SPRING 2007
EXAM MLC SOLUTIONS

Question # 1
Key: E

$$p_{70} = \frac{{}_3p_{70}}{{}_2p_{71}} = \frac{0.95}{0.96} = 0.9896$$

$$ {}_4p_{71} = e^{-\int_{71}^{75} \mu_x\, dx} = e^{-0.107} = 0.8985$$

$$ {}_5p_{70} = 0.9896 \times 0.8985 = 0.889$$

Question # 2
Key: B

$$\bar{A}_x = \mu / (\mu + \delta) = \mu / (\mu + 0.08) = 0.3443$$

$$\mu = 0.042$$

$$ {}^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = 0.2079$$

$$Var\left(\bar{a}_{\overline{T|}}\right) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2} = \frac{0.2079 - 0.3443^2}{0.08^2}$$

$$= 13.962$$

Question # 3
Key: D

$$\ddot{a}_{[60]} = \left(1 - A_{[60]}\right)/d = \left(1 - 0.359\right)/\left(0.06/1.06\right) = 11.3243$$

$$1000P_{[60]} = 1000A_{[60]}/\ddot{a}_{[60]} = 359/11.3243 = 31.70$$

$$1000\,{}_5V_{[60]} = 1000A_{65} - 1000P_{[60]} \times \ddot{a}_{65}$$

$$= 439.80 - 31.70 \times 9.8969$$

$$= 126.06$$

Alternatively,

$$1000\ {}_5V_{[60]} = 1000\left(\frac{A_{65} - A_{[60]}}{1 - A_{[60]}}\right)$$

$$= 1000\left(\frac{0.4398 - 0.359}{1 - 0.359}\right)$$

$$= 126.05$$

Question # 4
Key: E

Let K = curtate future lifetime random variable

$$\sigma_L = \sqrt{1000\,Var(L)}$$

$$L = 150000v^{K+1} - 150000P_x\ddot{a}_{\overline{K+1}|}$$

$$= 150000\left[\left(1+\frac{P_x}{d}\right)v^{K+1} - \frac{P_x}{d}\right]$$

$$Var(L) = Var\left[150000\left[\left(1+\frac{P_x}{d}\right)v^{K+1} - \frac{P_x}{d}\right]\right]$$

$$= \left[150,000\left(1+\frac{P_x}{d}\right)\right]^2 Var\left(v^{K+1}\right)$$

because $Var(aX+b) = a^2\,Var(x)$ if a and b are constants

P_x is a constant, a numeric value, not a random variable.

There are various ways to evaluate $\left(1+\frac{P_x}{d}\right)$.

One possibility is

$$\left(1+\frac{P_x}{d}\right) = \left(1 - \frac{\left(\frac{1}{\ddot{a}_x} - d\right)}{d}\right)$$

$$= 1 - \frac{1}{d\ddot{a}_x} - 1$$

$$= \frac{1}{d\ddot{a}_x}$$

$$= \frac{1}{1-A_x} = \frac{1}{1-0.0653} = 1.0699$$

$$Var(L) = \left[150,000\left(1+\frac{P_x}{d}\right)\right]^2 Var\left(v^{K+1}\right)$$

$$= \left[(150,000)(1.0699)\right]^2 \left({}^2A_x - A_x^2\right)$$

$$= (25,755,400,000)\left(0.0143 - 0.0653^2\right)$$

$$= 258,478,900$$

Standard deviation $= \sqrt{} = 16,077$

Question # 5
Key: C

Number of claims in 2 months is Poisson with $\lambda = 2$

n	$P(N = n)$
0	0.1353
1	0.2707
2	0.2707

$$\Pr\left(N \ge 3\right) = 1 - 0.1353 - 0.2707 - 0.2707$$

$$= 0.3233$$

Question # 6
Key: A

Donations = compound Poisson $\lambda = 7 \times 10 \times 0.8 = 56$

Withdrawals = compound Poisson $\lambda = 7 \times 10 \times 0.2 = 14$

expected donations = $56 \times 15 = 840$ where 15 = mean
expected withdrawals = $14 \times 40 = 560$ where 40 = mean
expected net = $840 - 560 = 280$

Variance of donations $= 56 \times \left(75 + 15^2\right)$
$= 16{,}800$
Variance of withdrawals $= 14 \times \left(533 + 40^2\right)$
$= 29{,}862$
Variance of net change $= 16{,}800 + 29{,}862$
$= 46{,}662$

$$\Pr\left[S > 600\right] = \Pr\left[N\left(0,1\right) > \frac{\left(600 - 280\right)}{\sqrt{46{,}662}}\right] = \Pr\left[N\left(0,1\right) > 1.48 = 1 - 0.93 = 0.07\right.$$

Question # 7
Key: E

All have the same benefits, so retrospectively the one with the highest accumulated value of premiums wil have the highest reserve.

