QFI-Quantitative Finance Formula Package
Spring and Fall 2019

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

In sources where some equations are numbered and others are not (nn) denotes that there is no number assigned to that particular equation.

An Introduction to the Mathematics of Financial Derivatives, 3rd Edition (second printing), A. Hirsa and S. Neftci

Chapter 2

(2.10) $$\begin{bmatrix} 1 \\ S(t) \\ C(t) \end{bmatrix} = \begin{bmatrix} (1+r\Delta) & (1+r\Delta) \\ S_1(t+\Delta) & S_2(t+\Delta) \\ C_1(t+\Delta) & C_2(t+\Delta) \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

(2.46) $$R_1(t+1) = \frac{S_1(t+1)}{S(t)}$$

(2.47) $$R_2(t+1) = \frac{S_2(t+1)}{S(t)}$$

(2.48) $$0 = ((1+r) - R_1)\psi_1 + ((1+r) - R_2)\psi_2$$

(2.66) $$S_t = \frac{1}{1+r}\left[\mathbb{Q}_{up}(S_t + \sigma\sqrt{\Delta}) + \mathbb{Q}_{down}(S_t - \sigma\sqrt{\Delta})\right]$$

(2.67) $$C_t = \frac{1}{(1+r)}\left[\mathbb{Q}_{up}C^{up}_{t+\Delta} + \mathbb{Q}_{down}C^{down}_{t+\Delta}\right]$$

(2.68) $$C_T = \max[S_T - C_0, 0]$$

(2.70) $$S = \frac{1+d}{1+r}\left[S^u\mathbb{Q}_{up} + S^d\mathbb{Q}_{down}\right]$$

(2.71) $$C = \frac{1}{1+r}\left[C^u\mathbb{Q}_{up} + C^d\mathbb{Q}_{down}\right]$$

(page 27) $$\mathbb{E}^{\mathbb{Q}}\left[\frac{C_{t+\Delta}}{C_t}\right] \approx 1 + r\Delta$$

Chapter 3

(3.37) $$\sum_{i=1}^{n} f\left(\frac{t_i + t_{i-1}}{2}\right)(t_i - t_{i-1}) \to \int_0^T f(s)ds$$

(3.49) $$\int_0^T g(s)df(s) \approx \sum_{i=1}^{n} g\left(\frac{t_i + t_{i-1}}{2}\right)(f(t_i) - f(t_{i-1}))$$

Chapter 4

(4.20) $$f(x) = \sum_{i=0}^{\infty} \frac{1}{i!} f^i(x_0)(x - x_0)^i$$

(4.23) $$dF(t) = F_S dS_t + F_r dr_t + F_t dt$$

(4.24) $$dF(t) = F_s dS_t + F_r dr_t + F_t dt + \frac{1}{2}F_{ss}dS_t^2 + \frac{1}{2}F_{rr}dr_t^2 + F_{sr}dS_t dr_t$$

Chapter 5

(5.11) $$\mathbb{E}[S_t|I_u] = \int_{-\infty}^{\infty} S_t f(S_t|I_u)dS_t, \qquad u < t$$

(5.18) $$P(\Delta F(t) = +a\sqrt{\Delta}) = p$$

(5.19) $$P(\Delta F(t) = -a\sqrt{\Delta}) = 1 - p$$

(5.37) $$P(\Delta N_t = 1) \approx \lambda\Delta$$

(5.38) $$P(\Delta N_t = 0) \approx 1 - \lambda\Delta$$

(5.39) $P(\Delta N_t = n) = \dfrac{e^{-\lambda\Delta}(\lambda\Delta)^n}{n!}$

(5.40) $f(x) = \dfrac{1}{\theta}\exp\left(-\dfrac{x}{\theta}\right), \quad x \geq 0$

(5.41) $F(x) = 1 - \exp(-x/\theta), \quad x \geq 0$

(5.42) $f(x) = \dfrac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$

(5.44) $P(X_{t+s} \leq x_{t+s} | x_t, \ldots, x_1) = P(X_{t+s} \leq x_{t+s} | x_t)$

(5.45) $r_{t+\Delta} - r_t = \mathbb{E}[(r_{t+\Delta} - r_t)|I_t] + \sigma(I_t, t)\Delta W_t$

(5.48) $dr_t = \mu(r_t, t)dt + \sigma(r_t, t)dW_t$

(5.49) $\begin{bmatrix} r_{t+\Delta} \\ R_{t+\Delta} \end{bmatrix} = \begin{bmatrix} \alpha_1 r_t + \beta_1 R_t \\ \alpha_2 r_t + \beta_2 R_t \end{bmatrix} + \begin{bmatrix} \sigma_1 W^1_{t+\Delta} \\ \sigma_2 W^2_{t+\Delta} \end{bmatrix}$

Chapter 6

(6.3) $\mathbb{E}_t[S_T] = \mathbb{E}[S_T|I_t], \quad t < T$

(6.4) $\mathbb{E}\,|S_t| < \infty$

(6.5) $\mathbb{E}_t\,[S_T] = S_t,$ for all $t < T$

(6.9) $\mathbb{E}^{\mathbb{Q}}_t[e^{-ru}B_{t+u}] = B_t, \quad 0 < u < T - t$ (The text formula is incorrect, the right-hand side should be B_t)

(6.10) $\mathbb{E}^{\mathbb{Q}}_t[e^{-ru}S_{t+u}] = S_t, \quad 0 < u$

(6.29) $V^1 = \sum_{i=1}^{n} |X_{t_i} - X_{t_{i-1}}|$

(6.30) $V^2 = \sum_{i=1}^{n} |X_{t_i} - X_{t_{i-1}}|^2$

(6.31) $V^4 = \sum_{i=1}^{n} |X_{t_i} - X_{t_{i-1}}|^4$

(6.36) $V^2 \leq \max_i |X_{t_i} - X_{t_{i-1}}| V^1$

(6.37) $\max_i |X_{t_i} - X_{t_{i-1}}| \to 0$

(6.38) $V^4 \leq \max_i |X_{t_i} - X_{t_{i-1}}|^2 V^2$

(6.44) $\Delta X_t \sim N(\mu\Delta, \sigma^2\Delta)$

(6.46) $X_{t+T} = X_0 + \int_0^{t+T} dX_u$

(6.49) $X_t + \mathbb{E}_t\left[\int_t^{t+T} dX_u\right] = X_t + \mu T$ (This is a correction to the text formula)

