Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities (ALDA)

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Abstract

This paper explores the financial properties of a concept product called an advanced-life delayed annuity (ALDA). The ALDA is a variant of a pure deferred annuity contract that is acquired by installments, adjusted for consumer price inflation, and pays off toward the end of the human life cycle. The ALDA concept is aimed at the growing population of North Americans without access to a traditional defined benefit (DB) pension plan and the implicit longevity insurance that a DB plan contains. I show that under quite reasonable pricing assumptions, a consumer can invest or allocate $1 per month, while saving for retirement, and receive between $20 and $40 per month in benefits, assuming the deductible in this insurance policy is set high enough. The ALDA concept might go a long way in mitigating the psychological barrier to voluntary lump-sum annuitization.

1. Introduction

There appears to be universal agreement among financial economists and pension actuaries about the substantial social welfare benefits from payout (or immediate) annuity contracts. But the public and media have yet to embrace this risk-management instrument as being equally important as a well-diversified retirement portfolio of stocks and bonds. Part of the low acceptance and take-up rates for annuitization can be attributed to the fact that a strong consensus has yet to emerge regarding the optimal age at which to annuitize as well as the optimal design of the ideal payout annuity. Indeed, the global trend away from defined benefit (DB) and toward defined contribution (DC) pension plans, in conjunction with exceptionally low levels of voluntary annuitization, cry out for a new way—or revisiting old ways—of thinking about the provision of lifetime retirement income.

This paper promotes, advocates, and explores the financial risk-and-return properties of a *concept product* called an advanced-life delayed annuity (ALDA), which is a variant of a *pure* deferred annuity contract that is paid by installments, linked to consumer price inflation, while locking in longevity insurance. Reduced to its essence, the ALDA would be acquired at a young age—and small premiums would be paid over a long period of time—but the ALDA would begin paying an inflation-adjusted life-contingent income only at the advanced age of 80, 85, or even 90. The product would contain zero cash value and no survival or estate benefits and could not be commuted for cash at any age. Of course, these stringent (no forfeiture benefits) design requirements might be impossible to attain, given the current regulatory environment. But, in theory, these features, combined with standard actuarial, interest, and (possibly) lapsation discounting, would reduce the ongoing premium for this insurance to mere cents on the dollar. The ALDA and its derivatives would be a close relative to a DB pension and are intended for those who don't have one, possibly even as an option within a DC (or 401k)–style pension.

From a slightly different perspective, this type of product is akin to buying auto, home, or health

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insurance with a large deductible, which is also the optimal strategy—and common practice—when dealing with catastrophic risk. By analogy, the ALDA's longevity insurance would kick in only once the longevity risk became substantial and financially unsupportable. Indeed, the raison d'être of life-contingent annuities is the acquisition of mortality credits, which at advanced ages are substantial and unavailable from any competing asset class. During the early years of retirement—when most pension decisions are made—the magnitude of these credits is quite small once survivor benefits, insurance fees, and antiselection (i.e., annuitant versus population) costs are included. In contrast, the ALDA would entitle the holder to insurance against the risk of outliving assets, but only when the assets actually run the risk of being depleted, which is later in life.

As King Solomon said, ''there is nothing new under the sun,'' and the intellectual origins of this particular idea can be traced to a 25-year-old article in the *Journal of Risk and Insurance* by Stephenson (1978). It has doubtlessly been toyed with, contemplated, and possibly even designed by many pension actuaries ever since. Currently, though, these products are unavailable on a stand-alone basis, although a number of companies in the United States—such as MetLife and GenWorth—have recently started offering variants of ALDA. Stephenson criticized existing annuity products in the marketplace and argued in favor of adopting designs that contain high ratios of ''protection to investment.'' He developed a concept called the *index of protection* and demonstrated that properly designed deferred annuities could provide greater inflation-protected value to consumers. This paper continues that line of thought by arguing that the fairly low actuarial premium for providing longevity insurance, together with well-known facts about individual behavior, make a compelling case for offering—and perhaps even imposing the purchase of—ALDAs in DC pension plans, as a substitute for a DB pension.

From a microeconomic (consumer welfare) framework, the ALDA would transform the portfolio choice and asset-allocation problem from a stochastic date of death to a deterministic one in which the terminal horizon becomes the annuity's payment commencement date. From a practical point of view, retirees no longer would have to worry about the ''risk'' of outliving their assets. They should be secure in the knowledge that if and when they reach an extreme (to be defined) age, their longevity insurance would begin. In fact, this might create interesting incentive effects in their own right.

I believe that the ALDA is preferable to a pure endowment policy that would (mature and) pay a lump sum at age 80, 85, or 90 since it would continue to provide periodic lifetime income regardless of how long the annuitant lived beyond the endowed age. This continual flow of income would serve as a better hedge for the household's liabilities and would reduce the (psychological, behavioral) risk of misspending the lump sum once it became available. Indeed, a pure endowment's embedded longevity insurance is terminated as soon as the policy matures. And it is highly unlikely the policyholder would (again) annuitize the lump sum at age 80, 85, or 90 for the same behavioral reasons individuals are reluctant to annuitize at retirement. The main thesis underpinning this paper is that engaging in irreversible financial transactions—that is, annuitization—involving large sums of money will *never* be appealing to individuals regardless of (whether they grasp) the importance of longevity insurance.