All have total premiums of 10, so the one with the premiums earliest will have the highest accumulated value. That is E.

Alternatively, all have the same benefits, so all have the same actuarial present value of premiums. E has the highest APV of the first 5 premiums, so it has the lowest APV (at both time 0 and time 5) of premiums for years 6 and later. Thus E has the highest prospective reserve.

Question #8
Key: E

Let ${}_t p_x$ = probability Kevin still there

${}_t p_y$ = probability Kira still there

Kira’s expected playing time = $\int_0^\infty \left(1 - {}_t p_x\right) {}_t p_y \, dt$

$$= \int_0^\infty \left(1 - e^{-0.7t}\right) e^{-0.6t} dt$$

$$= \int_0^\infty e^{-0.6t} dt - \int_0^\infty e^{-1.3t} dt$$

$$= \frac{1}{0.6} - \frac{0}{1.3} = 0.89744$$

Alternatively and equivalently $\mathring{e}_y - \mathring{e}_{xy}$

Alternatively, Kira has $\frac{7}{13}$ chance of “surviving” Kevin,

and (memoryless) $\frac{1}{0.6} = 1.667$ future lifetime then if she does

$\left(\frac{7}{13}\right)(1.667) = 0.89744$

Question # 9
Key: E

For decrement 1 in its associated single decrement table:

$${}_t p'^{(1)}_{25} = 1 - 0.1 \times t$$

Thus ${}_{0.2} p'^{(1)}_{25} = 0.98$; ${}_{0.6} p'^{(1)}_{25} = 0.94$; ${}_{0.4} p'^{(1)}_{25.2} = 0.94/0.98 = 0.9592$

For decrement 2 in its associated single decrement table:

No one age 25 dies before age 25.2.

$9\%\left(\frac{3}{4} \times 0.12\right)$ of the lives which reach 25.2 die at 25.2.

$3\%\left(\frac{1}{4} \times 0.12\right)$ of the lives age 25 die at 25.6.

Thus $0.03/(1-0.09) = 0.033$ is the probability a life which reaches 25.6 will die at 25.6.

For a life age 25 in the double decrement table:

- Probability of surviving until $t = 0.2$ is 0.98, because ${}_{0.2} p'^{(1)}_{25} = 0.98$ and no one dies from cause (2) before 0.2.
- Probability of dying at 25.2 from cause (2) is $0.98 \times 0.09 = 0.0882$, since 9% of those who reach 25.2 die then.
- Probability of surviving beyond $25.2 = 0.98 - 0.0882 = 0.8918$
- Probability of surviving to $t = 0.6 = {}_{0.2} p^{(\tau)}_{25} \times {}_{0.4} p'^{(1)}_{25.2}$, since no one dies from cause (2) between $t = 0.2$ and $t = 0.6$; probability $= 0.8918 \times 0.9592 = 0.8554$
- Probability of dying at 25.6 from cause (2) is $0.8554 \times 0.033 = 0.0282$, since 3.3% of those who reach 25.6 die then.

Total probability of dying from cause (2) $= 0.0882 + 0.0282$
$= 0.1164$

Question # 10
Key: C

Let A be values at 6% all years.
Let A* be values at 10% for one year, then 6%.

$$A_{66}^{*} = vq_{66} + vp_{66}A_{67}$$
$$0.300 = \frac{0.012}{1.1} + \frac{0.988A_{67}}{1.1}$$
$$(0.988)A_{67} = 0.318$$
$$A_{66} = \frac{0.012}{1.06} + \frac{(0.988)(A_{67})}{1.06} = 0.311$$
$$A_{65}^{*} = \frac{0.010}{1.1} + \frac{(0.99)(0.311)}{1.1} = 0.289$$