(6.50) $Z_t = X_t - \mu t$

(6.53) $\mathbb{E}_t[Z_{t+T}] = X_t + \mathbb{E}_t[(X_{t+T} - X_t)] - \mu(t+T)$ (This is a correction to the text formula)

(6.54) $\mathbb{E}_t[Z_{t+T}] = X_t - \mu t$ (This is a correction to the text formula)

(6.55) $\mathbb{E}_t[Z_{t+T}] = Z_t$

(6.64) $I_t \subseteq I_{t+1} \subseteq \cdots \subseteq I_{T-1} \subseteq I_T$

(6.65) $M_t = \mathbb{E}^{\mathbb{P}}[Y_T|I_t]$

(6.66) $\mathbb{E}^{\mathbb{P}}[M_{t+s}|I_t] = M_t$

(6.70) $G_T = f(S_T)$

(6.71) $B_T = e^{\int_t^T r_s ds}$

(6.72) $M_t = \mathbb{E}^{\mathbb{P}}\left[\frac{G_T}{B_T}|I_t\right]$

(6.105) $e^{-rt}S_t = A_t + Z_t$

(6.106) $M_{t_k} = M_{t_0} + \sum_{i=1}^{k} H_{t_{i-1}}[Z_{t_i} - Z_{t_{i-1}}]$

(6.108) $\mathbb{E}_{t_0}[M_{t_k}] = M_{t_0}$ (This is a correction to the text formula)

Chapter 7

(7.23) $\Delta W_k = [S_k - S_{k-1}] - \mathbb{E}_{k-1}[S_k - S_{k-1}]$

(7.26) $W_k = \sum_{i=1}^{k} \Delta W_i$

(7.28) $\mathbb{E}_{k-1}[W_k] = W_{k-1}$

(7.29) $\mathbb{V}^k = \mathbb{E}_0[\Delta W_k^2]$

(7.30) $\mathbb{V} = \mathbb{E}_0\left[\sum_{k=1}^{n} \Delta W_k\right]^2 = \sum_{k=1}^{n} \mathbb{V}^k$

(7.31) $\mathbb{V} > A_1 > 0$

(7.33) $\mathbb{V} < A_2 < \infty$

(7.34) $\mathbb{V}_{max} = \max_k[\mathbb{V}^k, k = 1, \ldots, n]$

(7.35) $\frac{\mathbb{V}^k}{\mathbb{V}_{max}} > A_3, \quad 0 < A_3 < 1$

(7.36) $\mathbb{E}[\Delta W_k]^2 = \sigma_k^2 h$

(7.56) $S_k - S_{k-1} = \mathbb{E}_{k-1}[S_k - S_{k-1}] + \sigma_k \Delta W_k$

(7.63) $\mathbb{E}_{k-1}(S_k - S_{k-1}) \approx a(I_{k-1}, kh)h$

(7.64) $S_{kh} - S_{(k-1)h} \approx a(I_{k-1}, kh)h + \sigma_k[W_{kh} - W_{(k-1)h}]$

Chapter 8

(8.7) $dN_t = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases}$

(8.8) $M_t = N_t - \lambda t$

(8.9) $\mathbb{E}[M_t] = 0$

(8.10) $\mathbb{E}[M_t]^2 = \lambda t$

(8.21) $\sigma_k \Delta W_k = \begin{cases} \omega_1 & \text{with probability } p_1 \\ \omega_2 & \text{with probability } p_2 \\ \vdots & \vdots \\ \omega_m & \text{with probability } p_m \end{cases}$

(8.22) $\mathbb{E}[\sigma_k \Delta W_k]^2 = \sigma_k^2 h$

(8.23) $\sum_{i=1}^{m} p_i \omega_i^2 = \sigma_k^2 h$

(8.29) $p_i(h) = \bar{p}_i h^{q_i}$

(8.33) $q_i + 2r_i = 1$

(8.34) $c_i = \bar{\omega}_i^2 \bar{p}_i$

(8.58) $J_t = (N_t - \lambda t)$

(8.60) $dS_t = a(S_t, t)dt + \sigma_1(S_t, t)dW_t + \sigma_2(S_t, t)dJ_t$

(8.61) $\mathbb{V}[X_t] = \mathbb{E}[X_t - \mathbb{E}[X_t]]^2$

(8.62) Higher-order (centered) moments are $\mathbb{E}[X_t - \mathbb{E}[X_t]]^k,\ k > 2$

(8.72) $\mathbb{E}[\sigma_2 \Delta J_k]^2 = h \left[\sum_{i=1}^{m} \omega_i^2 \bar{p}_i \right]$

(8.73) $\mathbb{E}[\sigma_2 \Delta J_k]^n = h \left[\sum_{i=1}^{m} \omega_i^n \bar{p}_i \right]$

(8.75) $t_0 = 0 < t_1 < \cdots < t_n = T$

(8.76) $n\Delta = T$

(8.77) $S_i = S_{t_i}, \quad i = 0, 1, \ldots, n$

(8.78) $S_{i+1} = \begin{cases} u_i S_i & \text{with probability } p_i \\ d_i S_i & \text{with probability } 1 - p_i \end{cases}$

(8.79) $u_i = e^{\sigma\sqrt{\Delta}}, \quad \text{for all } i$

(8.80) $d_i = e^{-\sigma\sqrt{\Delta}}, \quad \text{for all } i$

(8.81) $p_i = \frac{1}{2}\left[1 + \frac{\mu}{\sigma}\sqrt{\Delta}\right], \quad \text{for all } i$