The remainder of the paper is organized as follows. Section 2 provides a brief review of the academic literature on the gains from annuitization and discusses some of the empirical evidence regarding the aversion to voluntary annuitization. Section 3 is the theoretical core of the paper; it describes the financial, economic, and actuarial properties as well as the different design possibilities for the ALDA contract. Section 4 discusses some related annuity products that recently have been made available to the public and describes an attempt by the author to get the ALDA introduced by a large insurance company in Canada. Section 5 discusses some of the issues surrounding longevity risk management, and Section 6 concludes the paper with some general comments.

2. GAINS FROM ANNUITIZATION

The industry as well as scholars in the field are aware of—and continue to puzzle over—the extremely low levels of voluntary annuitization exhibited among new and elderly retirees. From a theoretical perspective, this aversion to annuitization is inconsistent with a standard Modigliani life-cycle model of savings and consumption as described by Yaari (1965). In a life-cycle model with no bequest motives, Yaari demonstrated that all consumers hold actuarial notes (a.k.a. life contingent annuities) as opposed to liquid and marketable assets. This implies that when given the chance, retirees should convert their liquid assets into lifetime payout annuities that provide longevity insurance and protection against outliving one's money. The rationale behind Yaari's results is that returns from actuarial notes (life annuities) dominate all other assets, since the *living* inherit the assets of the *deceased.* Moreover, having access to life annuities enables the rational consumer to better smooth his or her planned consumption over the life cycle and hence gain greater lifetime utility of wealth. And, although some might react to the failure of this utility-based argument to sway retirees, by dismissing the economic theory itself, there are a number of more intuitive ways to explain the benefits of annuitization.

A simple example can help convert the problem into the language of investments. Suppose, according to population mortality tables, that there is a 20% chance that a 95-year-old female will die during the next year and before she reaches her 96th birthday. If 1,000 such females enter into a one-year term-life annuity (a.k.a. tontine) agreement by investing $100 each in a pool yielding 5%, the funds will grow to $105,000 by the end of the year. Of the initial 1,000 females, 800 are expected to survive, with a rather small variance around the expected value, leaving an average of $105,000/800 = $131.25 per survivor. This is a total return of 31.25%. This quite obviously far exceeds the interest rate (or investment return) of 5% used to ''store'' the funds, because the annuitants have seceded control of assets in the event of death.

The powerful algebra of longevity credits can be stated symbolically as follows: If r denotes the effective interest rate per year and if p is the probability of survival per year, then the return for the survivors from the one-year annuity is expected to be $(1 + r)/p - 1 > r$. The expectation will become reality as long as the group of annuitants participating in this risk-mitigating scheme is large enough. The gap between the one-year returns to the survivors and the interest rate is the so-called mortality credits. Table 1 illustrates some numerical values for these credits at different ages, using a unisex annuitant mortality table.

To put these numbers in perspective, a (unisex) 85-year-old who decides *not* to annuitize and instead *take his or her chances* investing in traditional (nonmortality contingent) asset classes and finance consumption from discretionary wealth would have to earn 725 basis points (which is 7.25%) above the risk-free rate of 6% during the next year, in order to be as well-off as someone who decided to annuitize at age 85. Think of this (6% + 7.25% =) 13.25% number as a hurdle rate that must be earned by the self-annuitizer to keep up with the ''annuitizer.'' At age 90 this hurdle rate increases to 18.56%, which becomes virtually unachievable using any conventional investment products. Of course, different interest rates and mortality tables will lead to different numerical results, but the order of magnitude is always the same. *At advanced ages nothing beats the implied yield from a payout annuity.*

As many actuaries understand and appreciate, the risk-sharing principle of a tontine is in fact the concept underlying all immediate annuities, and all pension plans for that matter. In practice, the risk-sharing agreement is made over a series of years, as opposed to just one. Think of it as *term* longevity insurance versus *whole-life* longevity insurance. The mechanics remain the same, and the survivors derive a higher return—which is then amortized over one's life—compared to placing the funds in a conventional (nonmortality-contingent) asset. See the paper by Milevsky

Table 1
Investment Benefits from Annuitization Assuming 40/60 Male/Female Split for Annuity 2000 Table under 6% Interest

Age	Mortality Credits (b.p.)
55	35
60	52
65	83
70	138
75	237
80	414
85	725
90	1256

(2005) for an in-depth discussion of how to calculate the implied longevity yield—akin to the above mortality credits—from a multiple-period payout annuity versus a one-period tontine.

Also, while the example assumes that r is fixed, in theory, the exact same principle applies with a variable investment return. In fact, the ex post returns might be even higher. For example, the 1,000 females, who are 95 years old, can invest their $100 in a stock mutual fund that earns the random return R. They do not know in advance what the fund/pool will earn. At the end of the year the annuitants will learn (or realize) their investment returns, and then split the gains among the surviving pool. Moreover, in the event the investment earns a negative return—and loses money—the participants will share in the losses as well, but the effect will be mitigated by the mortality credits. Algebraically, the expected return will be the same $(1 + R)/p - 1 > R$. In fact, this concept is the foundation of a *variable* immediate annuity. See Milevsky (2006) for more detailed information about the mechanics of variable immediate annuities, which are also referred to as variable payout or index-linked annuities.

In practice, most insurance companies go one step further than the above (participating annuity) example and actually *guarantee* that the annuitant will receive the mortality credit enhancements even if the mortality experience of the participants is better than expected. In other words, in the above-mentioned example for fixed annuities, with an expected 20% mortality rate, the insurance company would guarantee that all survivors receive 31.25% on their money, regardless of whether or not 20% of the group died during the year. See Poterba (1997) for a history of the development of the variable annuity in the United States.