Question #11
Key: A

$$G\ddot{a}_{40:\overline{10}|} = 1000A^{1}_{40:\overline{20}|} + 0.04G\ddot{a}_{40:\overline{10}|} + \underbrace{0.25G + 0.05Ga_{40:\overline{9}|}}_{0.20G+0.05G\ddot{a}_{40:\overline{10}|}} + \underbrace{10 + 5a_{40:\overline{19}|}}_{5+5\ddot{a}_{40:\overline{20}|}}$$

$$G = \frac{1000A^{1}_{40:20} + 10 + 5a_{40:\overline{19}|}}{0.91\ddot{a}_{40:\overline{10}|} - 0.2}$$
$$\Rightarrow \quad = \frac{1000\left(A_{40} - {}_{20}E_{40}A_{60}\right) + 10 + 5\left(\ddot{a}_{40} - {}_{20}E_{40}\ddot{a}_{60}\right)}{0.91\left(\ddot{a}_{40} - {}_{10}E_{40}\,\ddot{a}_{50}\right) - 0.2}$$
$$= \frac{1000\left[0.16132 - 0.27414(0.36913)\right] + 5 + 5\left[14.8166 - 0.27414(11.1454)\right]}{0.91\left(14.8166 - 0.53667(13.2668)\right) - 0.2}$$
$$= \frac{60.127 + 5 + 58.806}{6.8040} = 18.21$$

Question # 12
Key: D

Possible outcomes:

Result	$_1L$	Probability
K(55) = 1, J = 1	$_1L = 2000v - 50 = 1836.79$	0.005
K(55) = 1, J = 2	$_1L = 1000v - 50 = 893.40$	0.04
K(55) = 1, J = 1	$_1L = 2000v^2 - 50\ddot{a}_{\overline{2}|} = 1682.82$	0.955(0.008) = 0.00764
K(55) = 2, J= 2	$_1L = 1000v^2 - 50\ddot{a}_{\overline{2}|} = 792.83$	0.955(0.06) = 0.0573
K(55) >2	$_1L = \quad -50\ddot{a}_{\overline{2}|} = -97.17$	0.955(0.932) = 0.89006

F(-97.17) = 0.89006
F(792.83) = 0.89006 + 0.0573 = 0.94736
F(893.40) = 0.94736 + 0.04 = 0.98736 which is > 95%

Question # 13
Key: C

$$v = \frac{1}{1.1}$$

The shortest solution uses Hattendorf. Here's one Hattendorf solution.

$Var\left({}_3L \middle| K(55) \geq 3\right) = 0$ since no coverage there.

$$Var\left({}_2L \middle| K(55) \geq 2\right) = \left[v\left(b_3 - {}_3V\right)\right]^2 \; p_{57}q_{57} + v^2 p_{57} \; Var\left({}_3L \middle| K(55) \geq 3\right)$$
$$= \left[v(1000 - 0)\right]^2 (0.5)(0.5) + v^2 (0.5)(0)$$
$$= 206{,}612$$

$$Var\left({}_1L \middle| K(55) \geq 1\right) = \left[v\left(b_2 - {}_2V\right)\right]^2 \; p_{56}q_{56} + v^2 p_{56} \; Var\left({}_2L \middle| K(55) \geq 2\right)$$
$$= \left[v(1000 - 120.833)\right]^2 (0.6)(0.4) + \left(v^2\right)(0.6)(206{,}612)$$
$$= 153{,}309 + 102{,}447$$
$$= 255{,}756$$

Without Hattendorf, you need to use the general formula for variance of any random variable, applied here to the contingent distribution:

$$Var\left({}_1L \middle| K(x) \geq 1\right) = E\left[\left({}_1L\right)^2 \middle| K(x) \geq 1\right] - \left[E\left({}_1L \middle| K(x) \geq 1\right)\right]^2$$

Before we can evaluate ${}_1L$ for any outcome, we must know the benefit premium *P*.