(8.91) $\log \frac{S_{i+n}}{S_i} = Z \log u + (n - Z) \log d$

(8.92) $\log \frac{S_{i+n}}{S_i} = Z \log \frac{u}{d} + n \log d$

(8.96) $\mathbb{E}\left[\log \frac{S_{i+n}}{S_i}\right] = \log \frac{u}{d} np + n \log d$

(8.97) $\mathbb{V}\left[\log \frac{S_{i+n}}{S_i}\right] = \left[\log \frac{u}{d}\right]^2 np(1-p)$

(8.98) $n = \frac{T}{\Delta}$

(8.99) $\log \frac{u}{d} np + n \log d \approx \mu T$

(8.100) $\left[\log \frac{u}{d}\right]^2 np(1-p) \approx \sigma^2 T$

(8.102) $[\log S_{i+n} - \log S_i] \sim \mathcal{N}\left(\mu(n\Delta), \sigma^2 \Delta\right)$

(8.103) $[\log S_{i+n} - \log S_i] \sim \text{Poisson}$

Chapter 9

(9.37) $$\lim_{n\to\infty} \mathbb{E}\left[\sum_{k=1}^{n} \sigma(S_{k-1},k)\,[W_k - W_{k-1}] - \int_0^T \sigma(S_u,u)dW_u\right]^2 = 0$$

(9.38) $$S_k - S_{k-1} = a(S_{k-1},k)h + \sigma(S_{k-1},k)[W_k - W_{k-1}], \quad k = 1,2,\ldots,n$$

(9.39) $$\mathbb{E}\left[\int_0^T \sigma(S_t,t)^2 dt\right] < \infty$$

(9.41) $$\sum_{k=1}^{n} \sigma(S_{k-1},k)[W_k - W_{k-1}] \to \int_0^T \sigma(S_t,t)dW_t \text{ as } n\to\infty (h\to 0)$$

(9.73) $$\int_0^T x_t dx_t = \frac{1}{2}\left[x_T^2 - T\right]$$

(9.74) $$\lim_{n\to\infty} \mathbb{E}\left[\sum_{i=0}^{n-1} \Delta x_{t_{i+1}}^2 - T\right]^2 = 0$$

(9.76) $$\text{If } \int_0^T (dx_t)^2 \text{ exists, then } \lim_{n\to\infty} \mathbb{E}\left[\sum_{i=0}^{n-1} \Delta x_{t_{i+1}}^2 - \int_0^T (dx_t)^2\right]^2 = 0$$

(9.77) $$\int_0^T dt = T$$

(9.78) $$\int_0^T (dx_t)^2 = \int_0^T dt$$

(9.79) $$(dW_t)^2 = dt$$

(9.85) $$\mathbb{E}_t\left[\int_0^{t+\Delta} \sigma_u dW_u\right] = \int_0^t \sigma_u dW_u$$ (This is a correction to the text formula)

(9.132) $$\mathbb{E}\left[\int_0^T f(W_t,t)dW_t \int_0^T g(W_t,t)dW_t\right] = \mathbb{E}\left[\int_0^T f(W_t,t)g(W_t,t)dt\right]$$

(This is a correction to the text formula)

(9.133) $$\mathbb{E}\left[\int_0^T f(W_t,t)dW_t\right]^2 = \mathbb{E}\left[\int_0^T f(W_t,t)^2 dt\right]$$ (This is a correction to the text formula)

Chapter 10

(page 170) $$dS_t = a_t dt + \sigma_t dW_t, \quad t \geq 0$$

(10.36) $$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\sigma_t^2 \frac{\partial^2 F}{\partial S_t^2} dt$$

(10.37) $$dF_t = \left[a_t \frac{\partial F}{\partial S_t} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma_t^2 \frac{\partial^2 F}{\partial S_t^2}\right] dt + \frac{\partial F}{\partial S_t}\sigma_t dW_t$$ (This is a correction to the text formula)

(10.64) $$\int_0^t F_s dS_u = [F(S_t,t) - F(S_0,0)] - \int_0^t \left[F_u + \frac{1}{2}F_{ss}\sigma_u^2\right] du$$

(10.69) $$dF = F_t dt + F_{s_1} dS_1 + F_{s_2} dS_2 + \frac{1}{2}\left[F_{s_1 s_1} dS_1^2 + F_{s_2 s_2} dS_2^2 + 2F_{s_1 s_2} dS_1 dS_2\right]$$

(10.72) $$dS_1(t)^2 = \left[\sigma_{11}^2(t) + \sigma_{12}^2(t)\right] dt$$

(10.73) $$dS_2(t)^2 = \left[\sigma_{21}^2(t) + \sigma_{22}^2(t)\right] dt$$

(10.74) $$dS_1(t)dS_2(t) = [\sigma_{11}(t)\sigma_{21}(t) + \sigma_{12}(t)\sigma_{22}(t)]\, dt$$

(10.79) $$Y(t) = \sum_{i=1}^{n} N_i(t)P_i(t)$$

(10.80) $$dY(t) = \sum_{i=1}^{n} N_i(t)dP_i(t) + \sum_{i=1}^{n} dN_i(t)P_i(t) + \sum_{i=1}^{n} dN_i(t)dP_i(t)$$

(10.81) $$dS_t = a_t dt + \sigma_t dW_t + dJ_t, \quad t \geq 0$$

(10.82) $$E[\Delta J_t] = 0$$

(10.83) $$\Delta J_t = \Delta N_t - \left[\lambda_t h \left(\sum_{i=1}^{k} a_i p_i\right)\right]$$

(10.84) $$a_t = \alpha_t + \lambda_t \left(\sum_{i=1}^{k} a_i p_i\right)$$

(10.85) $$dF(S_t, t) = \left[F_t + \lambda_t \sum_{i=1}^{k} (F(S_t + a_i, t) - F(S_t, t))p_i + \frac{1}{2} F_{ss}\sigma^2\right] dt + F_s dS_t + dJ_F$$

(10.86) $$dJ_F = [F(S_t, t) - F(S_t^-, t)] - \lambda_t \left[\sum_{i=1}^{k} (F(S_t + a_i, t) - F(S_t, t))p_i\right] dt$$