Nevertheless, despite the highly appealing arguments in favor of annuitization there is little evidence that retirees are voluntarily embracing this arrangement. Modigliani (1986), Friedman and Warshawsky (1990), and Brown (2001), among others, have carefully documented that very few people consciously choose to annuitize their marketable wealth. In the comprehensive Health and Retirement Survey (HRS) conducted in the United States, only 1.57% of the HRS respondents reported annuity income. Likewise, only 8.0% of respondents with a DC pension plan selected an annuity payout. The Society of Actuaries and LIMRA, as reported in Sondergeld (1997), conducted a study showing that only 0.3% of variable annuity contracts were annuitized during the 1992–94 period. According to the National Association of Variable Annuities, of the $909 billion invested in variable annuities during 2003, only 2% were annuitized. It remains to be seen whether the high take-up rates the industry has experienced lately for guaranteed minimum income benefits (GMIBs)—which are additional riders available on variable annuities—will translate into increased annuitization activity in the future.

Indeed, the frustratingly low appreciation of the welfare benefits of annuitization has led some researchers to advocate mandatory annuitization for a fraction of discretionary pension savings that benefit from income-tax sheltered growth. See, for example, Bateman, Kingston, and Piggott (2001).

In the face of poor empirical evidence, various theories have been proposed to salvage this aspect of the life-cycle hypothesis and to justify the low demand for longevity insurance. For example, in one of the earlier papers on this ‘‘puzzle,’’ Kotlikoff and Spivak (1981) argued that family-risk pooling may be preferred to public annuity markets, especially given the presence of adverse selection and transaction costs. Indeed, a married couple functions as a mini-annuity market, as elaborated by Brown and Poterba (2000). Friedman and Warshawsky (1990) showed that average yields on individual life annuities during the late 1970s and early 1980s were lower than plausible alternative investments. The reduced yield was largely attributed to actuarial loads and profits, which have declined over time, according to work by Mitchell et al. (1999).

Bernheim (1991) argues that large preexisting annuities in the form of Social Security and government pensions might serve as an additional deterrent to voluntary annuitization. In a distinct line of reasoning, Yagi and Nishigaki (1993) argue that the actual design of annuities impedes full annuitization. One cannot obtain a life annuity that provides arbitrary payments contingent on survival, which is dictated by Yaari's (1965) model. They must be either fixed (in nominal or real terms) or variable (linked to an index). This constraint forces consumers to hold both marketable wealth and annuities.

In summary, many explanations exist for why people do *not* annuitize further wealth. Although these justifications have explanatory power, they fail to provide financial advice on optimal product design as well as normative strategies for the elderly. Furthermore, they cannot account for the casual observation that most people shun life annuities simply because they want to maintain control of their assets.

This paper takes the approach that consumers will remain reluctant to annuitize a large lump sum at retirement—regardless of *if and when* academics manage to solve the so-called annuity puzzle. What is needed is to accept that a sudden irreversible transaction will never be popular, especially when the underlying funds were under complete discretionary control of the annuitant up to the point of retirement. The alternative is perhaps slow annuitization over a very long period of time[1] or the gradual purchase of longevity insurance that start providing income only at any advanced age. The ALDA could be offered as an additional rider on existing saving and insurance products or could be sold as a stand-alone product. The critical factor would be to take the edge off a daunting and irreversible annuitization decision.

3. PRICING THE ALDA

Although the pricing and valuation of life annuities are the bread and butter of the pension actuary, this section will briefly review the mathematics, if only to increase the accessibility and reach of this article.

With some slight abuse of actuarial notation—and my apology to the profession—I start by letting $\bar{a}_{x:u}$ denote the continuous time *annuity factor at age x*. In my simple model it represents the price, value, and cost of acquiring a financial contract that pays an inflation-adjusted life-contingent \$1 per annum from time zero (i.e., age x) to time u (i.e., age $x + u$). Implicit in the expression is a real interest rate (or curve) denoted by r. The retirement or pension income flow is adjusted for realized inflation each and every year. Thus, in nominal terms, the life annuity initially pays \$1 per annum and then increases by the realized rate of the consumer price index. When the pricing or valuation rate r is constant (i.e., no term structure effects) the expression for the annuity factor reduces to the familiar

$$\bar{a}_{x:u} := \int_0^u e^{-rs}({}_sp_x)\, ds, \qquad (3.1)$$

where $({}_sp_x)$ denotes the conditional survival probability, also in continuous time. Without any loss of generality, I will suppress the symbol $u = \infty$ and use $\bar{a}_x$ when I am dealing with a *complete life* annuity that pays until death. In this paper and the subsequent numerical examples, the ALDA purchase age will range from $x = 35$ to 45, while the ALDA commencement age will range from age $x + u = 70$ to 90.

By construction, the net single premium (NSP) at age $x < (x + u)$ for a \$1 per annum ALDA benefit is the annuity factor in equation (3.1) discounted for the probability of survival and the time value of money. Mathematically,

$$\text{NSP} = e^{-r(u)}\bar{a}_{x+u}({}_up_x), \qquad (3.2)$$

where the first term captures the u years of interest, the second term represents the annuity factor that commences at age $x + u$, and the third term is the conditional probability that someone currently aged x will survive for u more years. Note that equation (3.2) is consistent with the idea that there are no benefits provided to beneficiaries in the event the primary annuitant dies between the initial acquisition age x and the benefit commencement age $x + u$. Adding a survivorship benefit would increase the NSP and reduce the appeal of the product from a risk management perspective. This idea was also stressed by Stephenson (1978). Note that some of the ALDA-like products that have recently been created by insurance companies for the 401k (DC pension) market contain survivorship benefits and cashable options—for example, the ability to sell the units at some commuted value—that completely eliminate the mortality credits during the accumulation phase.