The shortest way is to calculate P from the benefit reserves.
E.g., $120.833 = 1000 \times 0.5v - P$; $P = 333.712$

Or (much more time consuming), you could calculate P as the actuarial present value of benefits divided by the actuarial present value of an annuity-due.

t	q_x	p_x	${}_t p_x$	$\ddot{a}_{x:\overline{3}\rvert} = \sum_{t=0}^{2} v^t \times {}_t p_x$	$A^1_{x:\overline{3}\rvert} = \sum_{t=0}^{2} v^{t+1} \times {}_{t\rvert}q_x$
0	0.3	0.7	1	1	0.272727
1	0.4	0.6	0.7	0.636364	0.231405
2	0.5	0.5	0.42	0.347107	0.157776
				$\sum \Rightarrow 1.983471$	$\sum \Rightarrow 0.661908$

$$\Rightarrow 1000 P^1_{x:\overline{3}\rvert} = \frac{1000(0.66198)}{1.983471} = 333.71212$$

Whichever way you solved for P, you can now finish the variance

K	${}_1L$	Conditional Prob	$({}_1L)^2$
1	$\frac{1000}{1.1} - P = 575.38$	0.4	331,062
2	$\frac{1000}{(1.1)^2} - P\left[1 + \frac{1}{1.1}\right] = 189.36$	0.3	35,857
≥ 3	$-P\left[1 + \frac{1}{1.1}\right] = -637.09$	0.3	405,884

$E\left({}_1L \middle| K(55) \geq 1\right) = 95.833$ (the benefit reserve)

$$E\left(({}_1L)^2 \middle| K(55) \geq 1\right) = (0.4)(331{,}062) + (0.3)(35{,}857) + (0.3)(405{,}884)$$
$$= 264{,}947$$

$$Var\left({}_1L \middle| K(55) \geq 1\right) = 264{,}947 - 95.833^2$$
$$= 255{,}763$$

Question # 14
Key: E

The intent of this problem, not the same as the wording used on the exam, is that the future lifetimes of (30) and (35) are independent and follow the same mortality table.

$${}_5p_{30} = a$$

$${}_\infty q^{1}_{30:35} = b$$

$$\Pr\left[(30)\text{dies within 5 years after (35)}\right] = \int_{t=0}^{\omega-35}\left({}_tp_{35}\,\mu_{35}^{(t)}\right)\left[{}_tp_{30} - {}_5p_{30}\times{}_tp_{35}\right]dt$$

$$= \int {}_tp_{30}\times{}_tp_{35}\times\mu_{35}(t)\,dt - {}_5p_{30}\int {}_tp_{35}\times{}_tp_{35}\times\mu_{35}(t)\,dt$$

$$= {}_\infty q_{30:\overset{1}{35}} - a\;{}_\infty q_{35:\overset{1}{35}}$$

$$= \left(1 - {}_\infty q^{1}_{30:35}\right) - a\frac{1}{2} \text{ (equally likely to die first since same age)}$$

$$= (1-b) - \frac{1}{2}a$$

Question # 15
Key: D

Present Value of Future Benefits

Year Benefit Paid	Pattern Necessary	Probability
Yr 1	$1 \to 2$	0.3
Yr 2	$1 \to 1 \to 2$	(0.6)(0.4)
Yr 3	$1 \to 1 \to 1 \to 2$	(0.6)(0.4)(0.1)

$$\text{PVFB} = 0.3(0.8)(1000) + 0.24(0.8)^2(1000) + 0.024(0.8)^3(1000)$$
$$= 405.888$$

Present Value of Future Premiums

Year Premium Paid	Pattern Necessary	Probability
Yr 1		P
Yr 2	$1 \to 1$	$(0.6)\times P$
Yr 3	$1 \to 1 \to 1$	$(0.6)(0.4)P$

$$PVFP = P + 0.8(0.6P) + (0.8)^2(0.24P)$$
$$= 1.6336\,P$$

$$PVFB = PVFP$$
$$405.888 = 1.6336\,P$$
$$P = 248.46$$

Question # 16
Key: B

Find 1, then find 2	(0.6)(0.3) = 0.18
Find 2, then find 1	0.3×0.5 = 0.15
Find 2, then find 2	0.3×0.4 = 0.12

Total = 0.18 + 0.15 + 0.12 = 0.45

Question # 17
Key: D

$$Q = \begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix} \qquad Q^2 = \begin{pmatrix} 0.78 & \dots \\ 0.77 & \dots \end{pmatrix} \qquad Q^3 = \begin{pmatrix} 0.778 & \dots \\ \dots & \dots \end{pmatrix}$$

