\mu_{y+t} = \mu, t \ge 0$.

(iv) $n < 95 - x$

Determine the probability that (x) dies within n years and also dies before (y).

(A) $\dfrac{e^{-\mu n}}{95 - x}$

(B) $\dfrac{ne^{-\mu n}}{95 - x}$

(C) $\dfrac{1 - e^{-\mu n}}{\mu(95 - x)}$

(D) $\dfrac{1 - e^{-\mu n}}{95 - x}$

(E) $1 - e^{-\mu n} + \dfrac{e^{-\mu n}}{95 - x}$

263. For (30) and (40), you are given:

(i) Their future lifetimes are independent.

(ii) Deaths of (30) and (40) are uniformly distributed over each year of age.

(iii) $q_{30} = 0.4$

(iv) $q_{40} = 0.6$

Calculate ${}_{0.25}q^{\,2}_{30.5:40.5}$.

(A) 0.0134

(B) 0.0166

(C) 0.0221

(D) 0.0275

(E) 0.0300

264. Removed

265. You are given:

(i) (x) and (y) are independent lives.

(ii) $\mu_{x+t} = 5t,\ t \geq 0$ is the force of mortality for (x).

(iii) $\mu_{y+t} = 1t,\ t \geq 0$ is the force of mortality for (y).

Calculate $q^{1}_{x:y}$.

(A) 0.16

(B) 0.24

(C) 0.39

(D) 0.79

(E) 0.83

266. For (80) and (85) with independent future lifetimes, you are given:

(i) Mortality follows ${}_t p_0 = 1 - \frac{t}{110}$, $0 \le t \le 110$.

(ii) G is the probability that (80) dies after (85) and before 5 years from now.

(iii) H is the probability that the first death occurs after 5 and before 10 years from now.

Calculate $G + H$.

(A) 0.25

(B) 0.28

(C) 0.33

(D) 0.38

(E) 0.41

267. You are given:

(i) $\mu_x = \sqrt{\frac{1}{80-x}},\ 0 \le x < 80$

(ii) F is the exact value of $S_0(10.5)$.

(iii) G is the value of $S_0(10.5)$ using the constant force assumption for interpolation between ages 10 and 11.

Calculate $F - G$.

(A) -0.01083

(B) -0.00005

(C) 0

(D) 0.00003

(E) 0.00172

268. Z is the present value random variable for an insurance on the lives of Bill and John. This insurance provides the following benefits:

(1) 500 at the moment of Bill's death if John is alive at that time; and

(2) 1000 at the moment of John's death if Bill is dead at that time.

You are given:

(i) Bill's survival function follows $l_x = 100(85 - x),\ 0 \le x \le 85$.

(ii) John's survival function follows $l_x = 100(84 - x),\ 0 \le x \le 84$

(iii) Bill and John are both age 80.

(iv) Bill and John have independent future lifetimes.

(v) i = 0.

Calculate E[Z].

(A) 600

(B) 650

(C) 700

(D) 750

(E) 800

269-273. Use the following information for questions 269-273. You are given:

(i) (30) and (50) have independent future lifetimes, each subject to a constant force of mortality equal to 0.05.

(ii) $\delta = 0.03$

269. Calculate ${}_{10}q_{\overline{30:50}}$.

(A) 0.155

(B) 0.368

(C) 0.424

(D) 0.632

(E) 0.845

270. Calculate $\mathring{e}_{\overline{30:50}}$.

(A) 10

(B) 20

(C) 30

(D) 40

(E) 50

271. Calculate $\bar{A}^{1}_{30:\overline{50}|}$.

(A) 0.23

(B) 0.38

(C) 0.51

(D) 0.64

(E) 0.77

272. Calculate $Var[T_{30:50}]$.

(A) 50

(B) 100

(C) 150

(D) 200

(E) 400

273. Calculate $Cov[T_{30:50}, T_{\overline{30:50}}]$.

(A) 10

(B) 25

(C) 50

(D) 100

(E) 200

274-277. Use the following information for questions 274-277.

For a special fully discrete whole life insurance on (x), you are given:

(i) Deaths are uniformly distributed over each year of age.