[1] This idea—which is a form of dollar cost averaging into annuities—has also been adopted by Principal Financial Group in their recently launched Income IRA program, in which tax sheltered funds are transferred over a 10–15 year period from equity-based investments to fixed payout annuities.

Table 2
Theoretical Net Single Premium for Advanced Life Delayed Annuity

Purchase Age (x)	Annuity Commencement Age			
	$x + u = 70$	$x + u = 75$	$x + u = 80$	$x + u = 85$
$r = 3.25\%$ (Real) Pricing Rate				
35	\$3.642	\$2.376	\$1.412	\$0.731
40	4.294	2.802	1.665	0.861
45	5.070	3.308	1.965	1.017
$r = 2\%$ (Real) Pricing Rate				
35	6.346	4.325	2.687	1.456
40	7.029	4.790	2.976	1.612
45	7.796	5.313	3.301	1.788
$r = 1\%$ (Real) Pricing Rate				
35	9.951	7.013	4.509	2.532
40	10.484	7.388	4.750	2.667
45	11.061	7.795	5.012	2.814

Note the focus on real (after inflation) versus nominal returns in the pricing and valuation of the annuity factor. The interest rate r is used in two places in equation (3.2). The first is to discount a single cash flow prior to the annuity commencement date—which covers the next u years—and the second is to price the annuity and discount the repeated cash flows that occur after age $x + u$. Thus, in practice one could envision using slightly different interest rates during the deferral period versus the payout period. Indeed, as I alluded to earlier, one could go a step further and use a real yield curve r_t—implied perhaps from REAL TIPS—as opposed to a single interest rate, which would conform to capital market pricing techniques.

To provide some numerical intuition for the NSP of the ALDA, I offer the following example under a continuous Gompertz approximation to discrete mortality. This law of mortality has been used extensively in the pricing of ''academic'' life annuities—see, for example, Frees, Carrière, and Valdez (1996)—and is a reasonable approximation for the purposes of the exposition in this paper. Recall that under a Gompertz law of mortality, the (natural logarithm) of the conditional survival probability is defined equal to

$$\ln({}_sp_x) = e^{\left(\frac{x-m}{b}\right)\left(1-\frac{s}{b}\right)}, \tag{3.3}$$

where m and b are the ''modal'' and ''scale'' parameters of the remaining lifetime distribution. Thus, for example, if we start (i.e., purchase the ALDA in one lump sum) at age $x = 35$, with benefits commencing at age $x + u = 85$ under Gompertz parameters of $m = 90$ and $b = 9.5$, and a real interest rate of $r = 3.25\%$, the NSP from equation (3.2) is \$0.731 in current dollars. This *pure* deferred lifetime annuity will pay \$1 in inflation-adjusted terms each year, commencing at age 85, in exchange for a premium payment of less than \$1 today. The \$0.731 came about from multiplying the age 85 annuity factor of $\bar{a}_{85} = 6.679$ by the 0.556 probability of survival to age 85 and then by the 0.1969 time-value-of-money factor.

Using the formula, Table 2 displays the NSP of a *unisex* annuity purchase age (x) and a variety of annuity commencement ages ($x + u$) under the same Gompertz approximation to mortality and a variety of different real interest rate assumptions r.

For reference purposes, the assumed life expectancy was 84.7, 84.8, and 84.9 at ages 35, 40, and 45, respectively. Likewise, the implied life expectancy at the annuity commencement age was 87.6, 88.9, 90.7, and 92.9 at ages 70, 75, 80, and 85, respectively. No improvement factors or any other dynamic projection methodologies were used to generate these (illustrative) numbers. The above calculation is trivial from an actuarial point of view since this type of ALDA, that is, one that is paid by a lump sum up front, is a well-known deferred annuity. I now proceed to computing the periodic premium for the ALDA, which eventually involves some subtle assumptions about lapsation behavior.

Payment for the ALDA will not be made in one lump sum; rather, the annuitant makes a series of real (after inflation) nonrefundable and non-cashable payments between the ages of x and $x + u$, which would then entitle him or her to a real \$1 per annum for life, commencing at age $x + u$. In practice, this would be implemented by linking both the periodic premiums and the benefits to the same consumer price index so that all cash flows could be discounted using the same unit of account. I emphasize that the pure actuarial pricing of this product would *not* require any assumptions about future inflation or nominal rates. The *premiums* would be variable in nominal terms, but fixed in real terms. Likewise, the *benefits* would be variable in nominal terms, but fixed in real terms. From a purely financial economic perspective, the lack of any asset-liability mismatch between the units of account should not require any additional reserves or capital requirements. Of course, the current regulatory environment might impede this theoretical invariance and further increase the cost of the product. A full discussion of these important yet complex issues would take us well beyond the scope of this brief article.

In either event, the NSP must be actuarially amortized over the next u years, contingent on survival. Using our previous notation and assuming no lapsation, the net periodic premium (NPP) for the ALDA is

$$\text{NPP} = \frac{e^{-r(u)}\bar{a}_{x+u}({}_u p_x)}{\bar{a}_{x:u}}, \qquad (3.4)$$

where the numerator is the NSP and the denominator effectively spreads these payments over the u years between the initial purchase age and the ALDA commencement period. Equation (3.4) is the standard method of converting single premiums into periodic life-contingent premiums. Note that the annuity factor in the denominator is subscripted by the purchase age x, while the factor in the numerator is subscripted by the commencement age $x + u$. Intuitively, for any given purchase age x, the longer the deferral period u, the greater the annuity factor $\bar{a}_{x:u}$, and the lower the ongoing periodic premium. Similarly, as emphasized in the earlier discussion, it is quite conceivable that the pricing interest rate r in the denominator's factor will differ from (be greater than) the pricing rate in the numerator's factor. This is because a non-flat-yield curve in practice will result in different (constant) interest rate approximations, depending on the period that is being discounted. Regardless, they are both real (after inflation) rates.