PV of option (i) $= 100\left(0.8v + 0.78v^2 + 0.778v^3\right) = 218.20$

PV of option (ii) $= 0.8^3 v^3 R = 0.4552R$

So $R = 218.20 / 0.4552 = 479$

Question # 18
Key: C

$$q_{[x+1]} = \frac{3}{4}\, q_{[x]+1}$$

$$q_{[x+1]+1} = \frac{4}{5}\, q_{x+2}$$

$$l_{[67]} = \frac{l_{69}}{{}_2 p_{[67]}} = \frac{7700}{{}_2 p_{[67]}}$$

$${}_2 p_{[67]} = p_{[67]} \cdot p_{[67]+1} = \left(1 - q_{[67]}\right)\left(1 - q_{[67]+1}\right)$$

$$q_{[67]+1} = \frac{4}{5} q_{68} = \frac{4}{5}\left[\frac{8000 - 7700}{8000}\right] = \frac{3}{100}$$

$$q_{[67]} = \frac{3}{4} q_{[66]+1} = \frac{3}{4} \cdot \frac{4}{5} q_{67} = \frac{3}{4} \cdot \frac{4}{5}\left[\frac{8200 - 8000}{8200}\right] = \frac{3}{205}$$

$$\Rightarrow {}_2 p_{[67]} = \left(1 - \frac{3}{205}\right)\left(1 - \frac{3}{100}\right) = .955805$$

$$\Rightarrow l_{[67]} = \frac{7700}{.955805} = 8056$$

Question # 19
Key: D

$$ {}_{10}V_{40} = {}_{13}V_{40} \Rightarrow 1-\frac{\ddot{a}_{50}}{\ddot{a}_{40}} = 1-\frac{\ddot{a}_{53}}{\ddot{a}_{40}} \Rightarrow \ddot{a}_{50} = \ddot{a}_{53} $$

$$ \ddot{a}_{50} = 1+vp_{50}+v^2\,{}_2p_{50}+v^3\,{}_3p_{50}\,\ddot{a}_{53} $$

$$ = 1+vp_{50}+(vp_{50})^2+(vp_{50})^3\,\ddot{a}_{50} \qquad * $$

$$ 10 = \ddot{a}_{50} = \frac{1+vp_{50}+(vp_{50})^2}{1-(vp_{50})^3} $$

Numerator is a geometric series, so

$$ 10 = \frac{\dfrac{1-(vp_{50})^3}{1-vp_{50}}}{1-(vp_{50})^3} = \frac{1}{1-vp_{50}} $$

$$ 1-vp_{50} = \frac{1}{10} $$

$$ p_{50} = 0.9\times 1.06 = 0.954 $$

*In a multiple choice test with numeric answers, you could have tried each of the answer choices and discovered 0.954 works.

Alternatively, consider the recursive reserve formula:

$$ {}_nV = \frac{({}_{n-1}V+\pi)(1+i)-1000q}{1-q} $$

The only difference in the recursion for ${}_{11}V_{40}$, ${}_{12}V_{40}$ and ${}_{13}V_{40}$ is the initial reserve.

<u>If</u> ${}_{11}V_{40}$ were larger than ${}_{10}V_{40}$, then ${}_{12}V_{40}$ would be even larger, and ${}_{13}V_{40}$ even larger still.

<u>If</u> ${}_{11}V_{40}$ were smaller than ${}_{10}V_{40}$, then ${}_{12}V_{40}$ would be even smaller, and ${}_{13}V_{40}$ even smaller still.

Therefore ${}_{10}V_{40} = {}_{11}V_{40} = {}_{12}V_{40} = {}_{13}V_{40}$

$$ 1-\frac{\ddot{a}_{50}}{\ddot{a}_{40}} = 1-\frac{\ddot{a}_{51}}{\ddot{a}_{40}} $$

$$ \ddot{a}_{50} = \ddot{a}_{51} $$

$$ \ddot{a}_{50} = 1+vp_{50}\,\ddot{a}_{51} = 1+vp_{50}\,\ddot{a}_{50} $$

$$ 10 = 1+10vp_{50} $$

$$ p_{50} = \frac{0.9}{v} = 0.9\times 1.06 = 0.954 $$

Question # 20
Key: C

Let $\mu^{(2)}$ be the force of all non-crash deaths.
$\mu^{(2)} = \mu^{(\tau)} - \mu^{(1)} = 0.001 - 0.0002 = 0.0008$
In year 1, $\mu_{D:A} = 0.0008 + 0.0008 + 0.0002 = 0.0018$
In year 2, $\mu_{D:A} = 0.001 + 0.001 = 0.002$
${}_2 p_{D:A} = e^{-0.0018} e^{-0.002} = 0.9962$

Question # 21
Key: B

For a model of the form $l_x = (\omega - x)^\alpha$ we have $\mathring{e}_x = (\omega - x)/(\alpha + 1)$

This can be verified by integration.