(ii)

k	Benefit premium at beginning of year k	Death benefit at end of year k	Interest rate used during year k	q_{x+k-1}	Benefit reserve at end of year k
2	---	---	---	---	84
3	18	240	0.07	---	96
4	24	360	0.06	0.101	---

274. Calculate q_{x+2} .

(A) 0.046

(B) 0.051

(C) 0.055

(D) 0.084

(E) 0.091

275. Calculate the benefit reserve at the end of year 4.

(A) 101

(B) 102

(C) 103

(D) 104

(E) 105

276. Calculate ${}_{0.5}q_{x+3.5}$.

(A) 0.046

(B) 0.048

(C) 0.051

(D) 0.053

(E) 0.056

277. Calculate the benefit reserve at the end of 3.5 years.

(A) 100

(B) 104

(C) 107

(D) 109

(E) 112

278-282. Use the following information for questions 278-282.

A 30-year term insurance on Janet age 30 and Andre age 40 provides the following benefits:

- A death benefit of 140,000 if Janet dies before Andre and within 30 years.
- A death benefit of 180,000 if Andre dies before Janet and within 30 years.

You are given:

(i) Mortality for both Janet and Andre follows $l_x = 100 - x,\ 0 \le x \le 100$.

(ii) Their future lifetimes are independent.

(iii) $i = 0$

(iv) The death benefit is payable at the moment of the first death.

(v) Premiums are payable continuously at rate P while both are alive, for a maximum of 20 years.

278. Calculate the probability that at least one of Janet and Andre will die within 10 years.

(A) 1/42

(B) 1/12

(C) 1/7

(D) 2/7

(E) 13/42

279. Calculate ${}_{10}q^{2}_{30:40}$.

(A) 0.012

(B) 0.024

(C) 0.042

(D) 0.131

(E) 0.155

280. Calculate the probability that the second death occurs between times 10 and 20.

(A) 0.071

(B) 0.095

(C) 0.293

(D) 0.333

(E) 0.357

281. Calculate the expected present value at issue of the death benefits.

(A) 81,000

(B) 110,000

(C) 116,000

(D) 136,000

(E) 150,000

282. Calculate the expected present value at issue of premiums in terms of *P*.

(A) 11.2*P*

(B) 14.4*P*

(C) 16.9*P*

(D) 18.2*P*

(E) 19.3*P*

283. For a four-state model with states numbered 0, 1, 2, and 3, you are given:

(i) The only possible transitions are 0 to 1, 0 to 2, and 0 to 3.

(ii) $\mu_{x+t}^{01} = 0.3, t \geq 0$

(iii) $\mu_{x+t}^{02} = 0.5, t \geq 0$

(iv) $\mu_{x+t}^{03} = 0.7, t \geq 0$

Calculate p_x^{02}

(A) 0.26

(B) 0.30

(C) 0.33

(D) 0.36

(E) 0.39

284. John approximates values of $\ddot{a}_{80}^{(m)}$ using Woolhouse's formula with three terms. His results are:
$\ddot{a}_{80}^{(2)} = 8.29340$ and $\ddot{a}_{80}^{(4)} = 8.16715$.

Calculate $\ddot{a}_{80}^{(12)}$ using Woolhouse's formula with three terms and using the same mortality and interest rate assumptions as John.

(A) 8.12525

(B) 8.10415

(C) 8.08345

(D) 8.06275

(E) 8.04135

285. You are given:

(i) The force of mortality follows Makeham's law where $A = 0.00020$, $B = 0.000003$ and $c = 1.10000$.

(ii) The annual effective rate of interest is 5%.

Calculate ${}_{1|}a_{70:\overline{2}|}$.

(A) 1.73

(B) 1.76

(C) 1.79

(D) 1.82

(E) 1.85

286. You are given:

(i) The force of mortality follows Gompertz's law with $B = 0.000005$ and $c = 1.2$.

(ii) The annual effective rate of interest is 3%.

Calculate $A^{1}_{50:\overline{2}|}$.

(A) 0.1024

(B) 0.1018

(C) 0.1009

(D) 0.1000

(E) 0.0994

287. For a special 3-year term life insurance on (50), you are given:

(i) The death benefit of 10,000 is paid at the end of the year of death.

(ii) The annual effective rate of interest is 4%.

(iii) The benefit premium in each of years 1 and 2 is one-half the benefit premium in year 3.

(iv) Benefit premiums are calculated using the equivalence principle.

(v) The mortality table has the following values:

x	q_x
50	0.05
51	0.06
52	0.07
53	0.08

Calculate the benefit reserve at the end of year 2.