(10.87) $$S_t^- = \lim_{s \to t} S_s, \quad s < t$$

Chapter 11

(11.24) $$dS_t = \mu S_t dt + \sigma S_t dW_t, \quad t \in [0, \infty)$$

(11.30) $$S_t = S_0 e^{\{(a - \frac{1}{2}\sigma^2)t + \sigma W_t\}}$$

(11.34) $$dS_t = r S_t dt + \sigma S_t dW_t, \quad t \in [0, \infty)$$

(11.38) $$S_T = \left[S_0 e^{(r - \frac{1}{2}\sigma^2)T}\right] \left[e^{\sigma W_T}\right]$$

(11.42) $$Z_t = e^{\sigma W_t}$$

(11.50) $$x_t = \mathbb{E}[Z_t] = e^{\frac{1}{2}\sigma^2 t}$$

(11.56) $$S_t = e^{-r(T-t)}\mathbb{E}_t[S_T]$$

(11.72) $$dS_t = \mu S_t dt + \sigma \sqrt{S_t} dW_t, \quad t \in [0, \infty)$$

(11.74) $$dS_t = \lambda(\mu - S_t)dt + \sigma S_t dW_t$$

(11.78) $$dS_t = -\mu S_t dt + \sigma dW_t$$

(11.79) $$dS_t = \mu dt + \sigma_t dW_{1t}$$

(page 192) $$d\sigma_t = \lambda(\sigma_0 - \sigma_t)dt + \alpha \sigma_t dW_{2t}$$

(page 193) $$dN_t = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases}$$

(11.83) $$\frac{dS_t}{S_t} = (\mu - \lambda\kappa)dt + \sigma dW_t + (e^J - 1)dN_t$$

(11.84) $$\frac{dS_t}{S_t} = (\mu - \lambda^*\kappa^*)dt + \sigma d\tilde{W}_t + (e^J - 1)dN_t$$

(page 194) $$S_t = S_0 e^{(r - q + \omega)t + X(t; \sigma, \nu, \theta)}$$

(page 194) $$f(x; \sigma, \nu, \theta) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi g}} \exp\left(-\frac{(x - \theta g)^2}{2\sigma^2 g}\right) \frac{g^{t/\nu - 1} e^{-g/\nu}}{\nu^{t/\nu}\Gamma(t/\nu)} dg$$

Chapter 12

(12.3) $P_t = \theta_1 F(S_t, t) + \theta_2 S_t$

(12.4) $dP_t = \theta_1 dF_t + \theta_2 dS_t$

(12.5) $dS_t = a(S_t, t)dt + \sigma(S_t, t)dW_t, \quad t \in [0, \infty)$

(12.6) $dF_t = F_t dt + \frac{1}{2} F_{ss} \sigma_t^2 dt + F_s dS_t$

(12.7) $dF_t = \left[F_s a_t + \frac{1}{2} F_{ss} \sigma_t^2 + F_t \right] dt + F_s \sigma_t dW_t$

(12.10) $\theta_1 = 1$

(12.11) $\theta_2 = -F_s$

(12.12) $dP_t = F_t dt + \frac{1}{2} F_{ss} \sigma_t^2 dt$

(12.16) $r(F(S_t, t) - F_s S_t) = F_t + \frac{1}{2} F_{ss} \sigma_t^2$

(12.17) $-rF + rF_s S_t + F_t + \frac{1}{2} F_{ss} \sigma_t^2 = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$

(12.20) $P_t = \theta_1 F(S_t, t) + \theta_2 S_t$

(12.23) $P_t = F(S_t, t) - F_s(S_t, t) S_t$

(12.24) $dP_t = dF(S_t, t) - F_s dS_t - S_t dF_s - dF_s(S_t, t) dS_t$

(12.26) $dP_t = dF(S_t, t) - F_s dS_t - S_t \left[\left[F_{st} + F_{ss} \mu S_t + \frac{1}{2} F_{sss} \sigma_t^2 S_t^2 \right] dt + F_{ss} \sigma S_t dW_t \right] - F_{ss} \sigma_t^2 S_t^2 dt$

(12.28) $dP_t = dF(S_t, t) - F_s dS_t - S_t [F_{ss}(\mu - r) S_t dt] - F_{ss} \sigma S_t^2 dW_t$

(page 202) $\mathbb{E}^{\mathbb{Q}} \left[S_t^2 F_{SS} (\sigma dW_t + (\mu - r)\Delta) \right] \approx 0$

(page 202) $dW_t^* = \sigma dW_t + (\mu - r) dt$

(12.29) $a_0 F + a_1 F_s S_t + a_2 F_t + a_3 F_{ss} = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$

(12.30) $F(S_T, T) = G(S_T, T)$

Chapter 13

(13.1) $a(S_t, t) = \mu S_t$

(13.2) $\sigma(S_t, t) = \sigma S_t, \quad t \in [0, \infty)$

(13.3) $-rF + rF_s S_t + F_t + \frac{1}{2} \sigma^2 F_{ss} S_t^2 = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$

(13.4) $F(T) = \max[S_T - K, 0]$

(13.6) $F(S_t, t) = S_t N(d_1) - K e^{-r(T-t)} N(d_2)$

(13.7) $d_1 = \frac{\ln(\frac{S_t}{K}) + (r + \frac{1}{2}\sigma^2)(T - t)}{\sigma \sqrt{T - t}}$

(13.8) $d_2 = d_1 - \sigma \sqrt{T - t}$

(13.9) $N(d_i) = \int_{-\infty}^{d_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx, \quad i = 1, 2$

(13.12) $a(S_t, t) = \mu S_t$

(13.13) $\sigma(S_t, t) = \sigma(S_t, t) S_t, \quad t \in [0, \infty)$

(13.14) $$-rF + rF_s S_t + F_t + \frac{1}{2}\sigma(S_t,t)^2 F_{ss} S_t^2 = 0, \quad 0 \le S_t, \quad 0 \le t \le T$$