For the De Moivre model $\mathring{e}_x = (\omega - x)/2$.

Consequently $4/3 = [(\omega - 30)/(\alpha + 1)]/[(\omega - 30)/2] = 2/[\alpha + 1] \Rightarrow \alpha = 0.5$
$20 = (\omega - 60)/(3/2) \Rightarrow \omega = 90$
Hence $(\omega - 70)/2 = 10$

Question # 22
Key: C

In first 10 years, $\Pr(Z > 700) = \Pr\left(1000\,e^{-0.1T} > 700\right)$

$$= \Pr(-0.1T > \ln 0.7)$$
$$= \Pr(T < 3.57)$$

After 10 years $\Pr(Z > 700) = \Pr\left(2500e^{-0.1T} > 700\right)$

$$= \Pr(-0.1T > \ln 0.28)$$
$$= \Pr(T < 12.73)$$

$$\Pr(Z > 700) = {}_{3.57}q_{40} + {}_{10|2.73}q_{40}$$
$$= 3.57 \times \frac{1}{60} + 2.73 \times \frac{1}{60} = 0.105$$

Question # 23
Key: B

$$p_x^{(\tau)} = \left(1 - q_x'^{(1)}\right)e^{-\mu_x^{(2)}} = (1-0.1)\left(e^{-0.20}\right) = 0.7369$$

$$\mu_{x+1}^{(2)} = -\ln\left(1 - q_{x+1}'^{(2)}\right) = -\ln(0.75) = 0.2877$$

$$q_{x+1}^{(2)} = \int_0^1 {}_t p_{x+1}^{(\tau)}\ \mu_{x+1}^{(2)}\ dt$$

$$= \int_0^1 e^{-\left[\mu_{x+1}^{(1)} + \mu_{x+1}^{(2)}\right]} \mu_{x+1}^{(2)}\ dt$$

$$= \int_0^1 e^{-(0.15+0.2877)t} \times 0.2877 dt$$

$$= \frac{-0.2877}{0.4377}\left(e^{-0.4377t}\right)\Big|_0^1$$

$$= \frac{0.2877}{0.4377}\left(1 - e^{-0.4377}\right)$$

$$= 0.233$$

$${}_{1|1}q_x^{(2)} = p_x^{(\tau)} q_{x+1}^{(2)} = (0.7369)(0.233) = 0.172$$

Question # 24
Key: A

$$S(x) = e^{-\int_0^x \mu(t)dt}$$

$$S(75) = e^{-0.01(75)^{1.2}} = 0.16888$$

$$S(76) = e^{-0.01(76)^{1.2}} = 0.16413$$

$$S(77) = e^{-0.01(77)^{1.2}} = 0.15951$$

$$100\ddot{a}_{75:\overline{3}|} = 100\left(1 + \frac{0.16413}{0.16888} \times \frac{1}{1.11} + \frac{0.15951}{0.16888} \times \frac{1}{1.11^2}\right)$$
$$= 264.21$$

Alternatively, $p_{75} = e^{\int_{75}^{76} \mu(t)dt} = e^{-0.01t^{1.2}} \Big|_{75}^{76}$
$$= e^{-0.028495}$$
$$= 0.9719$$

(same as $\frac{0.16413}{0.16888}$ similarly for ${}_2p_{75}$)

Question # 25
Key: B

$$m(25) = \int_0^{25} \lambda(t)\,dt = \int_0^{10} 0.05dt + \int_{10}^{20} \frac{t}{200}dt + \int_{20}^{25} 0.1dt$$

$$= 0.05 \times 10 + \frac{t^2}{400}\Big|_{10}^{20} + 0.1 \times 5 = 1.75$$

N is Poisson with mean 1.75

$$p(4) = e^{-1.75} \times 1.75^4 / 4!$$
$$p(4) = 0.068$$

Question # 26
Key: A

There are two ways to attack this problem.

