COMPUTING THE PRESENT VALUE OF LIFETIME INCOME

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INTRODUCTION

One of the approaches to determining a person's need for life insurance is to assess the loss of income to dependents in the event of the insured's death. This method involves projecting the insured's future income over the remainder of his expected working life and then discounting these future income amounts to their combined present value. By investing this present value at a rate of return equal to the rate employed in the discounting procedure, dependents can produce cash inflows equivalent to those which the income of the insured would have furnished.

The figure used as the insured's income need not be gross income. The figure can be adjusted for pertinent deductions and expenses which would cease with the insured's death. Indeed, a variant of this general approach is to judge the need for life insurance as the present value of the future dollar amounts which dependents require to achieve the life-style which the insured would have provided.

Projecting future incomes, or other, adjusted amounts, and discounting them to their total present value is tedious, at best. It is suggested here, that this computation can be achieved via a simple equation which is derived from proceedures used in valuing common stocks.

VALUING COMMON STOCKS

A principle of valuation is that an asset is worth the present value of the benefits it generates in its best use. The most anyone would pay for an asset is the present value of the benefits which are expected to result from owning the asset. The present value is calculated by discounting these expected benefits at investors' required rate of return which compensates investors for the time value of money and the uncertainty which is associated with the expected benefits.

For a share of common stock, dividends constitute stockholders' benefits. Typically, investors expect dividends on a common stock to rise through time. Hence, the current price per share can be viewed as the sum of the discounted, growing expected future dividends. That is,

(1) P{O} = D{1}/(1+K) + D{2}/((1+K)^2) + ... + D{N}/((1+K)^N)

where

P{O} = the stock price at time O (right now),

D{1} = the dividend expected one period from now,

K = investors' per period required rate of return,

N = time period N (for common stock valuation, theoretically, N equals infinity; as a practical matter, however, any large number, say above 100, will do), and

^N = raise to the Nth power.

In situations where

(1) dividends are expected to advance at a constant rate, G, through time,

(2) the discount rate is constant through time, and

(3) K exceeds G (if G were to forever exceed K, $P\{0\}$ would be infinite),

then equation (1) can be greatly simplified. Given that dividends grow through time at the constant rate, G, equation (1) can be expressed as

$$(2) \quad P\{0\} = D\{1\}/(1+K)$$

$$+ (D\{1\}(1+G))/((1+K)^2)$$

$$+ (D\{1\}((1+G)^2)/((1+K)^3)$$

$$+ \ldots + (D\{1\}((1+G)^{N-1}))/((1+K)^N).$$

Multiplying equation (2) by $(1+K)/(1+G)$ and subtracting equation (2) from the result yields

$$P\{0\}[(1+K)/(1+G)] - P\{0\} = D\{1\}/(1+G) - [(D\{1\}((1+G)^{N}-1))/((1+K)^{N})].$$

As N gets large, the value of the last term approaches zero and can be ignored.

Multiplying the simplified equation by $(1+G)$ and rearranging terms gives

(3) $P\{0\} = D\{1\}/(K-G)$.

That is, a current stock price equals next period's expected dividend divided by the remainder of investors' required rate of return minus the expected rate of growth of dividends.

THE PRESENT VALUE OF DIVIDENDS THROUGH SOME FUTURE PERIOD

Given that dividends rise through time, so will the stock price. For example, at period 10, the stock price will be

$$P_{10} = D_{11}/(K-G).$$

Since $D_{11} = D_{1}((1+G)^{10})$, it must be true that

$$P_{10} = P_{0}((1+G)^{10}).$$

The stock price appreciates at the same rate as dividends.

The present value of P_{10} is determined by merely discounting at K. Hence, the present value of P_{10} is

$$[P_{0}((1+G)^{10})]/[(1+K)^{10}].$$

or

$$(4) \quad [(D_{0}(1+G))/(K-G)] \; [((1+G)/(1+K))^{10}].$$

Given that P{0} is the present value of expected dividends in perpetuity (or, at least, a long time), and that the present value of P{10} is the present value of expected dividends beyond the tenth period, it follows that P{0} minus the present value of P{10} equals the present value of expected dividends from the first through the tenth periods.