(13.15) $$F(T) = \max[S_T - K, 0]$$

(13.34) $$rF - rF_s S_t - \delta - F_t - \frac{1}{2} F_{ss}\sigma_t^2 = 0$$

(13.35) $F(S_{1T}, S_{2T}, T) = \max[0, \max(S_{1T}, S_{2T}) - K]$ (multi-asset option)

(13.36) $F(S_{1T}, S_{2T}, T) = \max[0, (S_{1T} - S_{2T}) - K]$ (spread call option)

(13.37) $F(S_{1T}, S_{2T}, T) = \max[0, (\theta_1 S_{1T} + \theta_2 S_{2T}) - K]$ (portfolio call option)

(13.38) $F(S_{1T}, S_{2T}, T) = \max[0, (S_{1T} - K_1), (S_{2T} - K_2)]$ (dual strike call option)

(This is a correction to the text formula)

(13.47) $$\frac{\Delta F}{\Delta t} + rS\frac{\Delta F}{\Delta S} + \frac{1}{2}\sigma^2 S^2 \frac{\Delta^2 F}{\Delta S^2} \approx rF$$

(13.48) $$\frac{\Delta F}{\Delta t} \approx \frac{F_{ij} - F_{i,j-1}}{\Delta t}$$

(13.49) $$\frac{\Delta F}{\Delta S} \approx \frac{F_{ij} - F_{i-1,j}}{\Delta S}$$

(13.50) $$rS\frac{\Delta F}{\Delta S} \approx rS_j \frac{F_{i+1,j} - F_{ij}}{\Delta S}$$

(13.51) $$\frac{\Delta^2 F}{\Delta S^2} \approx \left[\frac{F_{i+1,j} - F_{ij}}{\Delta S} - \frac{F_{ij} - F_{i-1,j}}{\Delta S}\right]\frac{1}{\Delta S}$$

Chapter 14

(14.3) $$\mathbb{P}\left(\bar{z} - \frac{1}{2}\Delta < z_t < \bar{z} + \frac{1}{2}\Delta\right) = \int_{\bar{z}-\frac{1}{2}\Delta}^{\bar{z}+\frac{1}{2}\Delta} \frac{1}{\sqrt{2\pi}} e^{-\frac{z_t^2}{2}} dz_t$$

(14.6) $$d\mathbb{P}(\bar{z}) = \mathbb{P}\left(\bar{z} - \frac{1}{2}dz_t < z_t < \bar{z} + \frac{1}{2}dz_t\right)$$

(14.7) $$\int_{-\infty}^{\infty} d\mathbb{P}(z_t) = 1$$

(14.8) $$\mathbb{E}[z_t] = \int_{-\infty}^{\infty} z_t d\mathbb{P}(z_t)$$

(14.9) $$\mathbb{E}\left[z_t - E[z_t]\right]^2 = \int_{-\infty}^{\infty} \left[z_t - \mathbb{E}[z_t]\right]^2 d\mathbb{P}(z_t)$$

(14.29) $$\mathbb{E}_t\left[\frac{1}{1+R_t} S_{t+1}\right] = S_t$$

(14.31) $$\mathbb{E}_t^{\mathbb{Q}}\left[\frac{1}{1+r_t} S_{t+1}\right] = S_t$$

(14.41) $$z_t \sim N(0,1)$$

(14.42) $$d\mathbb{P}(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t)^2} dz_t$$

(14.43) $$\xi(z_t) = e^{z_t\mu - \frac{1}{2}\mu^2}$$

(14.44) $$[d\mathbb{P}(z_t)][\xi(z_t)] = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t^2) + \mu z_t - \frac{1}{2}\mu^2} dz_t$$

(14.45) $$d\mathbb{Q}(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t - \mu)^2} dz_t$$

(14.47) $$d\mathbb{Q}(z_t) = \xi(z_t)d\mathbb{P}(z_t)$$

(14.48) $$\xi(z_t)^{-1}d\mathbb{Q}(z_t) = d\mathbb{P}(z_t)$$

(14.53) $$f(z_{1t}, z_{2t}) = \frac{1}{2\pi\sqrt{|\Omega|}} e^{-\frac{1}{2}\left[\begin{array}{cc} z_{1t}-\mu_1, & z_{2t}-\mu_2 \end{array}\right]\Omega^{-1}\left[\begin{array}{c} z_{1t}-\mu_1 \\ z_{2t}-\mu_2 \end{array}\right]}$$

(This is a correction to the text formula)

(14.54) $$\Omega = \left[\begin{array}{cc} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{array}\right]$$

(14.55) $$|\Omega| = \sigma_1^2\sigma_2^2 - \sigma_{12}^2$$

(14.56) $$d\mathbb{P}(z_{1t}, z_{2t}) = f(z_{1t}, z_{2t})dz_{1t}dz_{2t}$$

(14.57) $$\xi(z_{1t}, z_{2t}) = \exp\left\{-\left[\begin{array}{cc} z_{1t}, & z_{2t} \end{array}\right]\Omega^{-1}\left[\begin{array}{c} \mu_1 \\ \mu_2 \end{array}\right] + \frac{1}{2}\left[\begin{array}{cc} \mu_1, & \mu_2 \end{array}\right]\Omega^{-1}\left[\begin{array}{c} \mu_1 \\ \mu_2 \end{array}\right]\right\}$$

(This is a correction to the text formula)

(14.58) $$d\mathbb{Q}(z_{1t}, z_{2t}) = \xi(z_{1t}, z_{2t})d\mathbb{P}(z_{1t}, z_{2t})$$

(14.59) $$d\mathbb{Q}(z_{1t}, z_{2t}) = \frac{1}{2\pi\sqrt{|\Omega|}} \exp\left\{-\frac{1}{2}\left[\begin{array}{cc} z_{1t}, & z_{2t} \end{array}\right]\Omega^{-1}\left[\begin{array}{c} z_{1t} \\ z_{2t} \end{array}\right]\right\} dz_{1t}dz_{2t}$$

(This is a correction to the text formula)