One approach focuses on the distribution of *T*. *T* is the sum of 289 independent exponential random variables, each with mean $\frac{1}{4}$ and therefore variance $\left(\frac{1}{4}\right)^2$.

$$E(T) = 289 \times \frac{1}{4} = 72.25$$

$$Var(T) = 289 \times \left(\frac{1}{4}\right)^2 = 18.0625$$

Using normal approximation

$$\Pr(T > 68) = \Pr\left(\frac{T - 72.25}{\sqrt{18.0625}} > \frac{68 - 72.25}{\sqrt{18.0625}} = 1\right)$$

$$1 - \Phi(-1) = 0.84$$

The other approach looks at the distribution of *N*, the number of cell divisions that occur in 68 days. *N* is Poisson with mean = variance $= 4 \times 68 = 272$.

$$\Pr(T > 68) = \Pr(N < 289) = \Pr(N < 288.5)$$

$$= \Pr\left(\frac{N - 272}{\sqrt{272}} < \frac{288.5 - 272}{\sqrt{272}} = 1.00046\right)$$

$$\Phi(1.00046) = 0.84$$

In evalutaing the normal approximation to *N*, we needed a continuity correction since *N* has a discrete distribution. We didn't need one in evaluating the normal approximation to *T*, a continuous distribution.

Question # 27
Key: A

$$E(Z)=\int_0^{\infty} e^{-t}e^{-\delta t}e^{-\mu t}\times\mu dt=\int_0^{\infty}\mu e^{-t(\mu+\delta+1)}dt$$

$$=-\frac{\mu}{\mu+\delta+1}e^{-t(\mu+\delta+1)}\Big|_0^{\infty}=\frac{\mu}{\mu+\delta+1}\Rightarrow\frac{\mu}{1.06+\mu}=0.03636 \qquad \mu=0.04$$

$$\text{E}\left(Z^2\right)=\int_0^{\infty} e^{-2t}e^{-2\delta}e^{-\mu t}\times\mu dt=\frac{\mu}{\mu+2\delta+2}\Rightarrow\frac{0.04}{0.04+0.12+2}=0.0185$$

$$Var(Z)=0.0185-(0.03636)^2=0.0172$$

Question # 28
Key: C

$${}_{16}AS=\frac{\left[1150+90(1-0.05)\right](1.08)-0.004(10,000)-0.05\,{}_{16}CV}{1-0.004-0.05}$$

$$1320=\frac{1334.34-40-0.05\,{}_{16}CV}{0.946}$$

$$0.05\,{}_{16}CV=1294.34-(0.946)(1320)$$

$${}_{16}CV=912.40$$

Question # 29
Key: A

For an annuity benefit of 1:

$$\text{Single Benefit Premium } (NSP) = {}_{30|}\ddot{a}_{30} + NSP\ A^{1}_{30:\overline{30}|}$$
$$= {}_{10}E_{30}\ {}_{20}E_{40}\ \ddot{a}_{60} + NSP\left(A_{30} - {}_{10}E_{30}\ {}_{20}E_{40}\ A_{60}\right)$$

$$=(0.54733)(0.27414)(11.1454)+NSP\left[0.10248-(0.54733)(0.27414)(0.36913)\right]$$

$$=1.67231-0.04709NSP$$

$$NSP=\frac{1.67231}{1-0.04709}=1.755$$

For benefit of 200, $1.755\times 200=351$

Question # 30
Key: D

$$\bar{a}_x=\frac{1}{\delta+\mu^{(\tau)}}$$
$$\bar{A}_x=\int_0^\infty 3e^{-\delta t}e^{-\left(\mu^{(1)}+\mu^{(2)}\right)}\mu^{(1)}dt+\int_0^\infty 1e^{-\delta t}e^{-\left(\mu^{(1)}+\mu^{(2)}\right)}\mu^{(2)}dt$$
$$=\frac{3\mu^{(1)}}{\delta+\mu^{(\tau)}}+\frac{\mu^{(2)}}{\delta+\mu^{(\tau)}}=\frac{3(0.02)+0.04}{\delta+\mu^{(\tau)}}=\frac{0.10}{\delta+\mu^{(\tau)}}$$

$$\bar{P}\left(\bar{A}_x\right)=\frac{\dfrac{0.10}{\delta+\mu^{(\tau)}}}{\dfrac{1}{\delta+\mu^{(\tau)}}}=0.10$$