Substituting D{0}(1+G) for D{1} in equation (3), the present value of the expected dividends for periods one through ten equals the right-hand side of amended equation (3) minus the right-hand side of equation (4):

$$D\{0\}(1+G)/(K-G)$$

$$- [D\{0\}(1+G)/(K-G)] [((1+G)/(1+K))\char`^10]$$

$$(5) \quad = \quad [D\{0\}(1+G)/(K-G)] [1-(((1+G)/(1+K))\char`^10)].$$

Substituting N for 10 generalizes equation (5) so that it can be employed to calculate the present value of expected dividends through any future period.

A COMPARISON

Analytically, the resemblance between computing the present value of dividends over some range of time periods and computing the present value of an individual's lifetime income (PVLI) is striking. The comparison is achieved by first capitalizing the individual's future income as though it were perpetual and then subtracting the present value of income beyond the individual's expected working life. The remainder must be the present value of the individual's expected lifetime income. The generalized equation for calculating PVLI is:

(6) PVLI = [I] [(1+G)/(K-G)] [1-(((1+G)/(1+K))^N)]

where

PVLI = the present value of lifetime income,

I = the annual income just received (this figure can be adusted for appropriate reductions),

G = the expected, constant annual rate of growth of income,

K = the discount rate, the rate currently available on safe, long-term investments (e.g., the long-term government bond rate),

^N = raise to the Nth power, and

N = the number of years until the expiration of the person's expected working life.

AN EXAMPLE

Hypothesize an individual with a current income of $70,000 per year which is expected to improve 11% per year over the remaining 20 years of his expected working life. Further, assume that his dependents could invest insurance proceeds in long-term government bonds which promise a 14% rate of return. Then, the individual's PVLI is approximately $1,070,000:

$$[\$70{,}000]\ [1.11/.03]\ [1-(1.11/1.14)^{20}]$$

$$= \$1{,}070{,}634.$$

SOME COMMENTS

First, equations (5) and (6) are general in the sense that they are valid whether G exceeds K or K exceeds G. If G equals K, PVLI simply equals (I)(N).

Second, if the analyst is uncertain of the values of the variables, he can apply their expected values or he can utilize probability distributions of possible values to derive a probability distribution of possible PVLI's. Moreover, the analyst can manipulate various values of variables to determine the sensitivity of PVLI to reasonable errors in estimating these input variables.

Third, equation (6) can be useful even in complex circumstances which entail a number of differing rates for G over subperiods of N. For example, assume that the circumstances of the individual are as above except that income is expected to leap 20% per year for the first five years before declining to the 11% level, thereafter.

In this situation, the present value of income over the first five years is computed using equation (6) as before. This figure is about $410,000:

[$70,000] [1.20/-.06] [1-((1.20/1.14)^5)]

= $409,298.

Next, calculate the value as of the fifth year of income for years six through 20. Note that, now, I is income as of the end of the fifth year and N equals 15.

[($70,000)(1.20^5)] [1.11/.03] [1-((1.11/1.14)^15)]

= $2,124,807.

Then, discount the $2,124,807 to its present value:

[$2,124,807] [(1/1.14)^5]

= $1,103,558.

Finally, add the two present values:

$409,298 + $1,103,558

= $1,512,856.

A fourth comment concerns the lack of widespread dissemination of equations (5) and (6). Equation (5) is hardly mentioned in literature from the field of investments. (A casual search turned-up only one passing reference to equation (5). See Sharpe, William F., Investments (second edition). Englewood Cliffs: Prentice-Hall, Inc., 1981, p. 380.) Applying the proceedure to gauging a person's PVLI, equation (6), appears unique.