(14.60) $$\xi(z_t) = e^{-z_t'\Omega^{-1}\mu + \frac{1}{2}\mu'\Omega^{-1}\mu}$$ (This is a correction to the text formula)

(14.69) $$\frac{d\mathbb{Q}(z_t)}{d\mathbb{P}(z_t)} = \xi(z_t)$$

(14.74) $$d\mathbb{Q}(z_t) = \xi(z_t)d\mathbb{P}(z_t)$$

(14.75) $$d\mathbb{P}(z_t) = \xi(z_t)^{-1}d\mathbb{Q}(z_t)$$

(14.76) $$\xi_t = e^{\left(\int_0^t X_u dW_u - \frac{1}{2}\int_0^t X_u^2 du\right)}, \quad t \in [0, T]$$

(14.77) $$\mathbb{E}\left[e^{\int_0^t X_u^2 du}\right] < \infty, \quad t \in [0, T]$$

(14.83) $$\mathbb{E}\left[\int_0^t \xi_s X_s dW_s | I_u\right] = \int_0^u \xi_s X_s dW_s$$

(14.84) $$W_t^* = W_t - \int_0^t X_u du, \quad t \in [0, T]$$ (This is a correction to the text formula)

(14.85) $$\mathbb{Q}(A) = \mathbb{E}^{\mathbb{P}}[1_A \xi_T]$$

(14.86) $$dW_t^* = dW_t - X_t dt$$

(14.93) $$d\mathbb{Q} = \xi_T d\mathbb{P}$$

(14.122) $$A_1 \cup A_2 \cup \cdots \cup A_n = \Omega$$

(14.123) $$1_{A_1} + 1_{A_2} + \ldots 1_{A_n} = 1_\Omega$$

(14.127) $$\mathbb{E}^{\mathbb{P}}[Z_t 1_{A_i}] = \mathbb{Q}(A_i)$$

(14.138) $$\mathbb{E}^{\mathbb{P}}[g(X_t)] = \int_\Omega g(x)f(x)dx$$

(page 249) $$g(X_t) = Z_t h(X_t)$$

(14.140) $$\mathbb{E}^{\mathbb{P}}[g(X_t)] = \int_\Omega h(x)\tilde{f}(x)dx = \mathbb{E}^{\mathbb{Q}}[h(X_t)]$$

Chapter 15

(15.2) $Y_t \sim N(\mu t, \sigma^2 t)$

(15.4) $M(\lambda) = \mathbb{E}[e^{Y_t \lambda}]$

(15.10) $M(\lambda) = e^{\left(\lambda \mu t + \frac{1}{2}\sigma^2 \lambda^2 t\right)}$

(15.15) $S_t = S_0 e^{Y_t} \quad , t \in [0, \infty)$

(15.25) $\mathbb{E}[S_t | S_u, u < t] = S_u e^{\mu(t-s) + \frac{1}{2}\sigma^2 (t-s)}$

(15.30) $Z_t = e^{-rt} S_t$

(15.31) $\mathbb{E}^{\mathbb{Q}}\left[e^{-rt} S_t | S_u, u < t\right] = e^{-ru} S_u$

(15.32) $\mathbb{E}^{\mathbb{Q}}[Z_t | Z_u, u < t] = Z_u$

(15.38) $\mathbb{E}^{\mathbb{Q}}\left[e^{-r(t-u)} S_t | S_u, u < t\right] = S_u e^{-r(t-u)} e^{\rho(t-u) + \frac{1}{2}\sigma^2 (t-u)}$ where under $\mathbb{Q}, Y_t \sim N(\rho t, \sigma^2 t)$

(This is a correction with $t - u$ replacing $t - s$)

(15.39) $\rho = r - \frac{1}{2}\sigma^2$

(15.42) $\mathbb{E}^{\mathbb{Q}}\left[e^{-rt} S_t | S_u, u < t\right] = e^{-ru} S_u$

(15.51) $dS_t = r S_t dt + \sigma S_t dW_t^*$

(15.58) $C_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(T-t)} \max\{S_T - K, 0\}\right]$

(15.88) $dS_t = \mu_t dt + \sigma_t dW_t$

(15.90) $d[e^{-rt} S_t] = e^{-rt}[\mu_t - r S_t] dt + e^{-rt} \sigma_t dW_t$

(15.92) $dW_t^* = dX_t + dW_t$

(15.97) $dX_t = \left[\frac{\mu_t - r S_t}{\sigma_t}\right] dt$

(15.98) $d[e^{-rt} S_t] = e^{-rt} \sigma_t dW_t^*$

(15.111) $d\left[e^{-rt} F(S_t, t)\right] = e^{-rt} \sigma_t F_s dW_t^*$

Chapter 17

(page 282) $R_{t_1} = (1 + r_{t_1} \Delta)$

(page 282) $R_{t_2} = (1 + r_{t_2} \Delta)$

(page 282) $B_{t_1}^s = B(t_1, t_3)$

(page 282) $B_{t_1} = B(t_1, T)$

(page 282) $B_{t_3} = B(t_3, T)$

(17.6)
$$\begin{bmatrix} 1 \\ 0 \\ B_{t_1}^s \\ B_{t_1} \\ C_{t_1} \end{bmatrix} = \begin{bmatrix} R_{t_1} R_{t_2}^u & R_{t_1} R_{t_2}^u & R_{t_1} R_{t_2}^d & R_{t_1} R_{t_2}^d \\ (F_{t_1} - L_{t_2}^u) & (F_{t_1} - L_{t_2}^u) & (F_{t_1} - L_{t_2}^d) & (F_{t_1} - L_{t_2}^d) \\ 1 & 1 & 1 & 1 \\ B_{t_3}^{uu} & B_{t_3}^{ud} & B_{t_3}^{du} & B_{t_3}^{dd} \\ C_{t_3}^{uu} & C_{t_3}^{ud} & C_{t_3}^{du} & C_{t_3}^{dd} \end{bmatrix} \begin{bmatrix} \psi^{uu} \\ \psi^{ud} \\ \psi^{du} \\ \psi^{dd} \end{bmatrix}$$

