QFI-Quantitative Finance Formula Package
Spring and Fall 2023

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

In sources where some equations are numbered and others are not, the page number is provided instead.

An Introduction to the Mathematics of Financial Derivatives, 3rd Edition (second printing), A. Hirsa and S. Neftci

Chapter 2

(2.10) $$\begin{bmatrix} 1 \\ S(t) \\ C(t) \end{bmatrix} = \begin{bmatrix} (1+r\Delta) & (1+r\Delta) \\ S_1(t+\Delta) & S_2(t+\Delta) \\ C_1(t+\Delta) & C_2(t+\Delta) \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

(2.46) $$R_1(t+1) = \frac{S_1(t+1)}{S(t)}$$

(2.47) $$R_2(t+1) = \frac{S_2(t+1)}{S(t)}$$

(2.48) $$0 = ((1+r) - R_1)\psi_1 + ((1+r) - R_2)\psi_2$$

(2.66) $$S_t = \frac{1}{1+r}\left[\mathbb{Q}_{up}(S_t + \sigma\sqrt{\Delta}) + \mathbb{Q}_{down}(S_t - \sigma\sqrt{\Delta})\right]$$

(2.67) $$C_t = \frac{1}{(1+r)}\left[\mathbb{Q}_{up}C^{up}_{t+\Delta} + \mathbb{Q}_{down}C^{down}_{t+\Delta}\right]$$

(2.68) $$C_T = \max[S_T - C_0, 0]$$

(2.70) $$S = \frac{1+d}{1+r}\left[S^u\mathbb{Q}_{up} + S^d\mathbb{Q}_{down}\right]$$

(2.71) $$C = \frac{1}{1+r}\left[C^u\mathbb{Q}_{up} + C^d\mathbb{Q}_{down}\right]$$

(page 27) $$\mathbb{E}^{\mathbb{Q}}\left[\frac{C_{t+\Delta}}{C_t}\right] \approx 1 + r\Delta$$

Chapter 3

(3.37) $$\sum_{i=1}^{n} f\left(\frac{t_i + t_{i-1}}{2}\right)(t_i - t_{i-1}) \to \int_0^T f(s)ds$$

(3.49) $$\int_0^T g(s)df(s) \approx \sum_{i=1}^{n} g\left(\frac{t_i + t_{i-1}}{2}\right)(f(t_i) - f(t_{i-1}))$$

Chapter 4

(4.20) $$f(x) = \sum_{i=0}^{\infty} \frac{1}{i!} f^i(x_0)(x - x_0)^i$$

(4.23) $$dF(t) = F_S dS_t + F_r dr_t + F_t dt$$

(4.24) $$dF(t) = F_s dS_t + F_r dr_t + F_t dt + \frac{1}{2}F_{ss}dS_t^2 + \frac{1}{2}F_{rr}dr_t^2 + F_{sr}dS_t dr_t$$

Chapter 5

(5.11) $$\mathbb{E}[S_t|I_u] = \int_{-\infty}^{\infty} S_t f(S_t|I_u)dS_t, \qquad u < t$$

(5.18) $$P(\Delta F(t) = +a\sqrt{\Delta}) = p$$

(5.19) $$P(\Delta F(t) = -a\sqrt{\Delta}) = 1 - p$$

(5.37) $$P(\Delta N_t = 1) \approx \lambda\Delta$$

(5.38) $$P(\Delta N_t = 0) \approx 1 - \lambda\Delta$$

(5.39) $$P(\Delta N_t = n) = \frac{e^{-\lambda\Delta}(\lambda\Delta)^n}{n!}$$

(5.40) $$f(x) = \frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right), \quad x \geq 0$$

(5.41) $$F(x) = 1 - \exp(-x/\theta), \quad x \geq 0$$

(5.42) $$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$$

(5.44) $$P(X_{t+s} \leq x_{t+s}|x_t, \dots, x_1) = P(X_{t+s} \leq x_{t+s}|x_t)$$

(5.45) $$r_{t+\Delta} - r_t = \mathbb{E}[(r_{t+\Delta} - r_t)|I_t] + \sigma(I_t, t)\Delta W_t$$

(5.48) $$dr_t = \mu(r_t, t)dt + \sigma(r_t, t)dW_t$$

(5.49) $$\begin{bmatrix} r_{t+\Delta} \\ R_{t+\Delta} \end{bmatrix} = \begin{bmatrix} \alpha_1 r_t + \beta_1 R_t \\ \alpha_2 r_t + \beta_2 R_t \end{bmatrix} + \begin{bmatrix} \sigma_1 W^1_{t+\Delta} \\ \sigma_2 W^2_{t+\Delta} \end{bmatrix}$$

Chapter 6

(6.3) $$\mathbb{E}_t[S_T] = \mathbb{E}[S_T|I_t], \quad t < T$$

(6.4) $$\mathbb{E}|S_t| < \infty$$

(6.5) $$\mathbb{E}_t[S_T] = S_t, \text{ for all } t < T$$

(6.9) $$\mathbb{E}^{\mathbb{Q}}_t[e^{-ru}B_{t+u}] = B_t, \quad 0 < u < T - t$$ (The text formula is incorrect, the right-hand side should be B_t)

(6.10) $$\mathbb{E}^{\mathbb{Q}}_t[e^{-ru}S_{t+u}] = S_t, \quad 0 < u$$

(6.29) $$V^1 = \sum_{i=1}^{n}|X_{t_i} - X_{t_{i-1}}|$$

(6.30) $$V^2 = \sum_{i=1}^{n}|X_{t_i} - X_{t_{i-1}}|^2$$

(6.31) $$V^4 = \sum_{i=1}^{n}|X_{t_i} - X_{t_{i-1}}|^4$$

(6.36) $$V^2 \leq \max_i|X_{t_i} - X_{t_{i-1}}|V^1$$

(6.37) $$\max_i|X_{t_i} - X_{t_{i-1}}| \to 0$$

(6.38) $$V^4 \leq \max_i|X_{t_i} - X_{t_{i-1}}|^2V^2$$

(6.44) $$\Delta X_t \sim N(\mu\Delta, \sigma^2\Delta)$$

(6.46) $$X_{t+T} = X_0 + \int_0^{t+T} dX_u$$

(6.49) $$X_t + \mathbb{E}_t\left[\int_t^{t+T} dX_u\right] = X_t + \mu T$$ (This is a correction to the text formula)

(6.50) $$Z_t = X_t - \mu t$$

(6.53) $$\mathbb{E}_t[Z_{t+T}] = X_t + \mathbb{E}_t[(X_{t+T} - X_t)] - \mu(t + T)$$ (This is a correction to the text formula)

(6.54) $$\mathbb{E}_t[Z_{t+T}] = X_t - \mu t$$ (This is a correction to the text formula)

(6.55) $$\mathbb{E}_t[Z_{t+T}] = Z_t$$

(6.64) $$I_t \subseteq I_{t+1} \subseteq \cdots \subseteq I_{T-1} \subseteq I_T$$

(6.65) $$M_t = \mathbb{E}^{\mathbb{P}}[Y_T|I_t]$$

(6.66) $\mathbb{E}^{\mathbb{P}}[M_{t+s}|I_t] = M_t$

(6.70) $G_T = f(S_T)$

(6.71) $B_T = e^{\int_t^T r_s ds}$

(6.72) $M_t = \mathbb{E}^{\mathbb{P}}\left[\frac{G_T}{B_T}|I_t\right]$

(6.105) $e^{-rt}S_t = A_t + Z_t$

(6.106) $M_{t_k} = M_{t_0} + \sum_{i=1}^{k} H_{t_{i-1}}[Z_{t_i} - Z_{t_{i-1}}]$

(6.108) $\mathbb{E}_{t_0}[M_{t_k}] = M_{t_0}$ (This is a correction to the text formula)

Chapter 7

(7.23) $\Delta W_k = [S_k - S_{k-1}] - \mathbb{E}_{k-1}[S_k - S_{k-1}]$

(7.26) $W_k = \sum_{i=1}^{k} \Delta W_i$

(7.28) $\mathbb{E}_{k-1}[W_k] = W_{k-1}$

(7.29) $\mathbb{V}^k = \mathbb{E}_0[\Delta W_k^2]$

(7.30) $\mathbb{V} = \mathbb{E}_0\left[\sum_{k=1}^{n} \Delta W_k\right]^2 = \sum_{k=1}^{n} \mathbb{V}^k$

(7.31) $\mathbb{V} > A_1 > 0$

(7.33) $\mathbb{V} < A_2 < \infty$

(7.34) $\mathbb{V}_{max} = \max_k[\mathbb{V}^k, k = 1, \dots, n]$

(7.35) $\frac{\mathbb{V}^k}{\mathbb{V}_{max}} > A_3, \quad 0 < A_3 < 1$

(7.36) $\mathbb{E}[\Delta W_k]^2 = \sigma_k^2 h$

(7.56) $S_k - S_{k-1} = \mathbb{E}_{k-1}[S_k - S_{k-1}] + \sigma_k \Delta W_k$

(7.63) $\mathbb{E}_{k-1}(S_k - S_{k-1}) \approx a(I_{k-1}, kh)h$

(7.64) $S_{kh} - S_{(k-1)h} \approx a(I_{k-1}, kh)h + \sigma_k[W_{kh} - W_{(k-1)h}]$

Chapter 8

(8.7) $dN_t = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases}$

(8.8) $M_t = N_t - \lambda t$

(8.9) $\mathbb{E}[M_t] = 0$

(8.10) $\mathbb{E}[M_t]^2 = \lambda t$

(8.21) $\sigma_k \Delta W_k = \begin{cases} \omega_1 & \text{with probability } p_1 \\ \omega_2 & \text{with probability } p_2 \\ \vdots & \vdots \\ \omega_m & \text{with probability } p_m \end{cases}$

(8.22) $\mathbb{E}[\sigma_k \Delta W_k]^2 = \sigma_k^2 h$

(8.23) $\sum_{i=1}^{m} p_i \omega_i^2 = \sigma_k^2 h$

(8.29) $p_i(h) = \bar{p}_i h^{q_i}$

(8.33) $q_i + 2r_i = 1$

(8.34) $c_i = \bar{\omega}_i^2 \bar{p}_i$

(8.58) $J_t = (N_t - \lambda t)$

(8.60) $dS_t = a(S_t, t)dt + \sigma_1(S_t, t)dW_t + \sigma_2(S_t, t)dJ_t$

(8.61) $\mathbb{V}[X_t] = \mathbb{E}[X_t - \mathbb{E}[X_t]]^2$

(8.62) Higher-order (centered) moments are $\mathbb{E}[X_t - \mathbb{E}[X_t]]^k,\ k > 2$

(8.72) $\mathbb{E}[\sigma_2 \Delta J_k]^2 = h\left[\sum_{i=1}^{m} \omega_i^2 \bar{p}_i\right]$

(8.73) $\mathbb{E}[\sigma_2 \Delta J_k]^n = h\left[\sum_{i=1}^{m} \omega_i^n \bar{p}_i\right]$

(8.75) $t_0 = 0 < t_1 < \cdots < t_n = T$

(8.76) $n\Delta = T$

(8.77) $S_i = S_{t_i}, \quad i = 0, 1, \ldots, n$

(8.78) $S_{i+1} = \begin{cases} u_i S_i & \text{with probability } p_i \\ d_i S_i & \text{with probability } 1 - p_i \end{cases}$

(8.79) $u_i = e^{\sigma\sqrt{\Delta}}, \quad \text{for all } i$

(8.80) $d_i = e^{-\sigma\sqrt{\Delta}}, \quad \text{for all } i$

(8.81) $p_i = \frac{1}{2}\left[1 + \frac{\mu}{\sigma}\sqrt{\Delta}\right], \quad \text{for all } i$

(8.91) $\log \frac{S_{i+n}}{S_i} = Z \log u + (n - Z) \log d$

(8.92) $\log \frac{S_{i+n}}{S_i} = Z \log \frac{u}{d} + n \log d$

(8.96) $\mathbb{E}\left[\log \frac{S_{i+n}}{S_i}\right] = \log \frac{u}{d} np + n \log d$

(8.97) $\mathbb{V}\left[\log \frac{S_{i+n}}{S_i}\right] = \left[\log \frac{u}{d}\right]^2 np(1 - p)$

(8.98) $n = \frac{T}{\Delta}$

(8.99) $\log \frac{u}{d} np + n \log d \approx \mu T$

(8.100) $\left[\log \frac{u}{d}\right]^2 np(1 - p) \approx \sigma^2 T$

(8.102) $[\log S_{i+n} - \log S_i] \sim \mathcal{N}\left(\mu(n\Delta), \sigma^2 \Delta\right)$

(8.103) $[\log S_{i+n} - \log S_i] \sim \text{Poisson}$

Chapter 9

(9.37) $$\lim_{n\to\infty} \mathbb{E}\left[\sum_{k=1}^{n} \sigma(S_{k-1},k)\left[W_k - W_{k-1}\right] - \int_0^T \sigma(S_u,u)dW_u\right]^2 = 0$$

(9.38) $$S_k - S_{k-1} = a(S_{k-1},k)h + \sigma(S_{k-1},k)[W_k - W_{k-1}], \quad k = 1,2,\dots,n$$

(9.39) $$\mathbb{E}\left[\int_0^T \sigma(S_t,t)^2 dt\right] < \infty$$

(9.41) $$\sum_{k=1}^{n} \sigma(S_{k-1},k)[W_k - W_{k-1}] \to \int_0^T \sigma(S_t,t)dW_t \text{ as } n \to \infty (h \to 0)$$

(9.73) $$\int_0^T x_t dx_t = \frac{1}{2}\left[x_T^2 - T\right]$$

(9.74) $$\lim_{n\to\infty} \mathbb{E}\left[\sum_{i=0}^{n-1} \Delta x_{t_{i+1}}^2 - T\right]^2 = 0$$

(9.76) $$\text{If } \int_0^T (dx_t)^2 \text{ exists, then } \lim_{n\to\infty} \mathbb{E}\left[\sum_{i=0}^{n-1} \Delta x_{t_{i+1}}^2 - \int_0^T (dx_t)^2\right]^2 = 0$$

(9.77) $$\int_0^T dt = T$$

(9.78) $$\int_0^T (dx_t)^2 = \int_0^T dt$$

(9.79) $$(dW_t)^2 = dt$$

(9.85) $$\mathbb{E}_t\left[\int_0^{t+\Delta} \sigma_u dW_u\right] = \int_0^t \sigma_u dW_u$$ (This is a correction to the text formula)

(9.132) $$\mathbb{E}\left[\int_0^T f(W_t,t)dW_t \int_0^T g(W_t,t)dW_t\right] = \mathbb{E}\left[\int_0^T f(W_t,t)g(W_t,t)dt\right]$$

(This is a correction to the text formula)

(9.133) $$\mathbb{E}\left[\int_0^T f(W_t,t)dW_t\right]^2 = \mathbb{E}\left[\int_0^T f(W_t,t)^2 dt\right]$$ (This is a correction to the text formula)

Chapter 10

(page 170) $$dS_t = a_t dt + \sigma_t dW_t, \quad t \geq 0$$

(10.36) $$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\sigma_t^2 \frac{\partial^2 F}{\partial S_t^2} dt$$

(10.37) $$dF_t = \left[a_t \frac{\partial F}{\partial S_t} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma_t^2 \frac{\partial^2 F}{\partial S_t^2}\right] dt + \frac{\partial F}{\partial S_t}\sigma_t dW_t$$ (This is a correction to the text formula)

