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APPLICATIONS OF GAME THEORY TO THE INSURANCE BUSINESS

Teaching Session

JEAN LEMAIRE*

PROFESSOR JEAN LEMAIRE: The two models I am going to present originate from game theory. They were devised as illustrations of game theoretical models to my operation research students. Game theory has not yet reached a point where it could be practically implemented in insurance companies. The first model I am going to describe is non-cooperative game theory. The second one uses utility theory and cooperative game theory, and this one comes nearer to practical implementation. Some big reinsurance companies, like the Swiss Reinsurance Company, have begun to use utility theory in their reinsurance policies and premium calculations.

The first model is certainly not yet ready to be implemented in insurance companies. Nevertheless, I think that game theory is major enough to enrich some of our reasonings. By using game theory we can try to introduce the opponent in our reasonings. Game theory is able to catch the flavor of competition between people. Game theory provides solutions to problems of conflicting interest, and that is why those models of game theory are certainly not completely useless to a practical paper.

I am going to explain what game theory is. First, non-cooperative game theory; second, cooperative game theory using insurance examples to illustrate. I will attempt to use some concepts of non-cooperative game theory in the problems of acceptance or rejection of life insurance proposals. The same ideas could be applied to about any other branch of insurance. I am going to formulate the underwriting problem of accepting or rejecting life insurance proposals as a two-person game theory problem.

Let me start with a very elementary model and try to progressively make it slightly more complicated. Consider two players. The first player is the insurance company and the second one is the set of potential policyholders that are filing a form in order to get a life insurance policy. We shall play this game many times. In fact, each time a prospective policyholder fills in a proposal, the game will be played once. We shall suppose that the proposer, the potential policyholder is either perfectly healthy, in which case he shall be accepted, or is not healthy, affected by a disease which should be detected, and cause rejection. For the moment, I shall only consider two strategies for the policyholders, to be healthy or non-healthy, and for the insurance companies to accept or to reject people. Clearly, in order to achieve more realism, one should introduce other strategies, like introducing a medical examination,

*Professor Lemaire, not a member of the Society, is Professor of Statistics, The Free University of Brussels, Brussels, Belgium.

or introduce the possibility of accepting the policyholder with a surcharge. Those little complications will be introduced later on.

Consider a two-by-two matrix the payoffs of the insurance company. We have two strategies for each and we have to assess a payoff to each of the four possible combinationa. Assume A, B, C, D represent those four payoffs. A is the payoff to the company when the company accepts a healthy proposer, B is the payoff on rejection of the healthy proposer. C & D are acceptance and rejection of any ill proposer. We can say that the worst thing that could happen to an insurance company would be to accept a non-healthy proposer, so C is the lowest figure. It is better to reject a non-healthy proposer than to accept him, so D should be larger than C. It is better to accept a healthy proposer than to reject him so A should be larger than B. Which one of the two good decisions is to be preferred? It might be a matter of taste. In the figures I'm going to present, I have assumed that D is larger than A, but this is not a necessary assumption. The same analysis could be performed if A is larger than D. In the figures I have assumed that D is larger than A, A is larger than B, and B in turn is larger than C.

So the first criterion, which is the minimax criterion, assimilates the set of the policyholders to a malevolent opponent whose unique goal is to deceive the insurer and to reduce his payoff. This is an extremely conservative approach, to be used by a very cautious and pessimistic insurer, which is concerned only by its security level. In this first approximation it is supposed that the policyholders will try to reduce your payoff as much as possible. The second approach will introduce a probability distribution on the behaviorial assumptions of the policyholders. The game becomes what is called a zero-sum game between two players. A gain for the policyholders means a loss for the insurance companies. Two extreme situations are all the policyholders are healthy or all of them are non-healthy. Of course, the policyholders might use a mixed strategy. One can say, for instance, that 15% of the policyholders are non-healthy, and 85% are healthy. This would lead to a mixed strategies. The use of mixed strategies is here rather justified, since the game is to be played many times.

The insurance company might also consider using mixed strategies, for example accept half of the people and reject half of the people. How do we find the value of the game and the optimal strategies of both players? The lowest point for the policyholders is, of course, C, to present always non-healthy policyholders, assuming that they would all be accepted. Of course the insurance company, is not going to play as bad as that. As soon as the company finds out that all of the players (all of the prospects) will be non-healthy, it will start rejecting everybody. So really, the policyholders cannot assume that this would be their payoff. Since the insurance company wants to maximize its payoff, to go as high as possible, those will be the strategies selected by the insurance company. The policyholders attempt to go as far down as possible, to minimize the payoff to the insurance company. The mini-max solution is the point that minimizes the maximal loss of the company. This would be the optimal strategy of the policyholders. They would present something like 75% of healthy proposers and 25% of non-healthy.