(A) 673.08

(B) 102.28

(C) 0.98

(D) -102.28

(E) -204.12

288. For a special 3-year term life insurance on (50), you are given:

(i) The death benefit of 10,000 is paid at the end of the year of death.
(ii) The annual effective rate of interest is 4%.
(iii) The benefit premium in year 1 is $10,000A^{1}_{50:\overline{1}|}$.
(iv) The benefit premiums in years 2 and 3 are equal.
(v) The mortality table has the following values:

x	q_x
50	0.05
51	0.06
52	0.07
53	0.08

Calculate the benefit reserve at the end of year 2.

(A) 0

(B) 48.56

(C) 50.51

(D) 52.52

(E) 53.16

289. For a 3-year term insurance on (60), you are given:

(i) The death benefit is 1,000,000.

(ii) The death benefit is payable at the end of the year of death.

(iii) $q_{60+t} = 0.014 + 0.001t$

(iv) Cash flows are accumulated at annual effective rate of interest of 0.06.

(v) The annual gross premium is 14,500.

(vi) Expenses prior to issue are 1000 and are paid at time 0.

(vii) Expenses after issue are 100 payable immediately after the receipt of each gross premium.

(viii) The reserve is 700 at the end of the first and second years.

(ix) Profits are discounted at annual effective rate of interest of 0.10.

Calculate the expected present value at issue of profits of the policy.

(A) -155

(B) -174

(C) -177

(D) -187

(E) -216

290. For a 10-year term life insurance (60), you are given:

(i) Mortality follows the Illustrative Life Table
(ii) Annual lapse rate is 0.05
(iii) The expected profit at the end of each year given that the insurance is in force at the beginning of the year:

Time in years	Profit
0	-700
1	180
2	130
3	130
4	135
5	135
6	140
7	140
8	140
9	135
10	130

(iv) Profits are discounted at an annual effective rate of 0.10.

Calculate the expected present value of future profits for a policy that is still in force immediately after the 7^{th} year end.

(A) 285

(B) 300

(C) 315

(D) 330

(E) 345

291. For a special term life insurance on (40) you are given:

(i) If the policyholder is diagnosed with a specified critical illness (SCI), a benefit of 50,000 is paid at the end of the month of diagnosis with the remaining 150,000 paid at the end of the month of death upon subsequent death.
(ii) If the policyholder dies without being diagnosed with a specified critical illness (SCI) a benefit of 200,000 is paid at the end of the month of death.
(iii) Premium is 700 per month payable at the beginning of each month.
(iv) Expenses are 10 per month payable at the beginning of each month.
(v) $i = 0.06$

The insurer profit tests the insurance using monthly time steps, and using a multiple state model with three states:

0 = Healthy (no SCI diagnosis); 1 = Diagnosed with a SCI, alive; 2 = Dead
and transition probabilities: ${}_{1/12}p_{41}^{00} = 0.9965$, ${}_{1/12}p_{41}^{01} = 0.0015$, ${}_{1/12}p_{41}^{02} = 0.0020$.

You are also given:

(i) Reserve at start of the 13th month: 6,000
(ii) Reserve at end of the 13th month: 6,200 in state 0, 15,000 in state 1

Calculate the expected profit for the 13th month, given that the policyholder is healthy at the start of the month.

(A) 32

(B) 47

(C) 69

(D) 77

(E) 96

292. For a fully discrete 3-year term life insurance policy on (40) you are given:

(i) All cash flows are annual.

(ii) The annual gross premium is 1000.

(iii) Profits and premiums are discounted at an annual effective rate of 0.12.

(iv) The expected profit at the end of each year given that the insurance is in force at the beginning of the year:

Time in years	Profit
0	-400
1	150
2	274
3	395

(v) The expected profit at the end of each year given that the insurance is in force at age 40:

Time in years	Profit
0	-400
1	150
2	245
3	300

Calculate the profit margin.

(A) 4.9%

(B) 5.3%

(C) 5.9%

(D) 6.6%

(E) 9.7%

293. For a fully discrete 3-year term life insurance policy on (60) you are given:

(i) The death benefit is 100,000.
(ii) Mortality follows the Illustrative Life Table.
(iii) The rate of interest is based on the yield curve at t = 0.

You are also given the following information about zero coupon bonds based on the yield curve at $t = 0$:

Years to Maturity	Price of 100 Bond
1	97.00
2	92.00
3	87.00

Calculate the benefit premium.