(10.64) $$\int_0^t F_s dS_u = \left[F(S_t,t) - F(S_0,0)\right] - \int_0^t \left[F_u + \frac{1}{2}F_{ss}\sigma_u^2\right] du$$

(10.69) $$dF = F_t dt + F_{s_1} dS_1 + F_{s_2} dS_2 + \frac{1}{2}\left[F_{s_1 s_1} dS_1^2 + F_{s_2 s_2} dS_2^2 + 2F_{s_1 s_2} dS_1 dS_2\right]$$

(10.72) $$dS_1(t)^2 = \left[\sigma_{11}^2(t) + \sigma_{12}^2(t)\right] dt$$

(10.73) $$dS_2(t)^2 = \left[\sigma_{21}^2(t) + \sigma_{22}^2(t)\right] dt$$

(10.74) $$dS_1(t) dS_2(t) = \left[\sigma_{11}(t)\sigma_{21}(t) + \sigma_{12}(t)\sigma_{22}(t)\right] dt$$

(10.79) $Y(t) = \sum_{i=1}^{n} N_i(t) P_i(t)$

(10.80) $dY(t) = \sum_{i=1}^{n} N_i(t) dP_i(t) + \sum_{i=1}^{n} dN_i(t) P_i(t) + \sum_{i=1}^{n} dN_i(t) dP_i(t)$

(10.81) $dS_t = a_t dt + \sigma_t dW_t + dJ_t, \quad t \geq 0$

(10.82) $E[\Delta J_t] = 0$

(10.83) $\Delta J_t = \Delta N_t - \left[\lambda_t h \left(\sum_{i=1}^{k} a_i p_i \right) \right]$

(10.84) $a_t = \alpha_t + \lambda_t \left(\sum_{i=1}^{k} a_i p_i \right)$

(10.85) $dF(S_t, t) = \left[F_t + \lambda_t \sum_{i=1}^{k} (F(S_t + a_i, t) - F(S_t, t)) p_i + \frac{1}{2} F_{ss} \sigma^2 \right] dt + F_s dS_t + dJ_F$

(10.86) $dJ_F = [F(S_t, t) - F(S_t^-, t)] - \lambda_t \left[\sum_{i=1}^{k} (F(S_t + a_i, t) - F(S_t, t)) p_i \right] dt$

(10.87) $S_t^- = \lim_{s \to t} S_s, \quad s < t$

Chapter 11

(11.24) $dS_t = \mu S_t dt + \sigma S_t dW_t, \quad t \in [0, \infty)$

(11.30) $S_t = S_0 e^{\{(a - \frac{1}{2}\sigma^2)t + \sigma W_t\}}$

(11.34) $dS_t = r S_t dt + \sigma S_t dW_t, \quad t \in [0, \infty)$

(11.38) $S_T = \left[S_0 e^{(r - \frac{1}{2}\sigma^2)T} \right] \left[e^{\sigma W_T} \right]$

(11.42) $Z_t = e^{\sigma W_t}$

(11.50) $x_t = \mathbb{E}[Z_t] = e^{\frac{1}{2}\sigma^2 t}$

(11.56) $S_t = e^{-r(T-t)} \mathbb{E}_t[S_T]$

(11.72) $dS_t = \mu S_t dt + \sigma \sqrt{S_t} dW_t, \quad t \in [0, \infty)$

(11.74) $dS_t = \lambda(\mu - S_t) dt + \sigma S_t dW_t$

(11.78) $dS_t = -\mu S_t dt + \sigma dW_t$

(11.79) $dS_t = \mu dt + \sigma_t dW_{1t}$

(page 192) $d\sigma_t = \lambda(\sigma_0 - \sigma_t) dt + \alpha \sigma_t dW_{2t}$

(page 193) $dN_t = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases}$

(11.83) $\frac{dS_t}{S_t} = (\mu - \lambda \kappa) dt + \sigma dW_t + (e^J - 1) dN_t$

(11.84) $\frac{dS_t}{S_t} = (\mu - \lambda^* \kappa^*) dt + \sigma d\tilde{W}_t + (e^J - 1) dN_t$

(page 194) $S_t = S_0 e^{(r - q + \omega)t + X(t; \sigma, \nu, \theta)}$

(page 194) $f(x; \sigma, \nu, \theta) = \int_0^\infty \frac{1}{\sigma \sqrt{2\pi g}} \exp\left(-\frac{(x - \theta g)^2}{2\sigma^2 g} \right) \frac{g^{t/\nu - 1} e^{-g/\nu}}{\nu^{t/\nu} \Gamma(t/\nu)} dg$

Chapter 12

(12.3) $P_t = \theta_1 F(S_t, t) + \theta_2 S_t$

(12.4) $dP_t = \theta_1 dF_t + \theta_2 dS_t$

(12.5) $dS_t = a(S_t, t)dt + \sigma(S_t, t)dW_t, \quad t \in [0, \infty)$

(12.6) $dF_t = F_t dt + \frac{1}{2} F_{ss} \sigma_t^2 dt + F_s dS_t$

(12.7) $dF_t = \left[F_s a_t + \frac{1}{2} F_{ss} \sigma_t^2 + F_t \right] dt + F_s \sigma_t dW_t$

(12.10) $\theta_1 = 1$

(12.11) $\theta_2 = -F_s$

(12.12) $dP_t = F_t dt + \frac{1}{2} F_{ss} \sigma_t^2 dt$

(12.16) $r(F(S_t, t) - F_s S_t) = F_t + \frac{1}{2} F_{ss} \sigma_t^2$

(12.17) $-rF + rF_s S_t + F_t + \frac{1}{2} F_{ss} \sigma_t^2 = 0, \quad 0 \le S_t, \quad 0 \le t \le T$

(12.20) $P_t = \theta_1 F(S_t, t) + \theta_2 S_t$

(12.23) $P_t = F(S_t, t) - F_s(S_t, t) S_t$

(12.24) $dP_t = dF(S_t, t) - F_s dS_t - S_t dF_s - dF_s(S_t, t) dS_t$

(12.26) $dP_t = dF(S_t, t) - F_s dS_t - S_t \left[\left[F_{st} + F_{ss} \mu S_t + \frac{1}{2} F_{sss} \sigma_t^2 S_t^2 \right] dt + F_{ss} \sigma S_t dW_t \right] - F_{ss} \sigma_t^2 S_t^2 dt$

(12.28) $dP_t = dF(S_t, t) - F_s dS_t - S_t [F_{ss}(\mu - r) S_t dt] - F_{ss} \sigma S_t^2 dW_t$

(page 202) $\mathbb{E}^{\mathbb{Q}} \left[S_t^2 F_{SS} (\sigma dW_t + (\mu - r)\Delta) \right] \approx 0$

(page 202) $dW_t^* = \sigma dW_t + (\mu - r) dt$

(12.29) $a_0 F + a_1 F_s S_t + a_2 F_t + a_3 F_{ss} = 0, \quad 0 \le S_t, \quad 0 \le t \le T$

(12.30) $F(S_T, T) = G(S_T, T)$

Chapter 13

(13.1) $a(S_t, t) = \mu S_t$

(13.2) $\sigma(S_t, t) = \sigma S_t, \quad t \in [0, \infty)$

(13.3) $-rF + rF_s S_t + F_t + \frac{1}{2} \sigma^2 F_{ss} S_t^2 = 0, \quad 0 \le S_t, \quad 0 \le t \le T$

(13.4) $F(T) = \max[S_T - K, 0]$

(13.6) $F(S_t, t) = S_t N(d_1) - K e^{-r(T-t)} N(d_2)$

(13.7) $d_1 = \frac{\ln(\frac{S_t}{K}) + (r + \frac{1}{2}\sigma^2)(T - t)}{\sigma \sqrt{T - t}}$

(13.8) $d_2 = d_1 - \sigma \sqrt{T - t}$

(13.9) $N(d_i) = \int_{-\infty}^{d_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2} dx, \quad i = 1, 2$

(13.12) $a(S_t, t) = \mu S_t$

(13.13) $\sigma(S_t, t) = \sigma(S_t, t) S_t, \quad t \in [0, \infty)$

(13.14) $-rF + rF_s S_t + F_t + \frac{1}{2}\sigma(S_t,t)^2 F_{ss} S_t^2 = 0, \quad 0 \le S_t, \quad 0 \le t \le T$

(13.15) $F(T) = \max[S_T - K, 0]$

(13.34) $rF - rF_s S_t - \delta - F_t - \frac{1}{2}F_{ss}\sigma_t^2 = 0$

(13.35) $F(S_{1T}, S_{2T}, T) = \max[0, \max(S_{1T}, S_{2T}) - K]$ (multi-asset option)

(13.36) $F(S_{1T}, S_{2T}, T) = \max[0, (S_{1T} - S_{2T}) - K]$ (spread call option)

(13.37) $F(S_{1T}, S_{2T}, T) = \max[0, (\theta_1 S_{1T} + \theta_2 S_{2T}) - K]$ (portfolio call option)

(13.38) $F(S_{1T}, S_{2T}, T) = \max[0, (S_{1T} - K_1), (S_{2T} - K_2)]$ (dual strike call option)

(This is a correction to the text formula)

(13.47) $\frac{\Delta F}{\Delta t} + rS\frac{\Delta F}{\Delta S} + \frac{1}{2}\sigma^2 S^2 \frac{\Delta^2 F}{\Delta S^2} \approx rF$

(13.48) $\frac{\Delta F}{\Delta t} \approx \frac{F_{ij} - F_{i,j-1}}{\Delta t}$

(13.49) $\frac{\Delta F}{\Delta S} \approx \frac{F_{ij} - F_{i-1,j}}{\Delta S}$

(13.50) $rS\frac{\Delta F}{\Delta S} \approx rS_j \frac{F_{i+1,j} - F_{ij}}{\Delta S}$

(13.51) $\frac{\Delta^2 F}{\Delta S^2} \approx \left[\frac{F_{i+1,j} - F_{ij}}{\Delta S} - \frac{F_{ij} - F_{i-1,j}}{\Delta S}\right]\frac{1}{\Delta S}$

Chapter 14

(14.3) $\mathbb{P}\left(\bar{z} - \frac{1}{2}\Delta < z_t < \bar{z} + \frac{1}{2}\Delta\right) = \int_{\bar{z}-\frac{1}{2}\Delta}^{\bar{z}+\frac{1}{2}\Delta} \frac{1}{\sqrt{2\pi}} e^{-\frac{z_t^2}{2}} dz_t$

(14.6) $d\mathbb{P}(\bar{z}) = \mathbb{P}\left(\bar{z} - \frac{1}{2}dz_t < z_t < \bar{z} + \frac{1}{2}dz_t\right)$

(14.7) $\int_{-\infty}^{\infty} d\mathbb{P}(z_t) = 1$

(14.8) $\mathbb{E}[z_t] = \int_{-\infty}^{\infty} z_t d\mathbb{P}(z_t)$

(14.9) $\mathbb{E}\left[z_t - E[z_t]\right]^2 = \int_{-\infty}^{\infty} \left[z_t - \mathbb{E}[z_t]\right]^2 d\mathbb{P}(z_t)$

(14.29) $\mathbb{E}_t\left[\frac{1}{1+R_t}S_{t+1}\right] = S_t$

(14.31) $\mathbb{E}_t^{\mathbb{Q}}\left[\frac{1}{1+r_t}S_{t+1}\right] = S_t$

(14.41) $z_t \sim N(0,1)$

(14.42) $d\mathbb{P}(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t)^2} dz_t$

(14.43) $\xi(z_t) = e^{z_t\mu - \frac{1}{2}\mu^2}$

(14.44) $[d\mathbb{P}(z_t)][\xi(z_t)] = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t^2) + \mu z_t - \frac{1}{2}\mu^2} dz_t$

(14.45) $d\mathbb{Q}(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t - \mu)^2} dz_t$

(14.47) $d\mathbb{Q}(z_t) = \xi(z_t)d\mathbb{P}(z_t)$

(14.48) $\xi(z_t)^{-1}d\mathbb{Q}(z_t) = d\mathbb{P}(z_t)$

(14.53) $$f(z_{1t}, z_{2t}) = \frac{1}{2\pi\sqrt{|\Omega|}} e^{-\frac{1}{2}\left[\begin{array}{cc} z_{1t}-\mu_1, & z_{2t}-\mu_2 \end{array}\right]\Omega^{-1}\left[\begin{array}{c} z_{1t}-\mu_1 \\ z_{2t}-\mu_2 \end{array}\right]}$$

(This is a correction to the text formula)

(14.54) $$\Omega = \left[\begin{array}{cc} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{array}\right]$$

(14.55) $|\Omega| = \sigma_1^2\sigma_2^2 - \sigma_{12}^2$

(14.56) $d\mathbb{P}(z_{1t}, z_{2t}) = f(z_{1t}, z_{2t})dz_{1t}dz_{2t}$

(14.57) $$\xi(z_{1t}, z_{2t}) = \exp\left\{-\left[\begin{array}{cc} z_{1t}, & z_{2t} \end{array}\right]\Omega^{-1}\left[\begin{array}{c} \mu_1 \\ \mu_2 \end{array}\right] + \frac{1}{2}\left[\begin{array}{cc} \mu_1, & \mu_2 \end{array}\right]\Omega^{-1}\left[\begin{array}{c} \mu_1 \\ \mu_2 \end{array}\right]\right\}$$

(This is a correction to the text formula)

(14.58) $d\mathbb{Q}(z_{1t}, z_{2t}) = \xi(z_{1t}, z_{2t})d\mathbb{P}(z_{1t}, z_{2t})$

(14.59) $$d\mathbb{Q}(z_{1t}, z_{2t}) = \frac{1}{2\pi\sqrt{|\Omega|}} \exp\left\{-\frac{1}{2}\left[\begin{array}{cc} z_{1t}, & z_{2t} \end{array}\right]\Omega^{-1}\left[\begin{array}{c} z_{1t} \\ z_{2t} \end{array}\right]\right\} dz_{1t}dz_{2t}$$

(This is a correction to the text formula)

(14.60) $\xi(z_t) = e^{-z_t'\Omega^{-1}\mu + \frac{1}{2}\mu'\Omega^{-1}\mu}$ (This is a correction to the text formula)

(14.69) $$\frac{d\mathbb{Q}(z_t)}{d\mathbb{P}(z_t)} = \xi(z_t)$$

(14.74) $d\mathbb{Q}(z_t) = \xi(z_t)d\mathbb{P}(z_t)$

(14.75) $d\mathbb{P}(z_t) = \xi(z_t)^{-1}d\mathbb{Q}(z_t)$

(14.76) $\xi_t = e^{\left(\int_0^t X_u dW_u - \frac{1}{2}\int_0^t X_u^2 du\right)}, \quad t \in [0, T]$

(14.77) $\mathbb{E}\left[e^{\int_0^t X_u^2 du}\right] < \infty, \quad t \in [0, T]$

(14.83) $$\mathbb{E}\left[\int_0^t \xi_s X_s dW_s | I_u\right] = \int_0^u \xi_s X_s dW_s$$

(14.84) $W_t^* = W_t - \int_0^t X_u du, \quad t \in [0, T]$ (This is a correction to the text formula)