(17.13) $1 = R_{t_1} R_{t_2}^u \psi^{uu} + R_{t_1} R_{t_2}^u \psi^{ud} + R_{t_1} R_{t_2}^d \psi^{du} + R_{t_1} R_{t_2}^d \psi^{dd}$

(17.14) $\mathbb{Q}_{ij} = (1 + r_{t_1})(1 + r^i_{t_2})\psi^{ij}$

(17.15) $1 = \mathbb{Q}_{uu} + \mathbb{Q}_{ud} + \mathbb{Q}_{du} + \mathbb{Q}_{dd}$

(17.16) $\mathbb{Q}_{ij} > 0$

(17.18) $B^s_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})}\right]$

(17.21) $B_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{B_{t_3}}{(1 + r_{t_1})(1 + r_{t_2})}\right]$ (This is a correction to the text formula)

(17.22) $0 = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})}[F_{t_1} - L_{t_2}]\right]$

(17.23) $C_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})}C_{t_3}\right]$

(17.31) $F_{t_1} = \frac{1}{\mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1+r_{t_1})(1+r_{t_2})}\right]}\mathbb{E}^{\mathbb{Q}}\left[\frac{L_{t_2}}{(1 + r_{t_1})(1 + r_{t_2})}\right]$

(17.36) $B^s_{t_1} = \psi^{uu} + \psi^{ud} + \psi^{du} + \psi^{dd}$

(17.38) $\pi_{ij} = \frac{1}{B^s_{t_1}}\psi^{ij}$

(17.39) $1 = \pi_{uu} + \pi_{ud} + \pi_{du} + \pi_{dd}$

(17.46) $F_{t_1} = \mathbb{E}^{\pi}[L_{t_2}]$

(17.52) $C_{t_3} = N \max[L_{t_2} - K, 0]$

(17.53) $C_{t_1} = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{(1 + r_{t_1})(1 + r_{t_2})} \max[L_{t_2} - K, 0]\right]$

(17.55) $C_{t_1} = B^s_{t_1}\mathbb{E}^{\pi}[\max[L_{t_2} - K, 0]]$

(17.56) $\frac{C_t}{S_t} = \mathbb{E}^{\mathbb{S}}_t\left(\frac{C_T}{S_T}\right)$

(17.57) $\frac{C(K)}{S_0} = \mathbb{E}^{\mathbb{S}}\left(\frac{(S_T - K)^+}{S_T}\right)$

(page 291) $y = \log\left(\frac{S_T}{K}\right)$

(page 291) $\frac{C(K)}{S_0} = \int_0^{\infty}(1 - F(y))e^{-y}dy$

(17.61) $\frac{C(K)}{S_0} = P(\ln S - \ln K > Y)$

(17.63) $F(t; T, S) = \frac{1}{S - T}\left(\frac{P(t,T)}{P(t,S)} - 1\right)$

(17.64) $0 = T_0 < T_1 < T_2 < \cdots < T_M$

(17.65) $\Delta_i = T_{i+1} - T_i, i = 0, 1, 2, \ldots, M - 1$

(17.66) $L_n(t) = \frac{P(t,T_n) - P(t,T_{n+1})}{\Delta_n P(t,T_{n+1})}$

(17.71) $P(T_i, T_{n+1}) = \prod_{j=1}^{n}\frac{1}{1 + \Delta_j L_j(T_i)}$

(17.72) $P(t, T_n) = P(t, T_l)\prod_{j=l}^{n-1}\frac{1}{1 + \Delta_j L_j(T_i)}, \qquad T_{l-1} < t \le T_l$

(17.75) $B_t^* = P(t,T_l)\prod_{j=0}^{l-1}(1+\Delta_j L_j(T_j))$

(17.77) $D_n(t) = \dfrac{\prod_{j=l}^{n-1}\frac{1}{1+\Delta_j L_j(t)}}{\prod_{j=0}^{l-1}(1+\Delta_j L_j(T_J))}$

(17.78) $dL_n(t) = \mu_n(t)L_n(t)dt + L_n(t)\sigma_n^\tau(t)dW_t, 0 \le t \le T_n, n = 1,2,\dots,M$

(17.103) $\mu_n(t) = \sum_{j=\ell}^{n}\dfrac{\Delta_j L_j(t)\sigma_n^\tau(t)\sigma_j(t)}{1+\Delta_j L_j(t)}$

(17.104) $dL_n(t) = \left(\sum_{j=\ell}^{n}\dfrac{\Delta_j L_j(t)\sigma_n^\top(t)\sigma_j(t)}{1+\Delta_j L_j(t)}\right)L_n(t)dt + L_n(t)\sigma_n^\top(t)dW_t, \quad 0\le t\le T_n, \quad n=1,\dots,M$

(17.105) $V_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{T+\delta} r_u du}(F_t - L_T)N\delta\right]$

(17.110) $V_t = \mathbb{E}_t^{\pi}[B(t,T+\delta)(F_t - L_T)N\delta]$

(17.111) $V_t = B(t,T+\delta)\mathbb{E}_t^{\pi}[(F_t - L_T)N\delta]$

(17.112) $F_t = \mathbb{E}_t^{\pi}[L_T]$

(page 296) $C_T = \max[L_{T-\delta} - K, 0]$

(17.113) $C_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{T+\delta} r_u du}\max[L_{T-\delta} - K, 0]\right]$

Chapter 18

(18.3) $B(t,T) = e^{-R(T,t)(T-t)}, \quad t < T$

(18.12) $B(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right]$

(18.20) $R(t,T) = \dfrac{-\log \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right]}{T-t}$

(18.33) $B(t,T) = e^{-\int_t^T F(t,s)ds}$

(18.40) $F(t,T) = \lim_{\Delta\to 0}\dfrac{\log B(t,T) - \log B(t,T+\Delta)}{\Delta}$