(A) 1410

(B) 1432

(C) 1455

(D) 1478

(E) 1500

294. An insurer issues a number of identical special 1-year term life insurance policies.

Each policy has a death benefit of 1000 payable at the end of the year of death, on condition that:

(i) The policyholder dies during the year; and

(ii) A stock index ends the year below its value at the start of the year.

Both conditions must be satisfied for the death benefit to be paid.

You are given:

(i) Future lifetimes of the policyholders are independent

(ii) $q_x = 0.05$ for all x.

(iii) The probability that the stock index ends the year below its value at the start of the year is 0.1 for all years.

(iv) Future lifetimes of the policyholders and the value of the stock index are independent.

(v) The annual effective rate of interest rate is 3%.

X_{10} denotes the total of the present value of benefits at issue for 10 policies.
X_N denotes the total present value of benefits for N policies.

Calculate $\dfrac{\sqrt{Var(X_{10})}}{10} - \lim_{N\to\infty} \dfrac{\sqrt{Var(X_N)}}{N}$.

(A) 11.1

(B) 16.3

(C) 21.2

(D) 25.7

(E) 31.4

295. An employee aged exactly 62 on January 1, 2010 has an annual salary rate of 100,000 on that date.

Salaries are revised annually on December 31 each year.
Future salaries are estimated using the salary scale given in the table below, where $S_y / S_x, y > x$ denotes the ratio of salary earned in the year of age from y to y+1 to the salary earned in the year of age x to x+1, for a life in employment over the entire period (x, y+1).

x	S_x
62	3.589
63	3.643
64	3.698
65	3.751

The multiple decrement table below models exits from employment:

(i) $d_x^{(1)}$ denotes retirements.

(ii) $d_x^{(2)}$ denotes deaths in employment.

(iii)There are no other modes of exit.

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
62	52,860	5,068	213
63	47,579	4,560	214
64	42,805	4,102	215
65	38,488	38,488	-

The employee has insurance that pays a death benefit equals to 4 times his salary at death if death occurs while employed and prior to age 65; and pays 0 otherwise. The death benefit is payable at moment of death. Assume deaths occur at mid-year.

The annual effective rate of interest is 0.05.

Calculate the actuarial present value of the death benefit.

(A) 4,389

(B) 4,414

(C) 4,472

(D) 4,518

(E) 4,585

296. For two universal life insurance policies issued on (60), you are given:

(i) Policy 1 has a level death benefit of 100,000.
(ii) Policy 2 has a death benefit equal to 100,000 plus the account value at the end of the month of death.

For each policy:
(i) Death benefits are paid at the end of the month of death.
(ii) Account values are calculated monthly.
(iii) Level monthly premiums of G are payable at the beginning of each month.
(iv) Mortality rates for calculating the cost of insurance:
   a. Follow the Illustrative Life Table.
   b. Assume UDD for fractional ages.
(v) Interest is credited at a monthly effective rate of 0.004.
(vi) The interest rate used for accumulating and discounting in the cost of insurance calculation is a monthly effective rate of 0.004.
(vii) Level expense charges of E are deducted at the beginning of each month.

At the end of the 36th month the account value for Policy 1 equals the account value for Policy 2.

Calculate the ratio of the account value for Policy 1 at the end of the 37th month to the account value of Policy 2 at the end of the 37th month.

(A) 1.0015

(B) 1.0035

(C) 1.0055

(D) 1.0075

(E) 1.0095

297. For a universal life insurance on (50) you are given:

(i) The death benefit is 100,000.
(ii) Death benefits are paid at the end of the year of death if (50) dies prior to age 70.
(iii) The account value is calculated annually.
(iv) Level annual premiums are payable at the beginning of each year.
(v) Mortality rates for calculating the cost of insurance follow the Illustrative Life Table.
(vi) Interest is credited at an annual effective rate of 0.06.
(vii) The interest rate used for accumulating and discounting in the cost of insurance calculation is an annual effective rate of 0.06.
(viii) Expense deductions are:
- 50 at the beginning of each year; and
- 5% of each annual contribution.

Calculate the level annual premium that results in an account value of 0 at the end of the 20^{th} year.

(A) 1155

(B) 1205

(C) 1212

(D) 1218

(E) 1268

298. For a fully discrete 3-year term life insurance on (50) you are given:

(i) The death benefit is 5000.