(14.85) $\mathbb{Q}(A) = \mathbb{E}^{\mathbb{P}}[1_A \xi_T]$

(14.86) $dW_t^* = dW_t - X_t dt$

(14.93) $d\mathbb{Q} = \xi_T d\mathbb{P}$

(14.122) $A_1 \cup A_2 \cup \cdots \cup A_n = \Omega$

(14.123) $1_{A_1} + 1_{A_2} + \ldots 1_{A_n} = 1_\Omega$

(14.127) $\mathbb{E}^{\mathbb{P}}[Z_t 1_{A_i}] = \mathbb{Q}(A_i)$

(14.138) $$\mathbb{E}^{\mathbb{P}}[g(X_t)] = \int_\Omega g(x)f(x)dx$$

(page 249) $g(X_t) = Z_t h(X_t)$

(14.140) $$\mathbb{E}^{\mathbb{P}}[g(X_t)] = \int_\Omega h(x)\tilde{f}(x)dx = \mathbb{E}^{\mathbb{Q}}[h(X_t)]$$

Chapter 15

(15.2) $Y_t \sim N(\mu t, \sigma^2 t)$

(15.4) $M(\lambda) = \mathbb{E}[e^{Y_t \lambda}]$

(15.10) $M(\lambda) = e^{(\lambda \mu t + \frac{1}{2}\sigma^2 \lambda^2 t)}$

(15.15) $S_t = S_0 e^{Y_t} \quad , t \in [0, \infty)$

(15.25) $\mathbb{E}[S_t | S_u, u < t] = S_u e^{\mu(t-s) + \frac{1}{2}\sigma^2(t-s)}$

(15.30) $Z_t = e^{-rt} S_t$

(15.31) $\mathbb{E}^{\mathbb{Q}}\left[e^{-rt} S_t | S_u, u < t\right] = e^{-ru} S_u$

(15.32) $\mathbb{E}^{\mathbb{Q}}[Z_t | Z_u, u < t] = Z_u$

(15.38) $\mathbb{E}^{\mathbb{Q}}\left[e^{-r(t-u)} S_t | S_u, u < t\right] = S_u e^{-r(t-u)} e^{\rho(t-u) + \frac{1}{2}\sigma^2(t-u)}$ where under $\mathbb{Q}, Y_t \sim N(\rho t, \sigma^2 t)$

(This is a correction with $t - u$ replacing $t - s$)

(15.39) $\rho = r - \dfrac{1}{2}\sigma^2$

(15.42) $\mathbb{E}^{\mathbb{Q}}\left[e^{-rt} S_t | S_u, u < t\right] = e^{-ru} S_u$

(15.51) $dS_t = r S_t dt + \sigma S_t dW_t^*$

(15.58) $C_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(T-t)} \max\{S_T - K, 0\}\right]$

(15.88) $dS_t = \mu_t dt + \sigma_t dW_t$

(15.90) $d[e^{-rt} S_t] = e^{-rt}[\mu_t - r S_t] dt + e^{-rt} \sigma_t dW_t$

(15.92) $dW_t^* = dX_t + dW_t$

(15.97) $dX_t = \left[\dfrac{\mu_t - r S_t}{\sigma_t}\right] dt$

(15.98) $d[e^{-rt} S_t] = e^{-rt} \sigma_t dW_t^*$

(15.111) $d\left[e^{-rt} F(S_t, t)\right] = e^{-rt} \sigma_t F_s dW_t^*$

Chapter 17

(page 282) $R_{t_1} = (1 + r_{t_1} \Delta)$

(page 282) $R_{t_2} = (1 + r_{t_2} \Delta)$

(page 282) $B_{t_1}^s = B(t_1, t_3)$

(page 282) $B_{t_1} = B(t_1, T)$

(page 282) $B_{t_3} = B(t_3, T)$

(17.6)
$$\begin{bmatrix} 1 \\ 0 \\ B_{t_1}^s \\ B_{t_1} \\ C_{t_1} \end{bmatrix} = \begin{bmatrix} R_{t_1} R_{t_2}^u & R_{t_1} R_{t_2}^u & R_{t_1} R_{t_2}^d & R_{t_1} R_{t_2}^d \\ (F_{t_1} - L_{t_2}^u) & (F_{t_1} - L_{t_2}^u) & (F_{t_1} - L_{t_2}^d) & (F_{t_1} - L_{t_2}^d) \\ 1 & 1 & 1 & 1 \\ B_{t_3}^{uu} & B_{t_3}^{ud} & B_{t_3}^{du} & B_{t_3}^{dd} \\ C_{t_3}^{uu} & C_{t_3}^{ud} & C_{t_3}^{du} & C_{t_3}^{dd} \end{bmatrix} \begin{bmatrix} \psi^{uu} \\ \psi^{ud} \\ \psi^{du} \\ \psi^{dd} \end{bmatrix}$$

(17.13) $1 = R_{t_1} R_{t_2}^u \psi^{uu} + R_{t_1} R_{t_2}^u \psi^{ud} + R_{t_1} R_{t_2}^d \psi^{du} + R_{t_1} R_{t_2}^d \psi^{dd}$

(17.14) $\mathbb{Q}_{ij} = (1 + r_{t_1})(1 + r^i_{t_2})\psi^{ij}$

(17.15) $1 = \mathbb{Q}_{uu} + \mathbb{Q}_{ud} + \mathbb{Q}_{du} + \mathbb{Q}_{dd}$

(17.16) $\mathbb{Q}_{ij} > 0$

(17.18) $B^s_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})}\right]$

(17.21) $B_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{B_{t_3}}{(1 + r_{t_1})(1 + r_{t_2})}\right]$ (This is a correction to the text formula)

(17.22) $0 = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})}[F_{t_1} - L_{t_2}]\right]$

(17.23) $C_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})}C_{t_3}\right]$

(17.31) $F_{t_1} = \frac{1}{\mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1+r_{t_1})(1+r_{t_2})}\right]}\mathbb{E}^{\mathbb{Q}}\left[\frac{L_{t_2}}{(1 + r_{t_1})(1 + r_{t_2})}\right]$

(17.36) $B^s_{t_1} = \psi^{uu} + \psi^{ud} + \psi^{du} + \psi^{dd}$

(17.38) $\pi_{ij} = \frac{1}{B^s_{t_1}}\psi^{ij}$

(17.39) $1 = \pi_{uu} + \pi_{ud} + \pi_{du} + \pi_{dd}$

(17.46) $F_{t_1} = \mathbb{E}^{\pi}[L_{t_2}]$

(17.52) $C_{t_3} = N \max[L_{t_2} - K, 0]$

(17.53) $C_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})} \max[L_{t_2} - K, 0]\right]$

(17.55) $C_{t_1} = B^s_{t_1}\mathbb{E}^{\pi}[\max[L_{t_2} - K, 0]]$

(17.56) $\frac{C_t}{S_t} = \mathbb{E}^{\mathbb{S}}_t\left(\frac{C_T}{S_T}\right)$

(17.57) $\frac{C(K)}{S_0} = \mathbb{E}^{\mathbb{S}}\left(\frac{(S_T - K)^+}{S_T}\right)$

(page 291) $y = \log\left(\frac{S_T}{K}\right)$

(page 291) $\frac{C(K)}{S_0} = \int_0^{\infty}(1 - F(y))e^{-y}dy$

(17.61) $\frac{C(K)}{S_0} = P(\ln S - \ln K > Y)$

(17.63) $F(t; T, S) = \frac{1}{S - T}\left(\frac{P(t, T)}{P(t, S)} - 1\right)$

(17.64) $0 = T_0 < T_1 < T_2 < \cdots < T_M$

(17.65) $\Delta_i = T_{i+1} - T_i, i = 0, 1, 2, \ldots, M - 1$

(17.66) $L_n(t) = \frac{P(t, T_n) - P(t, T_{n+1})}{\Delta_n P(t, T_{n+1})}$

(17.71) $P(T_i, T_{n+1}) = \prod_{j=1}^{n}\frac{1}{1 + \Delta_j L_j(T_i)}$

(17.72) $P(t, T_n) = P(t, T_l)\prod_{j=l}^{n-1}\frac{1}{1 + \Delta_j L_j(T_i)}, \qquad T_{l-1} < t \le T_l$

(17.75) $B_t^* = P(t, T_l) \prod_{j=0}^{l-1} (1 + \Delta_j L_j(T_j))$

(17.77) $D_n(t) = \frac{\prod_{j=l}^{n-1} \frac{1}{1+\Delta_j L_j(t)}}{\prod_{j=0}^{l-1} (1 + \Delta_j L_j(T_J))}$

(17.78) $dL_n(t) = \mu_n(t) L_n(t) dt + L_n(t) \sigma_n^\tau(t) dW_t, 0 \le t \le T_n, n = 1, 2, \ldots, M$

(17.103) $\mu_n(t) = \sum_{j=\ell}^{n} \frac{\Delta_j L_j(t) \sigma_n^\tau(t) \sigma_j(t)}{1 + \Delta_j L_j(t)}$

(17.104) $dL_n(t) = \left(\sum_{j=\ell}^{n} \frac{\Delta_j L_j(t) \sigma_n^\top(t) \sigma_j(t)}{1 + \Delta_j L_j(t)} \right) L_n(t) dt + L_n(t) \sigma_n^\top(t) dW_t, \quad 0 \le t \le T_n, \quad n = 1, \ldots, M$

(17.105) $V_t = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^{T+\delta} r_u du} (F_t - L_T) N \delta \right]$

(17.110) $V_t = \mathbb{E}_t^\pi [B(t, T + \delta)(F_t - L_T) N \delta]$

(17.111) $V_t = B(t, T + \delta) \mathbb{E}_t^\pi [(F_t - L_T) N \delta]$

(17.112) $F_t = \mathbb{E}_t^\pi [L_T]$

(page 296) $C_T = \max[L_{T-\delta} - K, 0]$

(17.113) $C_t = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^{T+\delta} r_u du} \max[L_{T-\delta} - K, 0] \right]$

Chapter 18

(18.3) $B(t, T) = e^{-R(T,t)(T-t)}, \quad t < T$

(18.12) $B(t, T) = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_s ds} \right]$

(18.20) $R(t, T) = \frac{-\log \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_s ds} \right]}{T - t}$

(18.33) $B(t, T) = e^{-\int_t^T F(t,s) ds}$

(18.40) $F(t, T) = \lim_{\Delta \to 0} \frac{\log B(t, T) - \log B(t, T + \Delta)}{\Delta}$

(page 311) $F(t, T, U) = \frac{\log B(t, T) - \log B(t, U)}{U - T}$

Chapter 19

(19.14) $dB_t = \mu(t, T, B_t) B_t dt + \sigma(t, T, B_t) B_t dV_t^T, \quad \text{where } B_t = B(t, T)$

(19.15) $dB_t = r_t B_t dt + \sigma(t, T, B_t) B_t dW_t^T$

(19.21) $dF(t, T) = \sigma(t, T, B(t, T)) \left[\frac{\partial \sigma(t, T, B(t, T))}{\partial T} \right] dt - \left[\frac{\partial \sigma(t, T, B(t, T))}{\partial T} \right] dW_t$

(This is a correction to the text formula)

(19.22) $dF(t, T) = a(F(t, T), t) dt + b(F(t, T), t) dW_t$

(19.25) $r_t = F(t, t)$

(page 325) $F(t, T) = F(0, T) + \int_0^t b(s, T) \left[\int_s^T b(s, u) du \right] ds + \int_0^t b(s, T) dW_s$

(19.26) $r_t = F(0, t) + \int_0^t b(s, t) \left[\int_s^t b(s, u) du \right] ds + \int_0^t b(s, t) dW_s$

(19.33) $dF(t,T) = b^2(T-t)dt + bdW_t$

(19.34) $dB(t,T) = r_t B(t,T)dt - b(T-t)B(t,T)dW_t$ (This is a correction to the text formula)

(19.35) $r_t = F(0,t) + \frac{1}{2}b^2t^2 + bW_t$

(19.36) $dr_t = (F_t(0,t) + b^2t)dt + bdW_t$

(19.37) $F_t(0,t) = \frac{\partial F(0,t)}{\partial t}$

Chapter 20

(20.5) $B^1 = B(t,T_1)$

(20.6) $B^2 = B(t,T_2)$

(20.7) $dB^1 = \mu(B^1,t)B^1dt + \sigma_1(B^1,t)B^1dW_t$

(20.8) $dB^2 = \mu(B^2,t)B^2dt + \sigma_2(B^2,t)B^2dW_t$

(20.9) $dr_t = a(r_1,t)dt + b(r_1,t)dW_t$

(20.10) $\mathcal{P} = \theta_1 B^1 - \theta_2 B^2$

(20.11) $\theta_1 = \frac{\sigma_2}{B^1(\sigma_2 - \sigma_1)}\mathcal{P}$ (This is a correction to the text formula)

(20.12) $\theta_2 = \frac{\sigma_1}{B^2(\sigma_2 - \sigma_1)}\mathcal{P}$ (This is a correction to the text formula)

(20.13) $d\mathcal{P} = \theta_1 dB^1 - \theta_2 dB^2$

(20.15) $\left(\theta_1\sigma_1B^1 - \theta_2\sigma_2B^2\right) = \left(\frac{\sigma_2}{B^1(\sigma_2-\sigma_1)}\sigma_1B^1 - \frac{\sigma_1}{B^2(\sigma_2-\sigma_1)}\sigma_2B^2\right)\mathcal{P} = 0$

(This is a correction to the text formula)

(20.16) $d\mathcal{P} = (\theta_1\mu_1B^1 - \theta_2\mu_2B^2)dt$

(20.17) $d\mathcal{P} = \frac{(\sigma_2\mu_1 - \sigma_1\mu_2)}{(\sigma_2-\sigma_1)}\mathcal{P}dt$

(20.18) $r_t\mathcal{P}dt = \frac{(\sigma_2\mu_1 - \sigma_1\mu_2)}{(\sigma_2-\sigma_1)}\mathcal{P}dt$ (This is a correction to the text formula)

(20.19) $\frac{\mu_1 - r_t}{\sigma_1} = \frac{\mu_2 - r_t}{\sigma_2}$

(20.20) $\frac{\mu_i - r_t}{\sigma_i} = \lambda(r_t,t)$

(20.21) $dB(t,T) = B_r dr_t + B_t dt + \frac{1}{2}B_{rr}b(r_t,T)^2dt$ (This is a correction to the text formula)

(20.23) $dB(t,T) = \left(B_r a(r_t,t) + B_t + \frac{1}{2}B_{rr}b(r_t,T)^2\right)dt + b(r_t,t)B_r dW_t$ (This is a correction to the text formula)

(20.31) $B_r(a(r_t,t) - b(r_t,t)\lambda_t) + B_t + \frac{1}{2}B_{rr}b(r_t,t)^2 - r_tB = 0$

(20.33) $dr_t = (a(r_t,t) - b(r_t,t)\lambda_t)dt + b(r_t,t)\widetilde{W}_t$

(20.39) $B(0,T) = e^{\frac{1}{\alpha}(1-e^{-\alpha T})(R-r) - TR - \frac{b^2}{4\alpha^3}(1-e^{-\alpha T})^2}$

