

ABSTRACTS

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EXAMPLES OF PARAMETRIC EMPIRICAL BAYES  
METHODS FOR THE ESTIMATION OF FAILURE  
PROCESSES FOR REPAIRABLE SYSTEMS

Asit P. Basu

Steven E. Rigdon

Suppose that several systems, e.g., airplanes, computers, etc., are operating independently and are subject to failure and repair. The failure times are assumed to have been generated by a stochastic point process, and the repair times are assumed to be negligible. In this paper the homogeneous Poisson process is assumed as the stochastic point process. If the systems are similar, for example, they might have been produced in the same factory in the same time period, then it seems appropriate that information from all systems could be used to estimate the failure intensities of one particular system. Here, we treat the failure intensities of the systems as random quantities which were generated from a gamma or a log-normal prior distribution whose parameters are considered fixed but unknown. The marginal maximum likelihood estimators of the parameters of the prior distribution are found by using numerical techniques. Point estimates for the failure intensities are then obtained by finding the mean of the posterior distribution given the estimated values of the prior distribution.

# ROBUST BAYESIAN ANALYSIS WITH APPLICATIONS IN RELIABILITY

L. Mark Berliner

Basic concepts of robust Bayesian analysis are reviewed. Special emphasis is given to discussion of general techniques discussed by Berger and Berliner (1983), (1984) involving  $\epsilon$ -contamination classes of priors and maximum likelihood, empirical Bayes analysis. Several examples concerning inferences for the parameters of one and two parameter exponential distributions with censoring are presented.

## A METHOD FOR CONSTRUCTING A UNIMODAL INFERENCEAL OR PRIOR DISTRIBUTION

Patrick L. Brockett

Abraham Charnes

Kwang Hun Paick

In this paper we show how to take personally assessed information and use it to develop a continuous unimodal prior density function, perhaps for subsequent Bayesian analysis. The method is completely nonparametric and uses only the furnished information and no other. The technique is easily computerized, and yields a closed analytical formula for the prior. The resulting distribution may be considered to be an inferential distribution.

## TOLERANCE INTERVALS IN RISK THEORY

Wolf-Rüdiger Heilmann

Up to now, the main aim of credibility theory has been to provide statistical models which allow for estimating (net) risk premiums appropriately. In the present note, a simple credibility model based on the percentile principle is introduced. It turns out that there are close connections between the resulting credibility premiums and statistical tolerance limits.

## BAYESIAN INFERENCE ABOUT THE TAIL OF A DISTRIBUTION

Bruce M. Hill

The methodology developed by Hill [1] for inference about the tails of a distribution is extended, and the diagnostic data-analytic Bayesian techniques in [1] are illustrated and their performance examined.

## BAYESIAN RELIABILITY ESTIMATION OF A TWO PARAMETER CAUCHY DISTRIBUTION

H. A. Howlader and G. Weiss

This paper gives an approximate Bayes procedure for the estimation of the reliability function of a two parameter Cauchy distribution using Jeffreys' non-informative prior under a squared-error loss function. A numerical example is given. Based on a Monte Carlo simulation, two such Bayes estimators of reliability are compared with the maximum likelihood estimator.

## PARAMETRIC EMPIRICAL BAYES AND CREDIBILITY THEORY

Carl N. Morris

The current view of empirical Bayes inference models data and parameters with two families of distributions: one family for data, and one for the unknown parameters, with the latter involving either parametric prior distributions (PEB) or nonparametric prior distributions (NPEB). The (prior) distributions of the unknown parameters is assumed to be restricted to a class  $\Pi$  having just one member, while frequentist statistical inference allows  $\Pi$  to contain all possible prior distributions.

This presentation shows how credibility theory and PEB theory are intimately related, and reviews and applies recent PEB theory to the credibility situation for the normal and other familiar distributions.

## AN ANALYSIS OF EXPERIENCE RATING

Glen Myers

Experience rating formulas that are currently in use have features that have no counterpart in the literature on Bayesian credibility. These features include the limiting of individual losses that go into the experience rating, separate treatment of primary and excess losses, and the gradual transition to self rating. This paper analyzes the effect of these features using the collective risk model.