(ii) An extract from a mortality table

x	q_x
50	0.005
51	0.006
52	0.007

(iii) The rate of interest is based on the yield curve at $t = 0$.

You are also given the following information based on the yield curve at $t = 0$:

t	Annual forward rate of interest
0	0.030
1	0.032
2	0.035

Calculate the second moment of the present value of the death benefit random variable.

(A) 392,000

(B) 406,000

(C) 419,000

(D) 432,000

(E) 446,000

299. For a special 20-year term life insurance on (40), you are given:

(i) The death benefit is 10,000.
(ii) The death benefit is payable at the moment of death.
(iii) During the 5th year the gross premium is 150 paid continuously at a constant rate
(iv) The force of mortality follows Gompertz's law with $B = 0.00004$ and $c = 1.1$
(v) The force of interest is 4%.
(vi) Expenses are:
- 5% of premium payable continuously
- 100 payable at the moment of death

(vii) At the end of the 5th year the expected value of the present value of future losses random variable is 1000.

Euler's method with steps of $h = 0.25$ years is used to calculate a numerical solution to Thiele's differential equation.

Calculate the expected value of the present value of future losses random variable at the end of 4.5 years.

(A) 975

(B) 962

(C) 949

(D) 936

(E) 923

300. Removed

301. For a universal life insurance policy on (70), you are given:

(i) The death benefit payable at the end of year 10 is the greater of 100,000 and 1.3 times the account value at the end of year 10.

(ii) $q_{79}^{(\text{death})} = 0.01$; $q_{79}^{(\text{withdrawal})} = 0.03$

(iii) The withdrawal benefit is the account value less a surrender charge of 1000.

(iv) A premium of 9000 and expenses of 900 were paid at the beginning of year 10.

(v) $i = 0.08$ is the earned interest rate in year 10.

(vi) The account value at the end of year 10 is 85,000.

(vii) ${}_9AS$, the asset share at the end of year 9, was 75,000.

Calculate ${}_{10}AS$, the asset share at the end of year 10.

(A) 85,700

(B) 86,700

(C) 87,700

(D) 88,700

(E) 89,700

302. For a fully discrete whole life insurance of 1000 on (70), you are given:

(i) The withdrawal benefit in year 10 is 110.
(ii) The gross annual premium is 16.
(iii) Expenses are incurred at the beginning of the year.
(iv) Withdrawals occur at the end of the year.
(v) 1000 such policies are in force at the beginning of year 10.
(vi)

	Anticipated experience	Actual experience
Mortality (d)	$q'^{(d)}_{79} = 0.01$	15 deaths
Withdrawal (w)	$q'^{(w)}_{79} = 0.10$	100 withdrawals
Interest	$i = 0.06$	$i = 0.05$
Expenses	3 per policy	5 per policy

(vii) Reserves are gross premium reserves.
(viii) The gross premium reserve at the end of year 9 is 115.

You calculate the combined gain from mortality and withdrawals during year 10 before calculating the gain from interest and expenses.

Calculate the combined gain from mortality and withdrawals.

(A) -4,360

(B) -4,340

(C) -4,320

(D) -4,300

(E) -4,280

303. (65) purchases a whole life annuity that pays 1000 at the end of each year. You are given:

(i) The gross single premium is 15,000.

(ii) 1000 such policies are in force at the beginning of year 10.

(iii)

	Anticipated experience	Actual experience
Mortality	$q_{74} = 0.01$	12 deaths
Interest	$i = 0.06$	$i = 0.05$
Expense	50 per policy	60 per policy

(iv) Expenses are paid at the end of each year for any policyholder who does not die during the year.

(v) Reserves are gross premium reserves.

(vi) The reserve at the end of the ninth year is 10,994.49.

You calculate the gain from interest during year 10, with the gain from interest calculated prior to the calculation of gain from any other sources.

Calculate the gain from interest.

(A) -112,000

(B) -111,000

(C) -110,000

(D) -109,000

(E) -108,000

304. (65) purchases a whole life annuity that pays 1000 at the end of each year. You are given:

(i) The gross single premium is 15,000.

(ii) 1000 such policies are in force at the beginning of year 10.

(iii)

	Anticipated experience	Actual experience
Mortality	$q_{74} = 0.01$	12 deaths
Interest	$i = 0.06$	$i = 0.05$
Expense	50 per policy	60 per policy

(iv) Expenses are paid at the end of each year for any policyholder who does not die during the year.