(20.40) $R = \kappa - \frac{b\lambda}{\alpha} - \frac{b^2}{\alpha^2}$

$$B(t,T) = A(t,T)e^{-C(t,T)r} \tag{20.48}$$

$$A(t,T) = \left(2\frac{\gamma e^{1/2(\alpha+\lambda+\gamma)T}}{(\alpha+\lambda+\gamma)(e^{\gamma T}-1)+2\gamma}\right)^{2\frac{\alpha\kappa}{b^2}} \tag{20.49}$$

$$C(t,T) = 2\frac{e^{\gamma T}-1}{(\alpha+\lambda+\gamma)(e^{\gamma T}-1)+2\gamma} \tag{20.50}$$

$$\gamma = \sqrt{(\alpha+\lambda)^2+2b^2} \tag{20.51}$$

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Chapter 14

(14.27) $dr_t = \gamma(\bar{r} - r_t)dt$

(14.29) $dr_t = \theta_t dt + \sigma dX_t$

(14.34) $r_t \sim \mathcal{N}(\mu(r_0,t), \sigma^2(t))$ where

(14.35) $\mu(r_0,t) = \bar{r} + (r_0 - \bar{r})e^{-\gamma t}$

(14.36) $\sigma^2(t) = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t})$

(14.37) $dP_t = \frac{1}{2}\left(\frac{d^2F}{dX^2}\right)dt + \left(\frac{dF}{dX}\right)dX_t$

(14.39) $dP_t = \left\{\left(\frac{\partial F}{\partial t}\right) + \frac{1}{2}\left(\frac{\partial^2 F}{\partial X^2}\right)\right\}dt + \left(\frac{\partial F}{\partial X}\right)dX_t$

Chapter 15

(15.9) $dr_t = \gamma(\bar{r} - r_t)dt + \sigma dX_t$

(15.22) $\frac{\left(\frac{\partial Z_1}{\partial t} + \frac{1}{2}\frac{\partial^2 Z_1}{\partial r^2}\sigma^2 - r_t Z_1\right)}{\partial Z_1/\partial r} = \frac{\left(\frac{\partial Z_2}{\partial t} + \frac{1}{2}\frac{\partial^2 Z_2}{\partial r^2}\sigma^2 - r_t Z_2\right)}{\partial Z_2/\partial r}$

(15.28) $Z(r,t;T) = e^{A(t;T) - B(t;T)\times r}$

(15.29) $B(t;T) = \frac{1}{\gamma^*}\left(1 - e^{-\gamma^*(T-t)}\right)$

(15.30) $A(t;T) = (B(t;T) - (T-t))\left(\bar{r}^* - \frac{\sigma^2}{2(\gamma^*)^2}\right) - \frac{\sigma^2 B(t;T)^2}{4\gamma^*}$

(15.31) $P_c(r,t;T) = \frac{100\times c}{2}\sum_{i=1}^{n} Z(r,t;T_i) + 100 \times Z(r,t;T_n)$

(15.34) $Z(r_t,\tau) = Z(r_t,t;T)$

(15.35) $A(\tau) = A(0;T-t)$

(15.36) $B(\tau) = B(0;T-t)$

(15.39) $D_Z(\tau) = -\frac{1}{Z}\frac{\partial Z}{\partial r} = B(\tau) = \frac{1}{\gamma^*}\left(1 - e^{-\gamma^*\times\tau}\right)$

(15.44) $V(r_0,0) = Z(r_0,0;T_B)\mathcal{N}(d_1) - KZ(r_0,0;T_O)\mathcal{N}(d_2)$ (This is a correction to the text formula)

(15.45) $d_1 = \frac{1}{S_Z(T_O;T_B)}\log\left(\frac{Z(r_0,0;T_B)}{KZ(r_0,0;T_O)}\right) + \frac{S_Z(T_O;T_B)}{2}$ (This is a correction to the text formula)

(15.46) $d_2 = d_1 - S_Z(T_O;T_B)$ (This is a correction to the text formula)

(15.47) $S_Z(T_O;T_B) = B(T_O;T_B)\times\sqrt{\frac{\sigma^2}{2\gamma^*}(1 - e^{-2\gamma^* T_O})}$ (This is a correction to the text formula)

(15.48) $V(r_0,0) = KZ(r_0,0;T_O)\mathcal{N}(-d_2) - Z(r_0,0;T_B)\mathcal{N}(-d_1)$ (This is a correction to the text formula)

(page 547) $P_c(r_K^*, T_0, T_B) = K$

(page 547) $K_i = Z(r_K^*, T_0; T_i),\ i = 1, 2, \ldots, n$

(15.51) $$\text{Call} = \sum_{i=1}^{n} c(i)(Z(r_0,0;T_i)\mathcal{N}(d_1(i)) - K_i Z(r_0,0;T_O)\mathcal{N}(d_2(i)))$$

(15.52) $$d_1(i) = \frac{1}{S_Z(T_O;T_i)} \log\left(\frac{Z(r_0,0;T_i)}{K_i Z(r_0,0;T_O)}\right) + \frac{S_Z(T_O;T_i)}{2}$$

(15.53) $$d_2(i) = d_1(i) - S_Z(T_O;T_i)$$

(15.55) $$\text{Put} = \sum_{i=1}^{n} c(i)(K_i Z(r_0,0;T_O)\mathcal{N}(-d_2(i)) - Z(r_0,0;T_i)\mathcal{N}(-d_1(i)))$$

(15.66) $$dr_t = \gamma(\bar{r} - r_t)dt + \sqrt{\alpha r_t}dX_t$$

(15.67) $$E[r_t|r_0] = \bar{r} + (r_0 - \bar{r})e^{-\gamma t}$$

(15.68) $$Var[r_t|r_0] = r_0\frac{\alpha}{\gamma}\left(e^{-\gamma t} - e^{-2\gamma t}\right) + \frac{\bar{r}\alpha}{2\gamma}\left(1 - e^{-\gamma t}\right)^2$$

(15.70) $$Z(r,t;T) = e^{A(t;T) - B(t;T)\times r}$$

(15.71) $$B(t;T) = \frac{2\left(e^{\psi_1(T-t)} - 1\right)}{(\gamma^* + \psi_1)\left(e^{\psi_1(T-t)} - 1\right) + 2\psi_1}$$

(15.72) $$A(t;T) = 2\frac{\bar{r}^*\gamma^*}{\alpha}\log\left(\frac{2\psi_1 e^{(\psi_1+\gamma^*)\frac{(T-t)}{2}}}{(\gamma^* + \psi_1)\left(e^{\psi_1(T-t)} - 1\right) + 2\psi_1}\right), \text{ and } \psi_1 = \sqrt{(\gamma^*)^2 + 2\alpha}$$

Chapter 16

(16.8) $$C_t = Z_1(r_t,t) - \Delta Z_2(r_t,t)$$

(16.9) $$P_t = \Delta Z_{2,t} + C_t$$

(16.10) $$dP_t = dZ_{1,t}$$

(16.18) $$\left(\frac{1}{\Pi}\frac{\partial \Pi}{\partial t}\right) + \frac{1}{2}\left(\frac{1}{\Pi}\frac{\partial^2 \Pi}{\partial r^2}\right)\sigma^2 = r$$

Chapter 18

(18.7) *Risk premium* $= E\left[\frac{dZ}{Z}\right]/dt - r = -B(t;T)(\gamma(\bar{r}-r) - \gamma^*(\bar{r}^*-r))$

(18.8) $\lambda(r,t) = \frac{1}{\sigma}(\gamma(\bar{r}-r) - \gamma^*(\bar{r}^*-r))$

(18.9) $\lambda(r,t) = \lambda_0 + \lambda_1 r$

(18.13) Risk premium $= E\left[\frac{dZ}{Z}\right]/dt - r = \sigma_Z \times \lambda(r,t)$

(18.16) *Risk premium* $= E\left[\frac{dZ}{Z}\right]/dt - r = \sigma_Z \lambda(r,t)$, where

(18.17) $\sigma_Z = \frac{1}{Z}\frac{\partial Z}{\partial r} s(r,t)$ and

(18.18) $\lambda(r,t) = \frac{1}{s(r,t)}(m(r,t) - m^*(r,t))$

(18.26) $V(r_{t+\delta}) \approx V(r_t) + \frac{\partial V}{\partial r}(r_{t+\delta} - r_t)$

(page 634) Standard deviation of $(V(r_{t+\delta}) - V(r_t)) \approx \frac{\partial V}{\partial r} \times$ Standard deviation of $(r_{t+\delta} - r_t)$

(18.28) $r_\delta - r_0 \sim N(\mu(r_0,\delta), \sigma^2(\delta))$

(18.29) $\mu(r_0,\delta) = (r_0 - \bar{r}) \times (e^{-\gamma\delta} - 1);\; \sigma(\delta) = \sqrt{\frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma\delta})}$

(18.32) $Z(t,T) = E_t\left[e^{-\rho(T-t)}\frac{Q_t Y_t^h}{Q_T Y_T^h}\right]$

(18.33) $Z(t,T) = E_t\left[e^{-\rho(T-t)-(q_T-q_t)-h(y_T-y_t)}\right]$

(18.34) $Z(i,t,T) = e^{A(t;T) - B(t;T)(i+c)}$

(18.35) $B(t;T) = \frac{1}{\gamma}\left(1 - e^{-\gamma(T-t)}\right)$

(18.36) $A(t;T) = (B(t;T) - (T-t))\left(\bar{r}^* - \frac{\sigma^2}{2\gamma^2}\right) - \frac{\sigma^2 B(t;T)^2}{4\gamma}$

(18.37) $c = \left(\rho + hg - \frac{1}{2}h^2\sigma_y^2\right) - h\sigma_y\sigma_q\rho_{qy} - \frac{1}{2}\sigma_q^2$

(18.38) $\bar{r}^* = \bar{r} - \frac{1}{\gamma}(h\sigma_i\sigma_y\rho_{yi} + \sigma_i\sigma_y\rho_{iq})$

(18.39) $\bar{r} = \bar{i} + c$

(18.40) $r_t = i_t + c$

(18.41) *Risk natural (true) dynamics:* $dr_t = \gamma(\bar{r} - r_t)dt + \sigma_i dX_i$

(18.42) *Risk neutral dynamics:* $dr_t = \gamma(\bar{r}^* - r_t)dt + \sigma_i dX_i$

(18.43) $\lambda = \frac{\gamma}{\sigma_i}(\bar{r} - \bar{r}^*) = h\sigma_y\rho_{yi} + \sigma_y\rho_{iq}$

Chapter 19

(19.7) $dr_t = \theta_t dt + \sigma dX_t$

(19.8) $Z(r, 0; T) = e^{A(0;T) - T \times r}$

(19.9) $A(0, T) = -\int_0^T (T - t)\theta_t dt + \frac{T^3}{6}\sigma^2$

(19.13) $\theta_t = \frac{\partial f(0, t)}{\partial t} + \sigma^2 \times t$

(19.14) *Payoff at* $T_O = \max(Z(T_O; T_B) - K, 0)$

(19.15) $V(r_0, 0) = Z(r_0, 0; T_B)\mathcal{N}(d_1) - KZ(r_0, 0; T_O)\mathcal{N}(d_2)$

Note: For this, and many of the following formulas, the text has $Z(0, r_0; \cdot)$ while $Z(r_0, 0; \cdot)$ is correct

(19.16) $d_1 = \frac{1}{S_Z(T_O; T_B)} \log\left(\frac{Z(r_0, 0; T_B)}{KZ(r_0, 0; T_O)}\right) + \frac{S_Z(T_O; T_B)}{2}$

(19.17) $d_2 = d_1 - S_Z(T_O; T_B)$

(19.18) $S_Z(T_O; T_B)^2 = \sigma^2 T_O (T_B - T_O)^2$

(19.19) *Payoff at* $T_O = \max(K - Z(T_O; T_B), 0)$

(19.20) $V(r_0, 0) = KZ(r_0, 0; T_O)\mathcal{N}(-d_2) - Z(r_0, 0; T_B)\mathcal{N}(-d_1)$

(19.25) $Z(r, 0; T) = e^{A(0;T) - B(0;T) \times r}$

(19.26) $B(0; T) = \frac{1}{\gamma^*}\left(1 - e^{-\gamma^* T}\right)$

(19.27) $A(0; T) = -\int_0^T B(0; T - t)\theta_t dt + \frac{\sigma^2}{2(\gamma^*)^2}\left(T + \frac{1 - e^{-2\gamma^* T}}{2\gamma^*} - 2B(0; T)\right)$

(This is a correction to the text formula)

(19.28) $\theta_t = \frac{\partial f(0, t)}{\partial t} + \gamma^* f(0, t) + \frac{\sigma^2}{2\gamma^*} \times \left(1 - e^{-2\gamma^* t}\right)$

(19.29) $\sigma_t(\tau) = \frac{B(\tau)}{\tau}\sigma$

(19.30) $V(r_0, 0) = Z(r_0, 0; T_B)\mathcal{N}(d_1) - KZ(r_0, 0; T_O)\mathcal{N}(d_2)$

(19.31) $d_1 = \frac{1}{S_Z(T_O; T_B)} \log\left(\frac{Z(r_0, 0; T_B)}{KZ(r_0, 0; T_O)}\right) + \frac{S_Z(T_O; T_B)}{2}$

(19.32) $d_2 = d_1 - S_Z(T_O; T_B)$

(19.33) $S_Z(T_O; T_B)^2 = B(T_O; T_B)^2 \frac{\sigma^2}{2\gamma^*}\left(1 - e^{-2\gamma^* T_O}\right)$

(19.34) $V(r_0, 0) = KZ(r_0, 0; T_O)\mathcal{N}(-d_2) - Z(r_0, 0; T_B)\mathcal{N}(-d_1)$

(19.36) *Payoff of call option at* $T_O = \max(P_c(r_{T_O}, T_O; T_B) - K, 0)$

(19.37) $$Call = \sum_{i=1}^{n} c(i)\left(Z(r_0,0;T_i)\mathcal{N}(d_1(i)) - K_i Z(r_0,0;T_O)\mathcal{N}(d_2(i))\right)$$

(19.38) $$d_1(i) = \frac{1}{S_Z(T_O;T_i)}\log\left(\frac{Z(r_0,0;T_i)}{K_i Z(r_0,0;T_O)}\right) + \frac{S_Z(T_O;T_i)}{2}$$

(19.39) $$d_2(i) = d_1(i) - S_Z(T_O;T_i)$$

(page 664) $$Z(r_t,t;T) = e^{A(t;T)-B(t;T)r_t}$$

(19.40) $$A(t;T) = -\int_t^T B(u;T)\theta_u du + \frac{\sigma^2}{2(\gamma^*)^2}\left((T-t) + \frac{1-e^{-2\gamma^*(T-t)}}{2\gamma^*} - 2B(t;T)\right)$$

(This is a correction to the text formula)

(19.41) $$A(t;T) = \log\left(\frac{Z(r_0,0;T)}{Z(r_0,0;t)}\right) + B(t;T)f(0,t) - \frac{\sigma^2}{4\gamma^*}B(t;T)^2\left(1-e^{-2\gamma^* t}\right)$$

(19.42) $$A(t;T) = \log\left(\frac{Z(r_0,0;T)}{Z(r_0,0;t)}\right) + (T-t)f(0,t) - \frac{\sigma^2}{2}(T-t)^2 t$$

(19.44) $$V(r_0,0) = M \times \left(KZ(r_0,0;T-\Delta)\mathcal{N}(-d_2) - Z(r_0,0;T)\mathcal{N}(-d_1)\right)$$

(19.45) $$d_1 = \frac{1}{S_Z(T-\Delta;T)}\log\left(\frac{Z(r_0,0;T)}{KZ(r_0,0;T-\Delta)}\right) + \frac{S_Z(T-\Delta;T)}{2}$$