Supplementary medical information can be used to improve the insurer's payoff. For our purposes, it's only necessary to characterize a medical examination by two probabilities. The first is the probability that the medical information works well. It is the probability of detecting a disease that the policyholder has. The second is the false alarm probability. It is the probability of rejecting a healthy proposer. One more strategy for the insurance company is simply to follow the indications of the medical information. Let's introduce payoffs E and F for this third pure strategy. E is the payoff if the proposer is not healthy. If he's not healthy, his illness is detected with a probability, then he's rejected, and the payoff has been denoted D. With the complementary probability the illness is not detected, in which case the payoff to the insurance company is C. In the case the policyholder is healthy, and there is no false alarm, then the proposer is accepted, as should be, and the payoff is A. If there is a false alarm, the policyholder is rejected, incorrectly, since he's healthy, and then the payoff is B. Assuming those probabilities are known, it's fairly straightforward to compute the payoffs for this third strategy. The calculation becomes a little more involved, since we now have three strategies for the insurance companies to accept, to reject, and to follow the advice of the medical information. One can see that introducing medical information has the effect of raising the payoff to the insurance company.

The insurance company has to mix strategies to accept and to follow the [illegible] advice, but of course other cases are possible, other mixed strategies a[illegible]ssible. It might even be the case that the medical information is s[illegible]k, that it might be disregarded and not used by the insurance company. [illegible] the questions that game theory is able to answer is: Did we make [illegible]mation we have? Did we select the best detector? A detector of medical information, as I told you, is characterized by a set of two probabilities. The underwriting department can try to modify those probabilities. They could try for instance to reject more people. If they reject more people, they would reject more healthy people and more non-healthy people. The success probability would increase; that's good. And the false alarm probability would increase too; that's bad. So by adjusting our underwriting procedures, we can work on those two probabilities. What we lose on one hand, will be gained on the other hand.

The question arises, how to select an optimal pair of probabilities. Must the company choose a very nervous detector, a high success probability, but also a high false alarm probability, or should the company select a phlegmatic, or a slow detector with low probabilities. For the examples I am going to use I have assumed that the insurance company knows regression analysis or discriminate analysis, and I am going to suppose that all of the information concerning the health of the policyholders could be aggregated into one single variable.

Each potential policyholder is characterized by a point of value of the discriminating variable. Of course all the healthy proposers do not have the same value, since they might have different weights, different blood pressures, and so on, so we have a distribution of the value of this variable for the healthy proposers and a distribution of the value of this variable for the non-healthy proposer. Usually those two distributions overlap. (If those two distributions did not overlap it would be easy to separate the good proposers from the bad ones.) Then, a medical examination or a detector could simply be characterized by a critical value. We just fix ourselves a limit. If the policyholder presents an index which is lower than the limit, we accept him; if it's higher than the limit, we reject him. Using another critical value, we may render the medical information more or less severe. If we increase the critical value, we shall have a lower false alarm probability, but also a lower success probability. So where is the optimal? For all of the critical values, we can plot the values of those publications in order to obtain what I call the efficiency curve. The best medical information would be a success probability equal to one and a false alarm probability equal to zero. A completely ineffective medical examination would result in nearly no differences between the success and the false alarm probability. If the detector is rejecting too many people, the policyholders might reduce the company's payoff by always presenting healthy proposers. If the detector is too slow, is too phlegmatic, the policyholders, by presenting more non-healthy proposers, could reduce the payoff. It can be shown mathematically, that all we have to do in order to find the optimal value is to equate the two payoffs, E and F.

The medical information could be improved. One could, for example, introduce a blood test, an electrocardiogram, or any complicated medical devide to improve the medical examination. Is it worthwhile? Is it worth the cost? If we complicate the medical examination, it is in the hope to have an improved discrimination ability. Introducing a more complicated medical examination will have the effect of separating the two distributions I presented to you, so it will be easier to discriminate between the two categories of policy holders. This will mean improved probabilities -- improved success probabilities, improved false alarm probabilities. If the cost of introducing the new system is less than the difference in payoff, then one should do it. The insurance company should be willing to pay any amount less than the difference in order to pay for its increased discrimination ability.

Editor's Note: The remainder of this teaching seminar was given to the development of specific examples of game theory application using techniques described above. A more complete text including these concepts is contained in "A GAME THEORETIC LOOK AT LIFE INSURANCE UNDERWRITING" by Professor Jean Lemaire.