(v) Reserves are gross premium reserves.

(vi) The reserve at the end of the ninth year is 10,994.49.

You calculate the gain from expenses during year 10, assuming the gains from interest has already been calculated and the gain from mortality is yet to be calculated.

Calculate the gain from expenses.

(A) -9,910

(B) -9,900

(C) -9,890

(D) -9,880

(E) -9,870

305. (65) purchases a whole life annuity that pays 1000 at the end of each year. You are given:

(i) The gross single premium is 15,000.

(ii) 1000 such policies are in force at the beginning of year 10.

(iii)

	Anticipated experience	Actual experience
Mortality	$q_{74} = 0.01$	12 deaths
Interest	$i = 0.06$	$i = 0.05$
Expense	50 per policy	60 per policy

(iv) Expenses are paid at the end of each year for any policyholder who does not die during the year.

(v) Reserves are gross premium reserves.

(vi) The reserve at the end of the ninth year is 10,994.49.

You calculate the gain from mortality during year 10, assuming that the gains from interest and expense have already been calculated.

Calculate the gain from mortality.

(A) 19,540

(B) 21,540

(C) 21,560

(D) 23,540

(E) 23,560

306. For a 5-year warranty on Kira's new cell phone, you are given:

(i) The warranty pays 100 at the moment of breakage, if the phone breaks. The warranty only pays for one breakage.

(ii) If the phone has not broken, the warranty pays 100 at the end of 5 years.

(iii) Premiums of G are payable continuously at an annual rate of 25 until the phone breaks.

(iv) The force of breakage for this phone is $\mu_t = 0.02t, t \geq 0$.

(v) $\delta = 0.05$

(vi) ${}_tV$ denotes the gross premium reserve at time t for this warranty.

(vii) At the end of year 4, Kira's cell phone has not broken.

(viii) You approximate ${}_4V$ using Euler's method, with step size h = 0.5 and using the derivatives of ${}_tV$ at times 4.0 and 4.5.

Calculate your approximation of ${}_4V$ using this methodology.

(A) 71.0

(B) 71.4

(C) 71.9

(D) 72.4

(E) 72.8

307. For a 5-year warranty on Kevin's new cell phone, you are given:

(i) The warranty pays 100 at the moment of breakage, if the phone breaks. The warranty only pays for one breakage.

(ii) If the phone has not broken, the warranty pays 100 at the end of 5 years.

(iii) Premiums of *G* are payable continuously at an annual rate of 25 until the phone breaks.

(iv) The force of breakage for this phone is $\mu_t = 0.02t, t \geq 0$.

(v) $\delta = 0.05$

(vi) ${}_tV$ denotes the gross premium reserve at time *t* for this warranty.

(vii) At the end of year 4, Kevin's cell phone has not broken.

(viii) You approximate ${}_4V$ using Euler's method, with step size *h* = 0.5 and using the derivatives of ${}_tV$ at times 4.5 and 5.0.

Calculate your approximation of ${}_4V$ using this methodology.

(A) 71.05

(B) 71.44

(C) 71.93

(D) 72.42

(E) 72.81

308. For a 4-year road hazard warranty on Elizabeth's new tire, you are given:

(i) The warranty pays $80(1-0.25t)$ at the moment of damage if the tire must be replaced. The factor of $0.25t$ reflects the decrease in value due to normal usage. The warranty only pays for one incident.

(ii) The force of damage requiring replacement is $\mu_t = 0.05 + 0.02t, t \geq 0$.

(iii) $\delta = 0.05$

You write the integral for the actuarial present value of the warranty in the form $\int_0^4 f(t)dt$ for an appropriate function $f(t)$.

Calculate $f(1)$.

(A) 3.76

(B) 3.78

(C) 3.80

(D) 3.82

(E) 3.84

309. For a special fully discrete 10-payment whole life insurance on (40), you are given:

(i) The death benefit in the first 10 years is the refund of all benefit premiums paid with interest at 6%.

(ii) The death benefit after 10 years is 1000.

(iii) Level benefit premiums are payable for 10 years.

(iv) Mortality follows the Illustrative Life Table.

(v) i = 0.06.

Calculate the benefit premium.

(A) 17.2

(B) 17.4

(C) 17.6

(D) 17.8

(E) 18.0