BAYESIAN PREDICTIONS: NON PARAMETRIC METHODS

AND LEAST-SQUARES APPROXIMATIONS

M. Mouchart and L. Simar

Bayesian prediction is analyzed in the i.i.d. case. In a search for robust methods we combine non-parametric methods - through Dirichlet processes - and Least-Squares Approximations. Autoprediction is first analyzed as a starting point. Then we consider the prediction of a variable when we are provided with observations of other associated variables. We first show the difficulties in conditioning in Dirichlet processes and thereafter propose various approximations for the posterior predictive conditional expectation.

## AUTOMATING PROBABILISTIC INFERENCE

Ross D. Shachter

An influence diagram is a network used to represent random variables, their conditional dependences, and their joint distribution. More compact and less cluttered than trees, influence diagrams are powerful communication tools when formulating a model, and are becoming increasingly powerful as solution tools as well.

An algorithm is developed that performs inference on a probabilistic model represented as an influence diagram, without constructing or manipulating the full joint distribution. In fact, the algorithm can detect which information is relevant and needed to solve a given problem. Applications include automatic inference on probabilistic data sets, symbolic analysis, and decision making under uncertainty.

## ASSUMPTIONS IN PENSION PLAN VALUATIONS

Arnold F. Shapiro

Many of the criteria for actuarial assumptions in pension plan valuations are only vaguely defined. This often impedes the attainment of a consensus regarding an appropriate mix of assumptions and jeopardizes the credibility of pension actuaries.

The purpose of this paper is to set the stage for further discussion and research into actuarial assumptions in pension plans by quantifying some of the criteria which underly the choice of these assumptions.

BAYESIAN ANALYSIS USING MONTE CARLO INTEGRATION  
WITH AN EXAMPLE OF THE ANALYSIS OF SURVIVAL DATA

Leland Stewart

Bayesian analysis using Monte Carlo integration is an effective approach for handling rich multiparameter families of distribution, nonconjugate priors, censored data, extrapolation uncertainty, and the computation of posterior distributions for parameters or predictions of interest. In the example, posterior percentile curves for a rate function are computed from survival data.

COMBINATION OF ESTIMATES OF OUTSTANDING CLAIMS IN NON-LIFE INSURANCE

G. C. Taylor

The paper is concerned with the estimation of outstanding claims of a non-life insurer. Typically, the actuary carries out this estimation by a number of different methods, and so arrives at a number of estimates. Each method will usually provide separate estimates in respect of separate years of origin. In addition, it is likely that physical estimates of outstandings prepared by the insurer will be available.

The question considered is: how should the various estimates of outstanding claims be combined to produce the actuary's final estimate.

# ACCURATE APPROXIMATIONS FOR POSTERIOR

## MOMENTS AND MARGINALS

Luke Tierney and Joseph B. Kadane

This paper describes approximations to the posterior means and variances of positive functions of a real or vector valued parameter. These approximations can be applied directly to compute approximations to the predictive density, and they can be modified for use in approximating marginal posterior densities in multi-parameter problems. To apply the proposed method one only needs to be able to maximize slightly modified likelihood functions and to evaluate the observed information at the maxima. Nevertheless, the resulting approximations are generally as accurate and in some cases more accurate than more conventional approximations based on third order expansions of the likelihood and requiring either the evaluation of third derivatives or the use of derivative-free maximization procedures. When used to obtain marginal posterior densities, this method behaves very much like the saddle point approximation method for sampling distributions. In particular, for several distributions, including the normal-gamma distribution and the Dirichlet distribution, the approximations to the marginal densities (renormalized to integrate exactly to one) are exact.



USING MODEL-BASED SMOOTHING AND EMPIRICAL BAYES ESTIMATION  
TECHNIQUES TO PREDICT AUTOMOBILE ACCIDENT FREQUENCIES FOR  
CALIFORNIA DRIVERS IN A TWO-WAY CLASSIFICATION

Thomas J. Tomberlin

For predicting accident frequencies, a succession of log-linear models for Poisson data, some of which include nested random effects, is introduced. By applying maximum likelihood and empirical Bayes estimation techniques to these models, the notions of risk classification, model-based smoothing, credibility theory and experience rating are incorporated under a unified statistical approach to loss prediction. The performance of these models is evaluated using accident data from California.

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