Here are some examples under the same pricing conditions that I considered in the NSP case. When the initial purchase age is $x = 35$ and the annuity commencement age is $x + u = 85$, then under an $r = 3.25\%$ real interest rate, the NPP needed to create a \$1 per annum real lifetime annuity is precisely \$0.0312 per annum. In other words, a mere three cents on the dollar per annum—paid over a period of 50 years—will generate an income flow of \$1 per year for life. This is a factor of 32 times the ongoing premium. I can scale this quantity up (or down) and declare that, for each \$100 of inflation-adjusted premium per week, month, or year, the ALDA will pay an inflation-adjusted pension of \$3,200 per week, month, or year. If instead of using ages 35 and 85, I use ages 40 and 80, while retaining the same interest rate of $r = 3.25\%$, the NPP becomes \$0.0779, which is a factor of 12.8 times the ongoing premium. Finally, if I increase the interest rate to 4%, the NPP becomes \$0.061, which is a factor of 16.2. Table 3 converts the NSP numbers in Table 2 into payout factors that are the reciprocal of the NPP.

Table 3 also includes the extreme case in which the commencement age is 90. In this case, a 35-year-old, for example, would receive \$77.7 real dollars starting at age 90 for each real dollar paid from age 35 under an $r = 3.5\%$ pricing rate. The number would drop by more than half to \$32.5 real dollars per year for life under a lower $r = 1\%$ pricing rate. Thus, with yields on inflation-protected zero-coupon bonds (a.k.a. TIPS) in the 2–2.5% vicinity at the time of writing, one would be expected to see ''real'' prices for ALDA in the marketplace somewhere between the lower and upper bounds of 3.5% and 1% provided in the table. Of course, whether or not a 35-year-old would actually persevere and pay premiums for 55 years is debatable, which leads us to the topic of lapsation, which I will return to later.

Table 3
Theoretical ALDA Income Payout Factors
Lifetime Retirement Income per Premium Dollar

Purchase Age (x)	Annuity Commencement Age				
	$x + u = 70$	$x + u = 75$	$x + u = 80$	$x + u = 85$	$x + u = 90$
$r = 3.25\%$ Pricing Rate					
35	5.6	9.2	16.1	32.0	77.7
40	4.4	7.2	12.8	25.7	62.6
45	3.3	5.6	10.1	20.4	49.9
$r = 2\%$ Pricing Rate					
35	3.9	6.2	10.5	20.2	47.3
40	3.1	5.1	8.7	17.0	39.9
45	2.4	4.1	7.1	14.0	33.2
$r = 1\%$ Pricing Rate					
35	2.9	4.5	7.6	14.3	32.5
40	2.4	3.8	6.5	12.4	28.3
45	1.9	3.2	5.5	10.5	24.3

3.1 Who Takes the Mortality and Interest Rate Risk?

The above description and pricing mechanics are predicated on the ability of the insurance company to guarantee the pricing rate and the mortality table. In practice, if the insurance company offering the ALDA were to earn less than the pricing rate, and/or experience mortality that was worse than assumed, the company would obviously face the potential of severe losses. This raises the question of whether the ALDA should have a participating structure in which a minimal income payout factor would be guaranteed, and then depending on investment performance and mortality experience, the income would be increased. Indeed, this kind of arrangement—which involves an additional level of risk sharing—is at the heart of some products that have recently been introduced in the North American marketplace. Thus, for example, a commercially viable version of the ALDA would guarantee an implicit real rate of *at least* 2% applied to the Annuity 2000 mortality table, and then, depending on future financial and economic conditions, the benefits could be ratcheted up (increased) on a periodic basis. The extent to which this minimum guarantee is calibrated would depend on a number of factors including the ability of the insurance company to hedge part of its mortality risk (i.e., the risk of underestimating longevity) using life and health insurance products in their portfolio with the opposite exposure. I will return to this issue in Section 5.

3.2 Lapsation Considerations

Although everyone who purchases (or starts) an ALDA likely has the full intention of holding the product to maturity, it is unreasonable to assume that 100% of all survivors will continue to pay premiums until the commencement date. In fact, if the product is structured with absolutely no cash value and/or no ability to scale down the income benefit by reducing premiums, there is a high probability that people will lapse the product prior to the benefit commencement age. Therefore, the lapsation phenomena might be taken into account in the original design. From a pricing perspective, one can assume the existence of an instantaneous lapse-rate curve, which is akin to a force of mortality, which determines the probability that the contract will be lapsed as a function of the number of years since initiation. This curve will most likely start at a level close to zero and then increase as time goes on, but will start to decline again as the ALDA nears the commencement date. The psychological justification would be that on an aggregate level as individuals ''see'' the payoff horizon approaching, they are likely to reduce the rate at which they become disillusioned with the product.

Lapse-adjusted pricing would impact the NPP of the ALDA in two partially offsetting ways. First, it would reduce the numerator in equation (3.4) by virtue of the smaller number of people who will utilize the product, but it would also reduce the denominator of equation (3.4) by virtue of the reduced size of the group who will actually cover the actuarial present value of the ALDA benefit. It is relatively easy to prove—and is a textbook case in multiple decrement theory—that the net effect will be a total reduction in the relevant NPP in equation (3.4), regardless of the precise shape of the lapsation curve. Indeed, for most reasonable specifications, the premiums will decline quite substantially.