(19.46) $$d_2 = d_1 - S_Z(T-\Delta;T)$$

(19.47) $$CF(T_j) = \Delta \times N \times \max(r_n(T_{j-1},T_j) - r_K, 0)$$

(19.48) $$Cap = \sum_{j=2}^{n} M \times \left(KZ(r_0,0;T_{j-1})\mathcal{N}(-d_2(j)) - Z(r_0,0;T_j)\mathcal{N}(-d_1(j))\right)$$

(19.49) $$d_1(j) = \frac{1}{S_Z(T_{j-1};T_j)}\log\left(\frac{Z(r_0,0;T_j)}{KZ(r_0,0;T_{j-1})}\right) + \frac{S_Z(T_{j-1};T_j)}{2}$$

(19.50) $$d_2(j) = d_1(j) - S_Z(T_{j-1};T_j)$$

(19.55) $$dy_t = \left(\theta_t + \frac{\partial\sigma_t/\partial t}{\sigma_t}y_t\right)dt + \sigma_t dX_t$$

(19.57) $$dy_t = (\theta_t - \gamma_t y_t)dt + \sigma_t dX_t$$

(19.58) $$dr_t = (\theta_t - \gamma_t r_t)dt + \sqrt{\sigma_t^2 + \alpha_t r_t}\,dX_t$$

(19.59) $$Z(r_t,t;T) = e^{A(t;T)-B(t;T)r_t}$$

(19.60) $$\frac{\partial B(t;T)}{\partial t} = B(t;T)\gamma_t + \frac{1}{2}B(t;T)^2\alpha_t - 1$$

(19.61) $$\frac{\partial A(t;T)}{\partial t} = B(t;T)\theta_t - \frac{1}{2}B(t;T)^2\sigma_t^2$$

Chapter 20

(20.3) $Caplet(0;T_{i+1}) = N \times \Delta \times Z(0,T_{i+1}) \times [f_n(0,T_i,T_{i+1})\mathcal{N}(d_1) - r_K\mathcal{N}(d_2)]$

(20.4) $d_1 = \frac{1}{\sigma_f\sqrt{T_i}}\log\left(\frac{f_n(0,T_i,T_{i+1})}{r_K}\right) + \frac{1}{2}\sigma_f\sqrt{T_i}$

(20.5) $d_2 = d_1 - \sigma_f\sqrt{T_i}$

(20.6) $Floorlet(0;T_{i+1}) = N \times \Delta \times Z(0,T_{i+1}) \times [r_K\mathcal{N}(-d_2) - f_n(0,T_i,T_{i+1})\mathcal{N}(-d_1)]$

(20.7) $Cap(0;T) = \sum_{i=1}^{n} Caplets(0;T_i)$

(20.17) $V(0,T_O;T_S) = N \times \Delta \times \left[\sum_{i=1}^{n} Z(0;T_i)\right] \times [r_K\mathcal{N}(-d_2) - f_n^s(0,T_O,T_S)\mathcal{N}(-d_1)]$

(20.18) $d_1 = \frac{1}{\sigma_f^s\sqrt{T_O}}\ln\left(\frac{f_n^s(0,T_O,T_S)}{r_K}\right) + \frac{1}{2}\sigma_f^s\sqrt{T_O}; \quad d_2 = d_1 - \sigma_f^s\sqrt{T_O}$

(This is a correction to the text formula)

(20.19) $V(0,T_O;T_S) = N \times \Delta \times \left[\sum_{i=1}^{n} Z(0;T_i)\right] \times [f_n^s(0,T_O,T_S)\mathcal{N}(d_1) - r_K\mathcal{N}(d_2)]$

Chapter 21

(21.2) $V(r,t;T) = E^*\left[e^{-\int_t^T r_u du} g_T\right]$

(21.3) $dr_t = m^*(r_t,t)dt + s(r_t,t)dX_t$

(21.4) $\widetilde{V}(r,t;T) = \frac{V(r,t;T)}{Z(r,t;T)}$

(21.5) $0 = \frac{\partial\widetilde{V}}{\partial t} + \frac{\partial\widetilde{V}}{\partial r}(m^*(r,t) + \sigma_Z(r,t)s(r,t)) + \frac{1}{2}\frac{\partial^2\widetilde{V}}{\partial r^2}s(r,t)^2$

(21.6) $\sigma_Z(r,t) = \frac{1}{Z}\frac{\partial Z}{\partial r}s(r,t)$

(21.7) $\frac{dZ}{Z} = \mu_Z(r,t)dt + \sigma_Z(r,t)dX_t$

(21.8) $\widetilde{V}(r,t;T) = E_f^*[g_T]$

(21.9) $dr_t = (m^*(r,t) + \sigma_Z(r,t)s(r,t))dt + s(r,t)dX_t$

(21.10) $V(r,t;T) = Z(r,t;T)E_f^*[g_T]$

(21.11) $E_f^*[\max(g_T - K, 0)] = F(0,T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)$

(21.12) $d_1 = \frac{1}{\sigma_T}\log\left(\frac{F(0,T)}{K}\right) + \frac{1}{2}\sigma_T$

(21.13) $d_2 = d_1 - \sigma_T$

(21.14) $Call = Z(0,T) \times [F(0,T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)]$

(21.15) $Put = Z(0,T) \times [K\mathcal{N}(-d_2) - F(0,T)\mathcal{N}(-d_1)]$

(21.27) $V^{fwd}(0;T) = Z(0,T)N\Delta E_f^*[r_n(\tau,T) - K]$

(21.28) $\frac{df_n(t,\tau,T)}{f_n(t,\tau,T)} = \sigma_f(t)dX_t$

(21.29) $r_n(\tau,T) \sim LogN\left(f_n(0,\tau,T), \int_0^\tau \sigma_f(t)^2dt\right)$

(21.32) $$Caplet(0;T_{i+1}) = N\Delta Z(0,T_{i+1})[f_n(0,T_i,T_{i+1})\mathcal{N}(d_1) - r_K\mathcal{N}(d_2)]$$

(21.33) $$d_1 = \frac{1}{\sigma_f\sqrt{T_i}}\log\left(\frac{f_n(0,T_i,T_{i+1})}{r_K}\right) + \frac{1}{2}\sigma_f\sqrt{T_i}$$

(21.34) $$d_2 = d_1 - \sigma_f\sqrt{T_i}$$

(21.37) $$\frac{df_n(t,T_i,T_{i+1})}{f_n(t,T_i,T_{i+1})} = \left(\sum_{j=\bar{i}}^{i}\frac{\Delta f_n(t,T_j,T_{j+1})\sigma_f^{i+1}(t)\sigma_f^{j+1}(t)}{1+\Delta f_n(t,T_j,T_{j+1})}\right)dt + \sigma_f^{i+1}(t)dX_t$$

(21.38) $$\frac{df_n(t,T_i,T_{i+1})}{f_n(t,T_i,T_{i+1})} = -\left(\sum_{j=i}^{\bar{i}-1}\frac{\Delta f_n(t,T_j,T_{j+1})\sigma_f^{i+1}(t)\sigma_f^{j+1}(t)}{1+\Delta f_n(t,T_j,T_{j+1})}\right)dt + \sigma_f^{i+1}(t)dX_t$$

(21.39) $$\sigma_f^{Fwd}(T_{i+1})^2 \times (T_i - t) = S_i^2 \times (T_1 - t) + S_{i-1}^2 \times \Delta + \ldots + S_1^2 \times \Delta$$

(page 722) $$S_1 = \sigma_f^{Fwd}(0.25)$$

(page 722) $$S_i = \sqrt{\frac{T_i}{\Delta}\left(\sigma_f^{Fwd}(T_{i+1})\right)^2 - \sum_{j=1}^{i-1}S_j^2}$$ (This is a correction to the text formula)

(21.42) $$f_n^s(t+\delta,T_i,T_{i+1}) = f_n^s(t,T_i,T_{i+1})e^{m_{i+1}^s(t)\delta + S(T_{i+1}-t)\sqrt{\delta}\epsilon_t^s}$$

(page 723) $$m_{i+1}^s(t) = \sum_{j=\bar{i}}^{i}\frac{\Delta f_n^s(t,T_j,T_{j+1})S(T_{i+1}-t)S(T_{j+1}-t)}{1+\Delta f_n^s(t,T_j,T_{j+1})} - \frac{1}{2}S(T_{i+1}-t)^2$$

(21.54) $$df(t,T) = m(t,T)dt + \sigma_f(t,T)dX_t$$

(21.55) $$m(t,T) = \sigma_f(t,T)\int_t^T \sigma_f(t,\tau)d\tau$$

(21.59) $$f(0,\tau,T) = f^{fut}(0,\tau,T) - \int_0^\tau \frac{\sigma_Z(t,T)^2 - \sigma_Z(t,\tau)^2}{2(T-\tau)}dt$$ (This is a correction to the text formula)

(21.60) $$f(0,\tau,T) = f^{fut}(0,\tau,T) - \frac{1}{2}\sigma^2\tau T$$

Chapter 22

(22.3) $$dP_t = \left\{\left(\frac{\partial F}{\partial t}\right) + \left(\frac{\partial F}{\partial \phi_1}\right) m_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) m_{2,t} + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_1^2}\right) s_{1,t}^2 + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_2^2}\right) s_{2,t}^2\right\} dt + \left(\frac{\partial F}{\partial \phi_1}\right) s_{1,t} dX_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) s_{2,t} dX_{2,t}$$

(22.13) $$R(\phi_1, \phi_2)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \phi_1} m_{1,t}^* + \frac{\partial V}{\partial \phi_2} m_{2,t}^* + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_1^2} s_{1,t}^2 + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_2^2} s_{2,t}^2$$

(22.19) $$Z(\phi_{1,t}, \phi_{2,t}, t; T) = e^{A(t;T) - B_1(t;T)\phi_{1,t} - B_2(t;T)\phi_{2,t}}$$

(22.20) $$B_i(t;T) = \frac{1}{\gamma_i^*}\left(1 - e^{\gamma_i^*(T-t)}\right)$$

(22.21) $$A(t;T) = (B_1(t;T) - (T-t))\left(\bar{\phi}_1^* - \frac{\sigma_1^2}{2(\gamma_1^*)^2}\right) - \frac{\sigma_1^2}{4\gamma_1^*} B_1(t;T)^2 + (B_2(t;T) - (T-t))\left(\bar{\phi}_2^* - \frac{\sigma_2^2}{2(\gamma_2^*)^2}\right) - \frac{\sigma_2^2}{4\gamma_2^*} B_2(t;T)^2$$

(22.27) $$Z(r_t, r_{\ell,t}, t; T) = e^{A_{\tau_\ell}(\tau) - B_{\tau_\ell,1}(\tau) r_t - C_{\tau_\ell}(\tau) r_{\ell,t}}, \text{ where } \tau = T - t$$

(22.28) $$A_{\tau_\ell}(\tau) = A(\tau) - C(\tau) \times \frac{A(\tau_\ell)}{C(\tau_\ell)}$$

(22.29) $$B_{\tau_\ell,1}(\tau) = B_1(\tau) - C(\tau) \times \frac{B_1(\tau_\ell)}{C(\tau_\ell)}$$

(22.30) $$C_{\tau_\ell}(\tau) = C(\tau) \times \frac{\tau_\ell}{C(\tau_\ell)}$$

(22.31) $$r_t(\tau) = -\frac{A_{\tau_\ell}(\tau)}{\tau} + \frac{B_{\tau_\ell,1}(\tau)}{\tau} r_t + \frac{C_{\tau_\ell}(\tau)}{\tau} r_{\ell,t}$$

(22.34) $$\sigma_{\ell,1} = \sigma_1 \frac{1 - e^{-\gamma_1^* \tau_\ell}}{\tau_\ell}; \quad \sigma_{\ell,2} = \sigma_2 \frac{1 - e^{-\gamma_2^* \tau_\ell}}{\tau_\ell}$$

(22.38) $$\text{Vasicek volatility of } dr_t(\tau) = \sigma_t(\tau) = \frac{\sigma}{\gamma^*}\frac{1 - e^{-\gamma^* \tau}}{\tau}$$

(22.39) $$\text{Volatility of } dr_t(\tau) = \sigma_t(\tau) = \sqrt{\sigma_1^2\left(\frac{B_1(\tau)}{\tau}\right)^2 + \sigma_2^2\left(\frac{B_2(\tau)}{\tau}\right)^2}$$

(22.41) $$V(\phi_{1,0}, \phi_{2,0}, 0) = Z(\phi_{1,0}, \phi_{2,0}, 0; T_B)\mathcal{N}(d_1) - KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O)\mathcal{N}(d_2)$$

(22.42) $$d_1 = \frac{1}{S_Z(T_O)} \log\left(\frac{Z(\phi_{1,0}, \phi_{2,0}, 0; T_B)}{KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O)}\right) + \frac{S_Z(T_O)}{2}$$

(22.43) $d_2 = d_1 - S_Z(T_O)$ (This is a correction to the text formula)

(22.44) $$V(\phi_{1,0}, \phi_{2,0}, 0) = -Z(\phi_{1,0}, \phi_{2,0}, 0; T_B)\mathcal{N}(-d_1) + KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O)\mathcal{N}(-d_2)$$

(22.46) $$d\phi_{1,t} = m_1(\phi_{1,t}, \phi_{2,t}, t)dt + s_1(\phi_{1,t}, \phi_{2,t}, t)dX_{1,t}$$

(22.47) $$d\phi_{2,t} = m_2(\phi_{1,t}, \phi_{2,t}, t)dt + s_2(\phi_{1,t}, \phi_{2,t}, t)dX_{2,t}$$

(22.48) $$dP_t = \left\{\left(\frac{\partial F}{\partial t}\right) + \left(\frac{\partial F}{\partial \phi_1}\right) m_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) m_{2,t} + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_1^2}\right) s_{1,t}^2 + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_2^2}\right) s_{2,t}^2 + \left(\frac{\partial^2 F}{\partial \phi_1 \phi_2}\right) s_{1,t} s_{2,t} \rho\right\} dt + \left(\frac{\partial F}{\partial \phi_1}\right) s_{1,t} dX_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) s_{2,t} dX_{2,t}$$

(22.49) $$R(\phi_1,\phi_2)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \phi_1} m^*_{1,t} + \frac{\partial V}{\partial \phi_2} m^*_{2,t} + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_1^2} s^2_{1,t} + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_2^2} s^2_{2,t} + \frac{\partial^2 V}{\partial \phi_1 \phi_2} s_{1,t}s_{2,t}\rho$$

(22.61) $$V(\phi_{1,t},\phi_{2,t},t) = E^*\left[e^{-\int_t^T R(\phi_{1,u},\phi_{2,u})du} g_T | \phi_{1,t},\phi_{2,t}\right]$$

(22.62) $$d\phi_{1,t} = m^*_{1,t}dt + s_{1,t}dX_{1,t}$$

(22.63) $$d\phi_{2,t} = m^*_{2,t}dt + s_{2,t}dX_{2,t}$$

The Volatility Smile, Derman and Miller

Chapter 3

(3.3) $$C(S,t) - P(S,t) = S - Ke^{-r(T-t)}$$

(3.9) $$C(S+dS,t+dt) = C(S,t) + \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}dS^2 + \cdots$$

(3.11) $$C(S+dS,t+dt) = C(S,t) + \Theta dt + \Delta dS + \frac{1}{2}\Gamma dS^2$$

(3.16) $$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

(3.17) $$\frac{\partial C}{\partial t} + \frac{1}{2}\Gamma\Sigma^2 S^2 = 0$$

Chapter 4

(4.1) $$C(S,K,\tau,\sigma,r) = SN(d_1) - Ke^{-r\tau}N(d_2), \quad d_{1,2} = \frac{\ln\left(\frac{S}{K}\right) + \left(r \pm \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \qquad N(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{1}{2}y^2}dy$$

(4.12) $$\kappa_\pi = \int_0^\infty \rho(xS)S^2 f(x,\nu,\tau)dx$$

(4.32) $$\frac{1}{T}\int_0^T \sigma_t^2 dt = \frac{2}{T}\left[\int_0^T \frac{1}{S}dS - \ln\left(\frac{S_T}{S_0}\right)\right]$$

(4.41) $$\pi(S_T,S_0,T,T) = \frac{2}{T}\left[\left(\frac{S_T - S_0}{S_0}\right) - \ln\left(\frac{S_T}{S_0}\right)\right]$$

(4.43) $$\sigma(K) = \sigma_F - b\frac{K - S_F}{S_F}$$

(4.44) $$\sigma_K^2 = \sigma_F^2(1 + 3Tb^2)$$

Chapter 5

(5.1) $dS = \mu_S S dt + \sigma_S S dZ, \quad dB = Brdt$

(5.2) $$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2 dt$$
$$= \left\{\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu_S S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2\right\}dt + \frac{\partial C}{\partial S}\sigma_S S dZ$$
$$= \mu_C C dt + \sigma_C C dZ$$

(5.3) $$\mu_C = \frac{1}{C}\left\{\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu_S S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2\right\}, \quad \sigma_C = \frac{S}{C}\frac{\partial C}{\partial S}\sigma_S = \frac{\partial \ln C}{\partial \ln S}\sigma_S$$

(5.10) $$\frac{(\mu_C - r)}{\sigma_C} = \frac{(\mu_S - r)}{\sigma_S}$$

(5.12) $$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

(5.13) $$C(S, K, t, T, \sigma, r) = e^{-r(T-t)}[S_F N(d_1) - KN(d_2)], \quad S_F = e^{r(T-t)}S$$

$$d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = \frac{\ln\left(\frac{S_F}{K}\right) - \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

(5.19) $$C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x)[dS_x - S_x r dx]e^{-rx}$$

(5.20) $$C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x)\sigma S_x e^{-rx} dZ_x$$

(5.21) $E[C_0] = E[C_T]e^{-rT}$

(5.22) $\pi(I, R) = V_I - \Delta_R S$

(5.23) $\mathrm{PV}[\mathrm{P\&L}(I, R)] = V(S, \tau, \sigma_R) - V(S, \tau, \Sigma)$

(5.25) $$\Delta_R = e^{-D\tau}N(d_1), \quad d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \frac{1}{2}\sigma_R^2\tau}{\sigma_R\sqrt{\tau}}, \quad S_F = Se^{(r-D)\tau}$$

(5.27) $d\mathrm{P\&L}(I, R) = dV_I - rV_I dt - \Delta_R[dS - (r - D)Sdt]$

(5.28) $d\mathrm{P\&L}(R, R) = 0 = dV_R - V_R r dt - \Delta_R[dS - (r - D)Sdt]$

(5.34) $$\mathrm{PV}[\mathrm{P\&L}(I, R)] = e^{rt_0}\left[e^{-rT}\cdot 0 - e^{-rt_0}(V_{I,t} - V_{R,t})\right] = V_{R,t} - V_{I,t}$$

(5.38) $$d\mathrm{P\&L}(I, R) = \frac{1}{2}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt + (\Delta_I - \Delta_R)[(\mu - r + D)Sdt + \sigma_R S dZ]$$

(page 100) The upper bound of the P&L is ... $(V_{R,0} - V_{I,0})$

(5.41) $$\mathrm{PV}[\pi(I, R)]_L = (V_{R,0} - V_{I,0}) - 2Ke^{-2r\tau}\left[N\left(\frac{1}{2}(\sigma_R - \Sigma)\sqrt{\tau}\right) - \frac{1}{2}\right]$$

(This is a correction to the text formula)

(5.42) $$d\mathrm{P\&L}(I, I) = \frac{1}{2}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt$$

(5.43) $$\mathrm{PV}[\mathrm{P\&L}(I, I)] = \frac{1}{2}\int_{t_0}^T e^{-r(t-t_0)}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt$$

(page 103, problem 5-4) $$\mathrm{PV}[\mathrm{P\&L}(I, H)] = V_h - V_I + \frac{1}{2}\int_{t_0}^T e^{-r(t-t_0)}\Gamma_h S^2(\sigma_R^2 - \sigma_h^2)dt$$

Chapter 6

(6.2) $$\pi = C - \frac{\partial C}{\partial S} S$$

(6.6) $$HE \approx \sum_{i=1}^{n} \frac{1}{2} \Gamma_i \sigma_i^2 S_i^2 (Z_i^2 - 1) dt$$

(6.7) $$\sigma_{HE}^2 \approx E\left[\sum_{i=1}^{n} \frac{1}{2} (\Gamma_i S_i^2)^2 (\sigma_i^2 dt)^2\right]$$

(6.12) $$\sigma_{HE} \approx \sqrt{\frac{\pi}{4}} \frac{\sigma}{\sqrt{n}} \frac{\partial C}{\partial \sigma}$$

(6.14) $$\sigma_{HE} \approx dC \approx \frac{\partial C}{\partial \sigma} d\sigma \approx \frac{\sigma}{\sqrt{n}} \frac{\partial C}{\partial \sigma}$$

(6.18) $$\frac{\sigma_{HE}}{C} \approx \sqrt{\frac{\pi}{4n}} \approx \frac{0.89}{\sqrt{n}}$$

Chapter 7

(7.14) $$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \frac{\partial C}{\partial t} - \sqrt{\frac{2}{\pi dt}} \left|\frac{\partial^2 C}{\partial S^2}\right| \sigma S^2 k = r\left(C - S\frac{\partial C}{\partial S}\right)$$

(7.18) $$\breve{\sigma}^2 = \sigma^2 + 2\sigma k \sqrt{\frac{2}{\pi dt}}$$

(7.19) $$\breve{\sigma} \approx \sigma \pm k \sqrt{\frac{2}{\pi dt}}$$

Chapter 8

(8.3) $$P[\ln(S_T) > \ln(K)] = P\left[Z > \frac{-\ln\left(\frac{S_t}{K}\right) - \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right] = N(d_2)$$

(8.6) $$\Delta_{\text{ATM}} \approx \frac{1}{2} + \frac{d_1}{\sqrt{2\pi}} \approx \frac{1}{2} + \frac{\sigma\sqrt{\tau}}{2\sqrt{2\pi}}$$

(8.9) $$\Delta \approx \Delta_{\text{ATM}} - \frac{1}{\sqrt{2\pi}} \frac{J}{\nu}$$

Chapter 10

(10.3) $$S = V - B,\ \frac{dS}{S} = \frac{dV}{S} = \frac{V\sigma dZ}{S} = \sigma \frac{S+B}{S} dZ,\ \sigma_S = \sigma\left(1 + \frac{B}{S}\right)$$

(10.4) $$\frac{dS}{S} = \mu(S,t)dt + \sigma S^{\beta-1} dZ$$

(10.5) $$dS = \mu S dt + \sigma S dZ,\ d\sigma = p\sigma dt + q\sigma dW,\ E[dW dZ] = \rho dt$$

(10.10) $$\text{Profit} = \frac{1}{2}\Gamma S^2(\sigma^2 - \Sigma^2)dt = \frac{1}{2}\Gamma(dS)^2 - \frac{1}{2}\Gamma S^2 \Sigma^2 dt$$

(10.15) $$D = -\frac{\partial C_{\text{BSM}}}{\partial K} - \frac{\partial C_{\text{BSM}}}{\partial \Sigma} \frac{\partial \Sigma}{\partial K}$$

Chapter 11

(11.28) $$x = \frac{\ln\left(\frac{S_T}{S_t}\right) - \left(r\tau - \frac{1}{2}\sigma^2\tau\right)}{\sigma\sqrt{\tau}}$$

(11.29) $$N'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$$

(11.34) $$p(S_t, t, S_T, T) = \frac{e^{-\frac{x^2}{2}}}{\sigma S_T\sqrt{2\pi\tau}}$$

Chapter 14

(14.4) $$F = Se^{(r-b)dt}$$

(14.5) $$F = qS_u + (1-q)S_d$$

(14.6) $$q = \frac{F - S_d}{S_u - S_d}$$

(14.11) $$S_u = Se^{\sigma(S,t)\sqrt{dt}},\ S_d = Se^{-\sigma(S,t)\sqrt{dt}}$$

(14.17) $$\Sigma(S,K) \approx \sigma_0 + \frac{\beta}{2}(S+K)$$

(14.18) $$\Sigma(S,K) \approx \sigma(S) + \frac{\beta}{2}(K-S)$$

Chapter 17

(17.6) $$d\pi_{\text{BSM}} = d\pi_{\text{loc}} - \varepsilon dS = \frac{1}{2}\Gamma_{\text{loc}}S^2\left[\sigma_R^2 - \sigma_{\text{loc}}^2(S,t)\right]dt - \varepsilon dS$$

Chapter 18

(18.6) $$\Sigma_{\text{ATM}}(S) = \Sigma(S,S) = \Sigma_0 - \beta(S - S_0)$$

(18.7) $$\Sigma(S,K) = \Sigma_0 - \beta(K-S)$$

(18.11) $$\Sigma(S,K,t,T) = \Sigma_0(t,T) - \beta'(t,T)[0.5 - \Delta(S,K,t,T,\Sigma_{\text{ATM}}(S))]$$

(18.12) $$\Sigma(S,K,\tau) = \Sigma_0 - \beta'(0.5 - \Delta(S,K,\tau,\Sigma_{\text{ATM}}))$$

Chapter 19

(19.8) $$\frac{dV}{V} = \alpha dt + \xi dW \text{ where } V = \sigma^2$$

(19.9) $$dY = \alpha(m - Y)dt + \beta dW$$

(19.15) $$Y_t = m + (Y_0 - m)e^{-\alpha t} + \beta\int_0^t e^{-\alpha(t-s)}dW_s$$

(19.21) $$\text{Var}[Y_t] = \frac{\beta^2}{2\alpha}\left(1 - e^{-2\alpha t}\right)$$

(19.24) $$d\sigma = \alpha(m-\sigma)dt + \beta dW$$

(19.25) $$dV = \alpha(m-V)dt + \beta dW$$

(19.26) $$dV = \alpha(m-V)dt + \beta V dW$$

(19.27) $$dV = \alpha(m-V)dt + \beta\sqrt{V}dW$$

QFIQ-120-19: Chapter 6 and 7 of Pricing and Hedging Financial Derivatives, Marroni and Perdomo

(page 124) Delta(call) = $N(d_1)$

(page 124) Delta(put) = $N(d_1) - 1$

(page 127) Gamma = $\dfrac{N'(d_1)}{S\sigma\sqrt{T-t}}$

(page 128) $d_1 = \dfrac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$

(page 128) $N'(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$

(page 133) Vega = $S\sqrt{T-t}N'(d_1)$

(page 137) Theta(call) = $-\dfrac{SN'(d_1)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$

(page 137) $d_1 = \dfrac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$

(page 137) $d_2 = \dfrac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$

(page 137) Theta(put) = $-\dfrac{SN'(d_1)\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}N(-d_2)$

(page 138) Theta $\Delta t + \dfrac{1}{2}$Gamma $\Delta S^2 \cong 0$

(page 139) $\dfrac{\Delta S}{S} \cong \dfrac{\sigma}{\sqrt{252}}$

(page 143) Rho(call) $= K(T-t)e^{-r(T-t)}N(d_2)$

(page 143) Rho(put) $= -K(T-t)e^{-r(T-t)}N(-d_2)$ (This is a correction to the text formula)

(page 180) $S(k) = S(k-1) * [\text{yield}(t) * dt + \text{vol}(t, S) * W * \text{sqrt}(dt)]$

(page 181) $\text{vol}^2 * (t-s) = \text{vol\_mkt}^2(t) * t - \text{vol\_mkt}^2(s) * s$

QFIQ-128-20: Mitigating Interest Rate Risk in Variable Annuities: An Analysis of Hedging Effectiveness under Model Risk

(1) $dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_t^S$

(2) $d\nu_t = \kappa(\theta - \nu_t)dt + \sigma_\nu \sqrt{\nu}_t dW_t^\nu$

(page 505) $dr_t^{(j)} = a_j(b_j - r_t^{(j)})dt + \sigma_{t,j}\sqrt{r_t^{(j)}} dW_{t,j}^r$

(page 506) $\Pi_{t+h} = (\Pi_t - \Delta_t S_t - n_t P_{t,t+T^B})B_{t+h}/B_t + \Delta_t S_{t+h} + n_t P_{t+h,t+T^B}$

(page 506) $dS_t = r_t S_t dt + \sigma_S S_t dZ_t^S$

(page 506) $dr_t = (\upsilon(t) - ar_t)dt + \sigma_r dZ_t^r$

(page 508) $dA_t = (\mu - \alpha)A_t dt - \omega_t dt + \sqrt{\nu_t} A_t dW_t^S$

(page 508) $L_T = L_T^{(D)} + L_T^{(A)}$

(page 508) $L_T^{(D)} = \int_0^T \left[\max(G_t - A_t, 0)B_{t,T} - \int_0^t \alpha A_s B_{s,T} ds\right] {}_t p_x u_{x+t} dt$

(page 509) $L_T^{(A)} = \left[\max(G_T - A_T, 0) - \int_0^T \alpha A_t B_{t,T} dt\right] {}_T p_x$

(3) $\Omega_t = \mathbb{E}^{\mathbb{Q}}\left[\int_t^T \beta_{t,v} \max(A_v, G_v)\ {}_{v-t}p_{x+t} u_{x+v} dv + \beta_{t,T} \max(A_T, G_T)\ {}_{T-t}p_{x+t} | \mathcal{F}_t\right]$

(page 510) $L_T = \frac{A_0}{T} \int_\tau^T B_{t,T} dt - \int_0^\tau \alpha A_t B_{t,T} dt$

(5) $\Omega_t = \frac{A_0}{T} \int_t^T P_{t,v} dv + \mathbb{E}^{\mathbb{Q}}[\beta_{t,T} A_T | \mathcal{F}_t]$

(page 512) $RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^N \left(HL_T^{(i)}\right)^2}$

(page 512) $CTE(1-p)\% = \frac{1}{N_p}\sum_{i=1}^{N_p} HL_T^{(i)}$

(page 522) $P_{t,T} = \mathcal{A}(t,T)e^{-\mathcal{B}(t,T)r_t}$

(page 522) $\mathcal{B}(t,T) = \frac{1}{a}\left[1 - e^{-a(T-t)}\right]$

(page 522) $\rho_t^B = \frac{\partial P_{t,t+T^B}}{\partial r_t} = -\mathcal{B}(t, t+T^B)P_{t,t+T^B}$