One could envision a wide number of specifications, each leading to their own premiums. For illustrative purposes, Table 4 takes a simple approach and displays the relevant income payout factors assuming a constant 2% lapse rate each year. In other words, the difference between Tables 4 and 3 includes the assumption that each year 2% (in continuous time) of the ALDA population ceases to make payments, but for nonmortality-driven reasons. I emphasize again that this is a very crude approximation, and that actual lapsation behavior and intensity in such a product would depend on the number of years remaining until the product commencement date as well as number of health-related factors. Despite the simplicity, a number of interesting facts emerge from Table 4. Income multiples increase by a factor of two to three, but this impact is even further pronounced as the commencement date becomes later.

3.3 Scaling Down Benefits

If the insurance company is unwilling to price the product using a lapse curve (assumption), one could envision an ALDA design in which the premiums could be voluntarily stopped at some age z prior to age $x + u$. The benefit would then be reduced accordingly, perhaps with the same benefit commencement date, to avoid antiselection problems. The benefit would be scaled down by computing the *ex post* actuarial present value of the premiums at the lapse age z and then scaled into the original NSP to arrive at a fractional scaled-down percentage of the originally guaranteed payout factor from Table 2. There are a number of compelling reasons why this particular incarnation of the ALDA would be the most popular from a consumer standpoint, and I envision variants of this design as having the best chance of survival in the marketplace.

In sum, I have described and motivated the basic *actuarial chassis* of the ALDA product, which, despite its actuarial simplicity, contains a number of important economic benefits. The main features can be summarized as follows: (a) real inflation-adjusted benefits, (b) an annuity commencement date that is irreversible and well into the retirement years akin to a deductible on an insurance policy, together with (c) slow and

Table 4
Lapse-Adjusted ALDA Income Payout Factor
Lifetime Retirement Income per Premium Dollar

Purchase Age (x)	Annuity Commencement Age				
	$x + u = 70$	$x + u = 75$	$x + u = 80$	$x + u = 85$	$x + u = 90$
r = 3.25% Pricing Rate; 2% Lapse Rate					
35	8.7	15.3	29.2	63.4	168.4
40	6.3	11.2	21.6	47.0	125.3
45	4.4	8.1	15.7	34.5	92.3
r = 2% Pricing Rate; 2% Lapse Rate					
35	5.9	10.0	18.4	38.5	98.0
40	4.4	7.7	14.3	30.1	76.8
45	3.3	5.8	10.9	23.2	59.5
r = 1% Pricing Rate; 2% Lapse Rate					
35	4.3	7.2	12.9	26.2	64.8
40	3.4	5.7	10.4	21.3	52.7
45	2.6	4.4	8.2	17.0	42.4

Table 5
Actual Payout Factors: Male/Female Income per Premium Dollar Quoted by Large Insurance Company in Canada (October 2003) Assuming 3.25% Real (after Inflation) Pricing Interest Rate

Purchase Age	Annuity Commencement Age			
	70	75	80	85
35	5.13/4.47	8.16/6.91	13.82/11.31	25.90/20.53
40	4.08/3.54	6.65/5.60	11.51/9.36	22.08/17.35
45	3.15/2.73	5.30/4.44	9.42/7.61	18.54/14.45

prolonged premium payments that counteract the ingrained reluctance of consumers to annuitize in one lump sum.

4. Does This Product Exist Already?

The answer to this question is *yes, but* . . . Indeed, as mentioned earlier, a number of North American insurance companies have recently launched variants of ALDA under numerous guises and incarnations. In fact, some of the older long-term-care (LTC) insurance policies also had an element of the ALDA as part of their benefit structure. I refer the reader to a recent article in *Best's Review* (February 2004, pp. 70–74) for a review of the industry in the payout annuity market. For example, Prudential Financial and Genworth as well as MetLife and Principal Financial are just some of the named companies that are in the process of developing, or already offer, a financial vehicle that allows one to acquire lifetime income using a dollar-cost averaging strategy. And, although it is beyond the mandate of this paper to critique the merits and pitfalls of each, it seems the emphasis on real (after-inflation) income has been neglected by most of the current manufacturers. Furthermore, as I mentioned above, some of the inherent flexibility and choice embedded in these products may ''kill'' the mortality credits and detract from the ultimate objective, which is to *encourage annuitization at the lowest possible cost.*

On a more pessimistic note, it seems that industry innovation around retirement income (payout) products has been taking place for decades, but with very few noticeable successes. In the late 1980s, the IDS Life Insurance company in Minneapolis (an American Express company) offered a variant of the ALDA called IDS retirement assurance. Under this product, the annuity premiums were paid in one lump sum upon initiation, the deferral or delay period lasted for 30 or 40 years, and the benefit commenced at age 80. This product paid out in nominal terms, included a survivor and/or surrender benefit of premiums paid (without interest), and included a participating structure linked to mortality credits. The policy statement contained a fairly complicated schedule of mortality credits that would be added to the account on attaining certain ages. And, despite the differences with the ALDA product described above, this product did in fact come close to achieving the objectives of longevity insurance with a deductible. The sales literature created by IDS stated that ''this product is designed for your later retirement years, and does this at a cost that is far lower than conventional annuities.'' Unfortunately, despite the sound theoretical foundations, this product was a commercial failure, and the company withdrew sales soon after.