(page 523)
$$\Delta_t = \frac{\partial \mathcal{L}_t}{\partial A_t} \times \frac{\partial A_t}{\partial S_t} = \left[\int_t^T \frac{\partial \Psi(t,v,A_t,G_v)}{\partial A_t}\ {}_{v-t}p_{x+t} u_{x+v} dv + \frac{\partial \Psi(t,T,A_t,G_T)}{\partial A_t}\ {}_{T-t}p_{x+t} - \left(1 - \int_t^T e^{-\alpha(v-t)}\ {}_{v-t}p_{x+t} u_{x+v} dv - e^{-\alpha(T-t)}\ {}_{T-t}p_{x+t}\right)\right] \times \frac{A_t}{S_t}$$

(This is a correction to the text formula)

(page 523) $\rho_t = \int_t^T \frac{\partial \Psi(t,v,A_t,G_v)}{\partial r_t}\ {}_{v-t}p_{x+t} u_{x+v} dv + \frac{\partial \Psi(t,T,A_t,G_T)}{\partial r_t}\ {}_{T-t}p_{x+t}$

(page 525) $\Delta_t = \left(e^{-\alpha(T-t)}\Phi(z) - 1\right) \times \frac{A_t}{S_t}$

(page 525) $\rho_t = -\frac{A_0}{T}\int_t^T \mathcal{B}(t,v)P_{t,v}dv + e^{-\alpha(T-t)}\frac{A_0}{T}\int_t^T e^{\alpha(v-t)}\mathcal{B}(t,v)P_{t,v}\Phi(z-m_v)dv$

(page 525) $dv_t = \tilde{\kappa}(\tilde{\theta} - v_t)dt + \sigma_v\sqrt{v_t}d\tilde{W}_t^v$

(page 525) $\text{1-year}VIX_t = \sqrt{\mathbb{E}^{\mathbb{Q}^{\mathbb{P}}}\left[\int_t^{t+1} v_s ds|\mathcal{F}_t\right]} = \sqrt{A + Bv_t}$

(page 525) $A = \tilde{\theta}\left(1 - \frac{1-e^{-\tilde{\kappa}}}{\tilde{\kappa}}\right), \quad B = \frac{1-e^{-\tilde{\kappa}}}{\tilde{\kappa}}$

QFIQ-130-21: Interest Rate Models-Theory and Practice, 2nd ed., Brigo and Mercurio

(4.4) $r(t) = x(t) + y(t) + \varphi(t),\ r(0) = r_0,$ where $\{x(t) : t \geq 0\}$ and $\{y(t) : t \geq 0\}$ satisfy

(4.5) $dx(t) = -ax(t)dt + \sigma dW_1(t),\ x(0) = 0$ and $dy(t) = -by(t)dt + \eta dW_2(t),\ y(0) = 0$

(page 144) $r(t) = x(s)e^{-a(t-s)} + y(s)e^{-b(t-s)} + \sigma\int_s^t e^{-a(t-u)}dW_1(u) + \eta\int_s^t e^{-b(t-u)}dW_2(u) + \varphi(t)$

(4.6) $E\{r(t)|\mathcal{F}_s\} = x(s)e^{-a(t-s)} + y(s)e^{-b(t-s)} + \varphi(t)$

$$\text{Var}\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2a}\left[1 - e^{-2a(t-s)}\right] + \frac{\eta^2}{2b}\left[1 - e^{-2b(t-s)}\right] + 2\rho\frac{\sigma\eta}{a+b}\left[1 - e^{-(a+b)(t-s)}\right]$$

(4.7) $r(t) = \sigma\int_0^t e^{-a(t-u)}dW_1(u) + \eta\int_0^t e^{-b(t-u)}dW_2(u) + \varphi(t)$

(4.10)
$$V(t,T) = \frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right]$$
$$+\frac{\eta^2}{b^2}\left[T - t + \frac{2}{b}e^{-b(T-t)} - \frac{1}{2b}e^{-2b(T-t)} - \frac{3}{2b}\right]$$
$$+2\rho\frac{\sigma\eta}{ab}\left[T - t + \frac{e^{-a(T-t)} - 1}{a} + \frac{e^{-b(T-t)} - 1}{b} - \frac{e^{-(a+b)(T-t)} - 1}{a+b}\right]$$

(4.11) $P(t,T) = \exp\left\{-\int_t^T \varphi(u)du - \frac{1-e^{-a(T-t)}}{a}x(t) - \frac{1-e^{-b(T-t)}}{b}y(t) + \frac{1}{2}V(t,T)\right\}$

(4.14) $P(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\exp\{\mathcal{A}(t,T)\}$

$$\mathcal{A}(t,T) := \frac{1}{2}[V(t,T) - V(0,T) + V(0,t)] - \frac{1-e^{-a(T-t)}}{a}x(t) - \frac{1-e^{-b(T-t)}}{b}y(t)$$

(page 162) $r(t) = \tilde{\chi}(t) + \psi(t) + \varphi(t)$

$$d\tilde{\chi}(t) = -\bar{a}\tilde{\chi}(t)dt + \sigma_3 dZ_3(t)$$
$$d\psi(t) = -\bar{b}\psi(t)dt + \sigma_4 dZ_2(t)$$
$$\varphi(t) = r_0 e^{-\bar{a}t} + \int_0^t \theta(v)e^{-\bar{a}(t-v)}dv$$

QFIQ-132-21: Investment Instruments with Volatility Target Mechanism, Albeverio, Steblovskaya, and Wallbaum

(2) $$\mathcal{O}_0 = \frac{1}{B_T}\mathbb{E}^*(f(S))$$

(3) $$V_t = \beta_k S_t + \gamma_k B_t$$

(5) $$\tilde{V}_t = \beta_k \tilde{S}_t + \gamma_k$$

(7) $$\hat{\mathcal{O}}_0 = \frac{1}{B_T}\mathbb{E}^*(f(V))$$

(8) $$dS_t = S_t(rdt + \sigma dW_t)$$

(10) $$P_{dp} = K + p_{dp}\cdot K \max\left\{\frac{S_T}{S_0} - 1, 0\right\}$$

(11) $$RB = K(1 - e^{-rT})$$

(12) $$g_{dp}(x) = \max\left\{\frac{K}{S_0}x - K, 0\right\}$$

(17) $$p_{dp} = \frac{1 - e^{-rT}}{\mathcal{O}_0(f_{dp})}$$

(18) $$P_{dp} = K + p_{dp}\cdot K \max\left\{\frac{S_T}{S_0} - 1, 0\right\}$$

(19) $$P_{dpa} = \max\left\{K; K\cdot\left(1 + p_{dpa}\cdot\frac{1}{S_0}\left(\frac{1}{n}\sum_{i=0}^{n} S_{t_i} - S_0\right)\right)\right\}$$

(20) $$g_{dpa}(S) = \max\left\{\frac{K}{S_0}\frac{1}{n}\sum_{i=0}^{n} S_{t_i} - K, 0\right\}$$

(25) $$p_{dpa} = \frac{1 - e^{-rT}}{\mathcal{O}_0(f_{dpa})}$$

QFIQ-134-22: An Introduction to Computational Risk Management of Equity-Linked Insurance, Feng Chapter 1

(page 15) $$G_{(k+1)/n} = G_{k/n}\left(1 + \frac{\rho}{n}\right), \quad \text{for } k = 0, 1, \cdots$$

(1.15) $$G_{(k+1)/n} = \max\left\{G_{k/n}, F_{(k+1)/n}\right\}, \quad \text{for } k = 0, 1, \cdots$$

(page 15) $$G_{(k+1)/n} = \frac{G_{k/n}}{F_{k/n}}\max\left\{F_{k/n}, F_{(k+1)/n}\right\}, \quad \text{for } k = 0, 1, \cdots$$

(1.16) $$\frac{G_{(k+1)/n} - G_{k/n}}{G_{k/n}} = \frac{(F_{(k+1)/n} - F_{k/n})_+}{F_{k/n}}$$

(page 16) $$G_{(k+1)/n} = G_0 \prod_{j=0}^{k-1} \max\left\{1, \frac{F_{(j+1)/n}}{F_{j/n}}\right\}$$

(1.17) $$G_{(k+1)/n} = \max\left\{G_{k/n}\left(1+\frac{\rho}{n}\right), F_{(k+1)/n}\right\}, \quad \text{for } k = 0, \ldots$$

(1.18) $$G_{k/n} = \left(1+\frac{\rho}{n}\right)^k \max_{j=0,\ldots,k}\left\{\left(1+\frac{\rho}{n}\right)^{-j} F_{j/n}\right\}$$

(1.19) $$G_t = \sup_{0\le s\le t}\{F_s\}$$

(1.23) $$L_e^{(\infty)}(T_x) = e^{-rT}(G - F_T)_+ I(T_x > T) - \int_0^{T\wedge T_x} e^{-rs} m_e F_s ds$$

(1.24) $$L_d^{(\infty)}(T_x) = e^{-rT_x}(Ge^{\rho T_x} - F_{T_x})_+ I(T_x \le T) - \int_0^{T\wedge T_x} e^{-rs} m_d F_s ds$$

(1.28) $$L_w^{(n)} := \sum_{k=n\tau}^{(n\tau-1)\vee\lceil nT\rceil} e^{-rk/n}\frac{w}{n} - \sum_{k=1}^{(n\tau-1)\wedge\lceil nT\rceil} e^{-r(k-1)/n} F_{(k-1)/n}\frac{m_w}{n}$$

(page 24) $$L_w^{(\infty)} := \int_\tau^{\tau\vee T} e^{-rt} w dt - \int_0^{t\wedge T} e^{-rt} m_w F_t dt, \quad \tau := \inf\{t > 0 : F_t \le 0\}$$

(page 27) $$L_{lw}^{(n)} := \sum_{k=n\tau}^{(n\tau-1)\vee\lfloor nT_x\rfloor} e^{-r(k+1)/n} G_{k/n}\frac{h}{n} - \sum_{k=0}^{(n\tau-1)\wedge\lfloor nT_x\rfloor} e^{-rk/n} G_{k/n}\frac{m_w}{n}$$

(page 27) $$L_{lw}^{(\infty)} := \int_\tau^{\tau\vee T_x} e^{-rt} G_t h dt - \int_0^{t\wedge T_x} e^{-rt} G_t m_w dt$$

(1.39) $$P\prod_{k=1}^{T}\max\left(\min\left(1+\alpha\frac{S_k - S_{k-1}}{S_{k-1}}, e^c\right), e^g\right)$$

(1.41) $$P\prod_{k=1}^{T}\max\left(\min\left(\left(\frac{S_k}{S_{k-1}}\right)^\alpha, e^c\right), e^g\right)$$

(1.42) $$\max\left(P\left(\frac{\max\{S_k : k = 1, \cdots, T\}}{S_0}\right)^\alpha, G_T\right)$$

(1.43) $$\max\left(P\left(\frac{\sup_{0\le t\le T}\{S_t\}}{S_0}\right)^\alpha, G_T\right)$$

Chapter 4

(4.52) $$dF_t = (r - m)F_t dt + \sigma F_t dW_t, \quad 0 < t < T$$

(4.53) $$B_e(t, F) = {}_T p_x\left[Ge^{-r(T-t)}\Phi\left(-d_2\left(T-t, \frac{F}{G}\right)\right) - Fe^{-m(T-t)}\Phi\left(-d_1\left(T-t, \frac{F}{G}\right)\right)\right]$$

(4.54) $$d_1(t, u) = \frac{\ln u + (r - m + \sigma^2/2)t}{\sigma\sqrt{t}}$$

(4.55) $$d_2(t, u) = d_1(t, u) - \sigma\sqrt{t}$$

(page 153) $$P_e(t, F) = m_e\, {}_t p_x F\int_0^{T-t} e^{-ms}\, {}_s p_{x+t} ds = m_e\, {}_t p_x F\bar{a}_{x+t:\overline{T-t}|m}$$

(page 153) $$ {}_Tp_x\left[e^{(\rho-r)T}\Phi\left(\frac{\rho-(r-m-\sigma^2/2)}{\sigma}\sqrt{T}\right)\right.$$
$$\left.-e^{-mT}\Phi\left(\frac{\rho-(r-m+\sigma^2/2)}{\sigma}\sqrt{T}\right)\right]=m_e\bar{a}_{x:\overline{T}|m}$$

(page 154) $$\tilde{\mathbb{E}}\left[e^{-r(T_2-t)}(G_{T_1}-F_{T_2})_+I(T_x>T_2)|\mathcal{F}_t\right]$$
$$={}_{T_2}p_xe^{-r(T_2-t)}\left[G_{T_1}\Phi(-d_2(T_2-t,F_t/G_{T_1}))-F_te^{(r-m)(T_2-t)}\Phi(-d_1(T_2-t,F_t/G_{T_1}))\right]$$

(This is a correction to the text formula)

(4.60) $$N_d(0,F_0):=B_d(0,F_0)-P_d(0,F_0)=\tilde{\mathbb{E}}\left[\int_0^T e^{-rt}{}_tp_x\mu_{x+t}(G_t-F_t)_+dt-\int_0^T e^{-rt}m_d\,{}_tp_xF_tdt\right]$$

(4.61) $$B_w(t,F)=\tilde{\mathbb{E}}[e^{-r(T-t)}F_{T-t}I(F_{T-t}>0)|F_0=F]+\frac{w}{r}(1-e^{-r(T-t)})$$

(4.63) $$N_w(t,F)=\tilde{\mathbb{E}}\left[w\int_{\tau-t}^{(\tau\vee T)-t}e^{-rs}ds-m_w\int_{(\tau\wedge t)-t}^{(\tau\wedge T)-t}e^{-rs}\tilde{F}_sds\middle|\tilde{F}_0=F\right]$$
$$=\tilde{\mathbb{E}}\left[w\int_{\tau}^{\tau\vee(T-t)}e^{-rs}ds-m_w\int_0^{\tau\wedge(T-t)}e^{-rs}F_sds\middle|F_0=F\right]$$

(page 158) $$e^{-rT}F_TI(\tau>T)+\frac{w}{r}(1-e^{-rT})-F_0$$

(page 158) $$w\int_{\tau\wedge T}^{T}e^{-rs}ds-m_w\int_0^{t\wedge T}e^{-rs}F_sds$$

(page 159) $$F_0-\mathbb{E}\left[\int_0^{\tau\wedge T}e^{-rs}(mF_s+w)ds\right]=\mathbb{E}[e^{-r(\tau\wedge T)}F_{\tau\wedge T}]=\mathbb{E}[e^{-rT}F_TI(\tau>T)]$$

(page 160) $$B_{lw}(F_0):=\tilde{\mathbb{E}}\left[\int_0^{T_x}we^{-rs}ds+e^{-rT_x}F_{T_x}I(F_{T_x}>0)\right]$$
$$=\frac{w}{r}-\frac{\delta w}{r(\delta+r)}+\delta\tilde{\mathbb{E}}\left[\int_0^{\tau}e^{-(\delta+r)t}F_tdT\right]$$

(4.75) $$N_{lw}(F_0)=\frac{w}{r+\delta}\tilde{\mathbb{E}}[e^{-(\delta+r)\tau}]+m_w\tilde{\mathbb{E}}\left[\int_0^{\tau}e^{-(\delta+r)u}F_udu\right]$$

(page 162) $$B_{lw}(t,F)=\frac{w}{r}(1-\tilde{q}_{x+t}(r))+\tilde{\mathbb{E}}\left[\int_0^{\tau}e^{-rs}F_s\,q_{x+t}(s)ds\right]$$

(page 163) $$N_{lw}(t,F)=\frac{w}{r}\tilde{\mathbb{E}}[e^{-r\tau}\bar{Q}_{x+t}(\tau)]-\frac{w}{r}\tilde{\mathbb{E}}\left[\int_{\tau}^{\infty}e^{-ru}q_{x+t}(u)du\right]$$
$$+m_w\tilde{\mathbb{E}}\left[\int_0^{\tau}e^{-ru}\bar{Q}_{x+t}(u)F_udu\right]$$

(page 166) Investment Income: $I[t]=A[t]\times\left(H[t]+U[t]-\frac{1}{2}L[t]\right)$

(page 167) Credited to Policholder Account: $J[t]=C[t]\times\left(Q[t]-\frac{1}{2}\times L[t]\right)$

(page 167) Risk Charges: $K[t]=B[t]\times\left(Q[t]-\frac{1}{2}\times L[t]\right)$

(page 167) Mortality: $L[t] = Q[t] \times F[t]$

(page 167) Lapses: $M[t] = S[t] \times D[t]$

(page 167) Surrender Charge: $N[t] = M[t] \times G[t]$

(page 167) Annuitization: $O[t] = R[t] \times E[t]$

(page 167) Current Inforce (as % of initial): $P[t] = P[t-1] \times (1 - D[t] - E[t] - F[t])$

(page 168) Policyholder Fund Value (BOY): $Q[t] = T[t-1] + H[t]$

(page 168) Policyholder Fund Value before Lapses & Annuitizations (EOY): $R[t] = Q[t]+J[t]-L[t]$

(page 168) Policyholder Fund Value before Lapses & after Annuitizations (EOY): $S[t] = R[t]-O[t]$

(page 168) Policyhodler Fund Value after Lapses & Annuitizations (EOY): $T[t] = S[t]-M[t]$

(page 168) Statutory Reserve (BOY): $U[t] = V[t-1]$

(page 168) Statutory Reserve (EOY): $V[t] = T[t]$

(page 168) GMDB Benefit (5% − roll-up rate): $W[t] = P[t-1] \times \max(Q[t], H[1] \times (1+5\%)^t)$

(page 168) GMDB Benefits: $X[t] = (W[t] - U[t])_+ \times F[t]$

(page 168) Poicy Fee Income (30 − annual policy fee: $AG[t] = 30 \times AD[t-1]$

(page 168) Total Revenues: $AH[t] = AE[t] + AF[t] + AG[t]$

(page 168) Premium-Based Administrative Expenses: $AO[t] = AE[t] \times AJ[t]$

(page 168) Per Policy Adminstrative Expenses (2% − inflation rate):

$AP[t] = AK[t] \times AM[t] \times (1 + 2\%)^{t-1}$

(page 169) Commissions: $AQ[t] = AE[t] \times AL[t]$

(page 169) GMDB Cost (0.4% of account value − GMDB cost): $AR[t] = T[t] \times 0.4\%$

(page 169) Total Expenses: $AS[t] = AO[t] + AP[t] + AQ[t] + AR[t]$

(page 169) Death Claims: $AT[t] = L[t]$

(page 169) Annuitization: $AU[t] = O[t]$

(page 169) Surrender Benefit: $AV[t] = M[t] - N[t]$

(page 169) Increase in Reserve: $AW[t] = V[t] - V[t-1]$

(page 170) GMDB Benefit: $AX[t] = (W[t] - U[t])_+ \times F[t]$

(page 170) Total Benefits: $AY[t] = AT[t] + AU[t] + AV[t] + AW[t] + AX[t]$

(page 170) Book Profit Before Tax: $AZ[t] = AH[t] - AS[t] - AY[t]$

(page 171) Taxes on Book Profit (37% − federal income tax rate): $BF[t] = BE[t] \times 37\%$

(page 171) Book Profits after Tax: $BD[t] = BE[t] - BF[t]$

(page 171) Target Surplus (BOY): $BI[t] = BJ[t-1]$

(page 171) Target Surplus (EOY)(0.85% − target surplus rate): $BI[t] = V[t] \times 0.85\%$

(page 171) Increase in Target Surplus: $BG[t] = BI[t] - BH[t]$

(page 172) Interest on Target Surplus (5% − interest rate on surplus): $BK[t] = BH[t] \times 5\%$

(page 172) Taxes on Interest on Target Surplus: $BL[t] = BK[t] \times 37\%$

(page 172) After Tax Interest on Target Surplus: $BJ[t] = BK[t] - BL[t]$

(page 172) Distributable Earnings: $BM[t] = BD[t] + BJ[t] - BG[t]$

(4.78) $$\mathfrak{B} = \sum_{k=1}^{nT} \left(1 + \frac{r}{n}\right)^k P_{k/n}$$

(4.79) $$\mathfrak{B} = \int_0^T e^{-rt} P_t dt = \int_0^T e^{-rt} k_t \mu^l_{x+t}\, {}_t p_x F_t dt + m \int_0^T e^{-rt}\, {}_t p_x F_t dt - \int_0^T e^{-rt} E_t dt - \int_0^T e^{-rt} \mu^d_{x+t}\, {}_t p_x (G_t - F_t)_+ dt$$

Chapter 6

(page 264) $$B_e(t, F_t) = {}_T p_x \times \left[G e^{-r(T-t)} \Phi\left(-d_2\left(T - t, \frac{F_t}{G}\right)\right) - F_t e^{-m(T-t)} \Phi\left(-d_1\left(T - t, \frac{F_t}{G}\right)\right)\right]$$

(6.16) $$c(t, s) = -\left(\frac{\partial}{\partial t} + rs\frac{\partial}{\partial s} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2}{\partial s^2} - r\right) f(t, s) = m\, {}_t p_x F(t, s)$$

(page 269) $$\Delta_t = \frac{\partial}{\partial s} f(t, S_t) = -\frac{F_t}{S_t}\left[{}_T p_x e^{-m(T-t)} \Phi\left(-d_1(T - t, \frac{F_t}{G})\right) + m \int_t^T e^{-m(s-t)}\, {}_s p_x ds\right]$$

(page 275 Greeks)

$$\Delta_t = -{}_T p_x \frac{F}{s} e^{-m(T-t)} \Phi\left(-d_1(T - t, \frac{F}{G})\right) - \frac{P_e}{s}$$

$$\Gamma_t = {}_T p_x \frac{F}{s^2} e^{-m(T-t)} \frac{\phi\left(d_1(T - t, \frac{F}{G})\right)}{\sigma\sqrt{T - t}}$$

$$\Theta_t = {}_T p_x \left[rG e^{-r(T-t)} \Phi\left(-d_2(T - t, \frac{F}{G})\right) - mF e^{-m(T-t)} \Phi\left(-d_1(T - t, \frac{F}{G})\right) - e^{-m(T-t)} \frac{\sigma F \phi\left(d_1(T - t, \frac{F}{G})\right)}{2\sqrt{T - t}}\right] + m\, {}_t p_x F$$ (This is a correction to the text formula)

$$\mathcal{V}_t = {}_T p_x F e^{-m(T-t)} \phi\left(d_1(T - t, \frac{F}{G})\right) \sqrt{T - t}$$

QFIQ-135-22: Structured Product Based Variable Annuities, Deng, et. al.

(1) $$PV = I_0 e^{-r(t_n - t_0)} \mathbb{E}\left[\prod_{i=1}^n (1 + R_i)\right] = I_0 e^{-r(t_n - t_0)} \prod_{i=1}^n \mathbb{E}[1 + R_i]$$

(2) $$\mathbb{E}[1+R_i] = 1 - \left(\frac{FV_{\text{Put}}(S_0, K_b, \tau, r, q, \sigma)}{S_0}\right) + \left(\frac{FV_{\text{Call}}(S_0, K = S_0, \tau, r, q, \sigma)}{S_0}\right) - \left(\frac{FV_{\text{Call}}(S_0, K_c, \tau, r, q, \sigma)}{S_0}\right)$$

(3) $$K_b = S_0(1 - \text{Buffer})$$

(4) $$K_c = S_0(1 + \text{Cap})$$

(5) $$\mathbb{E}[1+R_i] = 1 + \left(\frac{FV_{\text{Call}}(S_0, K = S_0, \tau, r, q, \sigma)}{S_0}\right) - \left(\frac{FV_{\text{Call}}(S_0, K_c, \tau, r, q, \sigma)}{S_0}\right)$$

(6) $$\mathbb{E}[1+R_i] = 1 + \left(\frac{FV_{\text{Binary Call}}(S_0, K = S_0, \tau, r, q, \sigma)}{S_0}\right)$$

(7) $$FV_{\text{Call}}(S_0, K, \tau, r, q, \sigma) = FN(d_1) - KN(d_2)$$

(8) $$FV_{\text{Put}}(S_0, K, \tau, r, q, \sigma) = KN(-d_2) - FN(-d_1)$$

(9) $$FV_{\text{Binary Call}}(S_0, K, \tau, r, q, \sigma) = KN(d_2)$$

(10) $$d_1 = \frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2\tau}{2}}{\sqrt{\sigma^2\tau}} \qquad d_2 = d_1 - \sqrt{\sigma^2\tau} \qquad F = e^{(r-q)\tau}$$

QFIQ-136-23: Calibrating Interest Rate Models

(page 8)

$$P[r_{t+s} < 0|r_t] = \Phi\left(-\frac{\bar{r} + (r_t - \bar{r})e^{-\gamma s}}{\sigma\sqrt{\frac{1-e^{-2\gamma s}}{2\gamma}}}\right)$$

(page 18)

$$\hat{\beta} = \frac{\sum_{i=1}^{n} r_{i\Delta} r_{(i-1)\Delta} - \frac{1}{n}\sum_{i=0}^{n-1} r_{i\Delta}\sum_{i=1}^{n} r_{i\Delta}}{\sum_{i=0}^{n-1} r_{i\Delta}^2 - \frac{1}{n}\left(\sum_{i=0}^{n-1} r_{i\Delta}\right)^2}$$

$$\hat{\alpha} = \frac{1}{n}\left(\sum_{i=1}^{n} r_{i\Delta} - \hat{\beta}\sum_{i=0}^{n-1} r_{i\Delta}\right)$$

$$\hat{\sigma^{*2}} = \frac{1}{n-2}\sum_{i=1}^{n}\left(r_{i\Delta} - \hat{\alpha} - \hat{\beta} r_{(i-1)\Delta}\right)^2$$

$$\gamma = -\frac{\ln(\hat{\beta}^*)}{\Delta}$$

$$\bar{r} = \frac{\hat{\alpha}^*}{1 - \hat{\beta}^*}$$

$$\sigma = \sqrt{\frac{2\hat{\gamma}\hat{\sigma}^{*2}}{1 - \hat{\beta}^{*2}}}$$

(page 23)

$$
\begin{aligned}
\gamma &= 2\left(\frac{1.96\sigma}{\hat{q}_{0.95}-\hat{q}_{0.05}}\right)^2 \\
\bar{r} &= \frac{\hat{q}_{0.05}+\hat{q}_{0.95}}{2}
\end{aligned}
$$

(page 26-27)

$$
\begin{aligned}
r(i) &= \alpha_1 + \beta_1 r(i-1) + \sqrt{r(i-1)}\epsilon_i, \quad i = 1,2,\ldots \\
\frac{r(i)}{\sqrt{r(i-1)}} &= \alpha_1\left(\frac{1}{\sqrt{r(i-1)}}\right) + \beta_1\sqrt{r(i-1)} + \epsilon_i
\end{aligned}
$$

$$
\begin{aligned}
y_i &= \frac{r(i)}{\sqrt{r(i-1)}} \\
x_{1i} &= \frac{1}{\sqrt{r(i-1)}} \\
x_{2i} &= \sqrt{r(i-1)} \\
y_i &= \alpha_1 x_{1i} + \beta_1 x_{2i} + \epsilon_i, i = 1,2,\ldots
\end{aligned}
$$

$$
\begin{aligned}
\gamma &= \frac{1-\beta_1}{\Delta} \\
\bar{r} &= \frac{\alpha_1}{1-\beta_1} \\
\alpha &= \frac{\sigma^2}{\Delta}
\end{aligned}
$$

(page 36)

$$
\begin{aligned}
r_{t+s} &= r_t\exp(-\gamma^* s) + \exp(-\gamma^*(t+s))\int_t^{t+s}\theta_u\exp(\gamma^* u)du + \sigma\exp(-\gamma^* s)\int_0^s\exp(\gamma^* u)dX_u \\
E[r_{t+s}|r_t] &= r_t\exp(-\gamma^* s) + \exp(-\gamma^*(t+s))\int_t^{t+s}\theta_u\exp(\gamma^* u)du \\
Var[r_{t+s}|r_t] &= \frac{\sigma^2}{2\gamma^*}(1-e^{-2\gamma^* s})
\end{aligned}
$$

(page 38)

$$
\theta_T = \frac{\partial f(0,T)}{\partial T} + \gamma^* f(0,T) + \frac{\sigma^2}{2\gamma^*}(1-\exp(-2\gamma^* T)
$$

$$
\begin{aligned}
E[r_{t+s}|r_t] = {} & r_t\exp(-\gamma^* s) + f(0,s+t) - f(0,t)\exp(-\gamma^* s) + \\
& \frac{\sigma^2}{2(\gamma^*)^2}\left[1-\exp(-\gamma^* s) + \exp(-2\gamma^*(t+s)) - \exp(-\gamma^*(2t+s))\right]
\end{aligned}
$$

(page 42)

$$\begin{aligned} r(0,t) &= \sum_{i=0}^{n} a_i t^i \\ f(0,t) &= \sum_{i=0}^{n} a_i (i+1) t^i \\ \frac{\partial f(0,t)}{\partial t} &= \sum_{i=1}^{n} a_i i (i+1) t^{i-1} \\ \theta_t &= \sum_{i=1}^{n} a_i i (i+1) t^{i-1} + \sum_{i=0}^{n} \gamma^* a_i (i+1) t^i + \frac{\sigma^2}{2\gamma^*} \left(1 - e^{-2\gamma^* t}\right) \end{aligned}$$

(page 44-45)

$$f(0,t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\tau}\right) + \frac{\beta_2 t}{\tau} \exp\left(-\frac{t}{\tau}\right)$$

$$r(0,t) = \beta_0 + \beta_1 \frac{1 - \exp\left(-\frac{t}{\tau}\right)}{\frac{t}{\tau}} + \beta_2 \frac{1 - \exp\left(-\frac{t}{\tau}\right) - \frac{t}{\tau} \exp\left(-\frac{t}{\tau}\right)}{\frac{t}{\tau}}$$

$$\frac{\partial}{\partial t} f(0,t) = -\frac{\beta_1}{\tau} \exp\left(-\frac{t}{\tau}\right) + \frac{\beta_2}{\tau} \exp\left(-\frac{t}{\tau}\right) - \frac{\beta_2 t}{\tau^2} \exp\left(-\frac{t}{\tau}\right)$$

(page 49)

$$\begin{aligned} r_t = {} & r_0 \exp(-\gamma_1^* t) + \int_0^t \theta_s \exp(-(t-s)\gamma_1^*) ds \\ & + \phi_{2,0} (\exp(-\gamma_2^* t) - \exp(-\gamma_1^* t)) \\ & + \sigma_1 \int_{s=0}^{t} \exp(-\gamma_1^* (t-s)) dX_s^1 \\ & + \sigma_2 \int_{u=0}^{t} \left[\exp(-\gamma_1^*(t-u)) + \exp(-\gamma_2^*(t-u)) - \exp(-(\gamma_1^* - \gamma_2^*)(t-u))\right] dX_u^2 \end{aligned}$$