In the same spirit, in the lead-up to the writing of this paper, the author approached one of the largest insurance companies in Canada with a proposal to develop an ALDA product. The author also volunteered to be the first to purchase the product (at age 35) and assist in the public marketing and promotion campaign once the product was launched. Initially, there was much excitement about the concept, and the insurance company's actuaries produced the pricing schedule displayed in Table 5, the number of which are well within the range of the numbers presented in Table 3. In general, the payout multiples were lower than the numbers obtained using our theoretical model, although at higher ages the numbers do seem excessively lower than what theory would dictate. For example, an ALDA purchased at age $x = 35$, whose benefit would commence at age $x + u = 85$ would entitle a male to 25.9 times multiple and a female to a 20.5 multiple. Note that these are not model values but actual prices at which the insurance company was (initially) willing to sell the ALDA to the public.

Unfortunately, as the ALDA proposal made its way up the chain of command it encountered a number of institutional and regulatory obstacles, and finally the initiative was abandoned in early 2004. The general concerns offered by the company can be broadly categorized as follows:

Monthly or weekly premiums: When long-dated annuities are sold, these types of annuities are based on the payment of one single lump-sum premium. In the ALDA case, the (small) premiums would be paid monthly or weekly until the annuity commencement date. This is an administration limitation since most insurance company software systems are not currently set up to handle such a long period of premium collection, or determine the new premium each year based on the current inflation rates.

Delayed period: The delayed period is the period between the payment of premiums and the commencement of annuity payments. In this case the annuity payments commence at age 70 to 90, which results in a deferred period of up to 55 years. Currently, the maximum deferred period of any annuity product offered by insurance companies in North America is 30 years. Most ALDAs are over this limit, and thus very long horizons result in both pricing and administrative issues since the company must track the annuity for quite a long period, and finding matching long-term investment is unlikely.

Inflation indexing: The fact that the annuity in question is an inflation-indexed annuity causes additional complications. For these annuities, the usual deferred (or delay) period accepted is even shorter—10 years. Again, this is due to the availability of matching investments (vis-à-vis reserving requirements), which would be limited to real bonds or taking on the risk component of inflation predictions.

No death benefit: Although this is possible, it means that the annuitant can be paying premiums for up to age x minus one day, pass away, and receive nothing. After 40, 45, or 50 years of premium payments, the product provides no death benefit. Most insurance companies do not feel comfortable from a public relations (a.k.a. legal or possibly fiduciary) perspective offering such a product and go so far as to argue that it would have limited popularity in the general marketplace.

In sum, there seem to be a number of institutional and regulatory impediments to offering such long-dated inflation-adjusted products. Furthermore, even if these obstacles can be overcome in an economically viable manner, it remains to be seen whether there is a market for the ALDA. Quite likely, a costly and prolonged marketing effort—undertaken by the industry as a whole as opposed to a particular company—will be required to make this concept a commercial success. Corporate patience and long managerial horizons will be necessary, but not sufficient, for success in this market.

5. MORTALITY RISK CONSIDERATIONS

As mentioned earlier, the insurance company selling an ALDA would be taking a *long* position in mortality rates by fixing the life-contingent payments for up to half a century in advance. Indeed, if experienced mortality (hazard) rates were to decline to a level that is lower than what was priced in advance—that is, if people live longer than expected—the insurance company could be facing the potential for substantial losses. Thus, even if the pricing assumed a very conservative (real) interest rate, and even if the reinvestment risk were mitigated by hedging in the capital markets, it would be difficult if not impossible to do so with uncertain mortality rates.

In fact, this is not just a concern for ALDAs. Insurance companies and reinsurers alike are concerned about guaranteeing mortality on the sale of immediate (let alone delayed) annuities. This is due to the perceived risk that unknown (and nonquantifiable) medical discoveries might increase human lifespans beyond currently projected mortality tables, perhaps even leaving the insurance company paying annuities to infinitely lived Methuselahs. It is common to see insurance companies imposing an explicit mortality risk charge, on a perpetual asset basis, to cover this contingency when selling variable payout annuities.

And, although some actuaries and financial economists argue that in-force life insurance might serve as a hedge against this (diversifiable) risk, others are quick to dismiss the so-called basis risk implicit in this strategy since the target group for both class of policies is distinct. Immediate annuities are sold to the old, while life insurance is purchased by the young (for the most part). Thus, it is plausible that an increase in population longevity will adversely impact the liabilities of the annuity book of business, but marginally impact the profitability of the insurance book. Furthermore, another concern is that the duration and especially the lapsation behavior of the opposing liabilities are mismatched and hence cannot properly hedge each other. Thus, it is unclear to what extent one side of the business could offset the other, and I therefore leave this particular issue for further research.

Yet, oddly enough—and this is the point of the current section—ALDAs might not be terribly sensitive to changes (or misestimates) in mortality assumptions and hence might not pose as much longevity risk to the insurance company as one would expect a priori. Most actuaries will be familiar with the counterintuitive argument that a book of payout annuities sold to a 35-year-old is less exposed to mortality risk compared to selling payout annuities to a 75-year-old. The former's price or value is similar to that of a fixed-income perpetuity—where the annuity factor is: $\bar{a}_x \approx 1/r$—whereas the latter is closer to a medium-term bond. At early issue ages and for long deferral periods, the dominant concern is reinvestment and interest rate risk. The same is true for ALDAs, and I offer the following numerical example to illustrate this concept.

Assume that an insurance company has just sold an ALDA to a (unisex) 45-year-old, whose benefit pays an inflation-adjusted \$10,000 per year starting at age 90. Long-term interest rates in the market are 3% (real), and the insurance company prices the ALDA by subtracting off a spread or profit margin of 1 percentage point from the 3% to arrive at annual premium of \$301.47 per year for the next 45 years: that is, using equation (3.2), under the same Gompertz (without lapsation) parameters displayed in Tables 2 and 3, but under an adjusted $r = 2\%$ pricing rate.

Now, let us further assume that the insurance company misestimated mortality and, in fact, hazard rates decline by 20% more than anticipated. Or, stated differently, mortality improves by 20% more than what was projected at the time of sale. The 20% can be modeled as a shock to the instantaneous force of mortality (IFM) curve, one that immediately shifts the IFM from μ_x to a modified 0.8 μ_x at all ages. This might appear simplistic, but it has the desired effect. To put this in perspective, the shifting of the hazard rate curve translates the conditional probability of survival to age 90, from the assumed (${}_{45}p_{45} = 37.11\%$) to a realized (${}_{45}p_{45} = 45.25\%$), for an individual who is currently 45 years old. These numbers are obtained under the usual methods, by integrating only 80% of the IFM curve in equation (3.3), and then evaluating the integral between zero and the survival time and then raising to the exponent.

If we translate this into prices—under the same $r = 2\%$, which is 3% minus the 100 basis point spread—the insurance company *should have* charged a \$412.15 premium for the ALDA as opposed to the \$301.47 per year it is committed to. Stated differently, if we solve for the implied interest rate that equates the \$301.47 premium to the model price under the modified mortality curve 0.8 μ_x, the insurance company's 100 basis point profit spread is reduced to a mere 4.2 basis points. This should not come as a surprise since a 20% improvement in experienced mortality—a.k.a. reduction in hazard rates—will obviously reduce profits. Our model simply quantifies this intuition by converting the 20% number into basis points.

However, the interesting fact is what happens when I do the exact same exercise—pricing the ALDA under one mortality assumption and then immediately shocking the IFM curve to lower level—at younger issue ages. One would think that the longer the deferral period, the greater the so-called risk to the insurance company in misestimating the true curve. It turns out that all else being equal, the situation is reversed, which is my main point. An ALDA that commences paying \$10,000 at age 90, but assuming a purchase age of 35 (instead of 45), leads to an annual premium of \$211.50 (instead of \$301.47) under the full μ_x curve used earlier. If the company misestimates mortality by the same 20% factor, with hindsight the ALDA premiums should have been $291.13 at age 35. In other words, under the true (new) mortality curve, the insurance company undercharged the 35-year-old by the difference between $291.13 and $211.50 per year. The company is ''losing'' $79.63 per year, relative to what they should have charged. Finally, if we invert and solve for the implied interest rate under the shifted IFM curve, the equivalent profit spread drops from 100 to 19 basis points. Obviously, the product is less profitable ex post, but the interesting and relevant fact is that the spread has dropped to by *less* than when the ALDA was sold to the 45-year-old. Recall that for the 45-year-old the same ''mistake'' led to a 4 basis points profit spread. And, although there are many ways to quantify the profitability—or lack thereof—of an ALDA, I interpret this evidence to imply that a longer deferral period per se does not necessarily lead to greater longevity risk for the insurance company.

Table 6 provides a summary of this analysis by comparing the revised profit spread under a variety of ALDA purchase and commencement ages. Thus, although misestimating mortality obviously can be very costly—and should be a concern in the pricing of any life contingent instrument—my main argument is as follows. All else being equal, an earlier ALDA purchase age reduces the sensitivity to misestimating experienced mortality. Longer deferral periods do not necessarily translate into greater mortality risk for the insurance company. I refer the interested reader to the report by Fliegelman, Robinson, and Milevsky (2002) for a broader analysis of this important topic.

Table 6

You Sold an ALDA under a 100 Basis Point Spread: What Is the Actual Spread if You Misestimate Mortality by 20%?

Purchase Age	Starting Age 85	Starting Age 90
35	38.4 b.p.	19.0 b.p.
40	32.9	12.2
45	26.6	4.1

Note: Assumptions: Gompertz mortality with $m = 90$ and $b = 9.5$, with 0.8 μ_x curve.

6. Conclusion

Despite valiant efforts by finance and insurance professionals to educate the public about the benefits of annuitization, the industry must recognize that few people will consciously choose to hand over a lump sum in exchange for lifetime income when given the choice. Numerous experiments involving ''live'' money have consistently documented consumers' hyperbolic levels of implied time preference when discounting future needs and cash flows during retirement. This is not to say that all consumers *take the money and run* when offered the choice to leave a DB pension plan. Rather, when the default status quo option is to continue maintaining full control of the funds—as in most DC plans—it is extremely hard to give up such control.

Therefore, in the face of a continuing erosion of traditional DB pension plans with their implicit life annuities, the industry must do more to create, promote, and explain viable alternatives. This paper provides another step in that direction by describing the actuarial mechanics of a product called ALDA. In its simple form, the ALDA would allow individuals to voluntarily acquire a lifetime payout annuity in small increments over long periods of preretirement saving. The ALDA could be offered as an additional rider on existing saving and insurance products or together with a phased withdrawal program, or it could be sold as a stand-alone product. The critical factor would be to take the edge off a daunting and irreversible annuitization decision. Likewise, this article emphasized the importance of framing the discussion in real (after inflation) terms, even though the extent to which the current CPI-U captures the basket of goods demanded by retirees is debatable.

While an introductory (motivational) article such as this leaves many details to complete, it is hoped that the ensuing dialogue will move the industry away from yet another generation of complex secondary guarantees such as GLBs on variable annuities—or finite maturity withdrawal benefits that masquerade as longevity insurance—toward a strategy that recognizes the consumer's ingrained reluctance to annuitize.

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