(page 311) $F(t,T,U) = \dfrac{\log B(t,T) - \log B(t,U)}{U-T}$

Chapter 19

(19.14) $dB_t = \mu(t,T,B_t)B_t dt + \sigma(t,T,B_t)B_t dV_t^T, \quad \text{where } B_t = B(t,T)$

(19.15) $dB_t = r_t B_t dt + \sigma(t,T,B_t)B_t dW_t^T$

(19.21) $dF(t,T) = \sigma(t,T,B(t,T))\left[\dfrac{\partial\sigma(t,T,B(t,T))}{\partial T}\right]dt - \left[\dfrac{\partial\sigma(t,T,B(t,T))}{\partial T}\right]dW_t$

(This is a correction to the text formula)

(19.22) $dF(t,T) = a(F(t,T),t)dt + b(F(t,T),t)dW_t$

(19.25) $r_t = F(t,t)$

(page 325) $F(t,T) = F(0,T) + \int_0^t b(s,T)\left[\int_s^T b(s,u)du\right]ds + \int_0^t b(s,T)dW_s$

(19.26) $r_t = F(0,t) + \int_0^t b(s,t)\left[\int_s^t b(s,u)du\right]ds + \int_0^t b(s,t)dW_s$

(19.33) $dF(t,T) = b^2(T-t)dt + bdW_t$

(19.34) $dB(t,T) = r_t B(t,T)dt - b(T-t)B(t,T)dW_t$ (This is a correction to the text formula)

(19.35) $r_t = F(0,t) + \frac{1}{2}b^2t^2 + bW_t$

(19.36) $dr_t = (F_t(0,t) + b^2 t)dt + bdW_t$

(19.37) $F_t(0,t) = \frac{\partial F(0,t)}{\partial t}$

Chapter 20

(20.5) $B^1 = B(t, T_1)$

(20.6) $B^2 = B(t, T_2)$

(20.7) $dB^1 = \mu(B^1,t)B^1dt + \sigma_1(B^1,t)B^1dW_t$

(20.8) $dB^2 = \mu(B^2,t)B^2dt + \sigma_2(B^2,t)B^2dW_t$

(20.9) $dr_t = a(r_1,t)dt + b(r_1,t)dW_t$

(20.10) $\mathcal{P} = \theta_1 B^1 - \theta_2 B^2$

(20.11) $\theta_1 = \frac{\sigma_2}{B^1(\sigma_2 - \sigma_1)}\mathcal{P}$ (This is a correction to the text formula)

(20.12) $\theta_2 = \frac{\sigma_1}{B^2(\sigma_2 - \sigma_1)}\mathcal{P}$ (This is a correction to the text formula)

(20.13) $d\mathcal{P} = \theta_1 dB^1 - \theta_2 dB^2$

(20.15) $\left(\theta_1\sigma_1B^1 - \theta_2\sigma_2B^2\right) = \left(\frac{\sigma_2}{B^1(\sigma_2-\sigma_1)}\sigma_1B^1 - \frac{\sigma_1}{B^2(\sigma_2-\sigma_1)}\sigma_2B^2\right)\mathcal{P} = 0$

(This is a correction to the text formula)

(20.16) $d\mathcal{P} = (\theta_1\mu_1B^1 - \theta_2\mu_2B^2)dt$

(20.17) $d\mathcal{P} = \frac{(\sigma_2\mu_1 - \sigma_1\mu_2)}{(\sigma_2 - \sigma_1)}\mathcal{P}dt$

(20.18) $r_t\mathcal{P}dt = \frac{(\sigma_2\mu_1 - \sigma_1\mu_2)}{(\sigma_2 - \sigma_1)}\mathcal{P}dt$ (This is a correction to the text formula)

(20.19) $\frac{\mu_1 - r_t}{\sigma_1} = \frac{\mu_2 - r_t}{\sigma_2}$

(20.20) $\frac{\mu_i - r_t}{\sigma_i} = \lambda(r_t, t)$

(20.21) $dB(t,T) = B_r dr_t + B_t dt + \frac{1}{2}B_{rr}b(r_t,T)^2dt$ (This is a correction to the text formula)

(20.23) $dB(t,T) = \left(B_r a(r_t,t) + B_t + \frac{1}{2}B_{rr}b(r_t,T)^2\right)dt + b(r_t,t)B_r dW_t$ (This is a correction to the text formula)

(20.31) $B_r(a(r_t,t) - b(r_t,t)\lambda_t) + B_t + \frac{1}{2}B_{rr}b(r_t,t)^2 - r_tB = 0$

(20.33) $dr_t = (a(r_t,t) - b(r_t,t)\lambda_t)dt + b(r_t,t)\widetilde{W}_t$

(20.40) $R = \kappa - \frac{b\lambda}{\alpha} - \frac{b^2}{\alpha^2}$

(20.48) $B(t,T) = A(t,T)e^{-C(t,T)r}$

(20.49) $A(t,T) = \left(2\frac{\gamma e^{1/2(\alpha+\lambda+\gamma)T}}{(\alpha+\lambda+\gamma)(e^{\gamma T}-1)+2\gamma}\right)^{2\frac{\alpha\kappa}{b^2}}$

(20.50) $C(t,T) = 2\frac{e^{\gamma T}-1}{(\alpha+\lambda+\gamma)(e^{\gamma T}-1)+2\gamma}$

(20.51) $\gamma = \sqrt{(\alpha+\lambda)^2+2b^2}$

Chapter 21

(21.41) $B(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right]$

(21.42) $dr_t = (a(r_t,t) - \lambda_t b(r_t,t))dt + b(r_t,t)dW_t$

(21.43) $dr_t = a(r_t,t)dt + b(r_t,t)dW_t^*$

(21.45) $B(t,T) = \mathbb{E}_t^{\mathbb{P}}\left[\exp\left(-\int_t^T r_s ds\right)\exp\left(\int_t^T\left[\lambda_s(r_s,s)dW_t^* - \frac{1}{2}\lambda(r_s,s)^2 ds\right]\right)\right]$

(21.68) $dS_t = a(S_t,t)dt + \sigma(S_t,t)dW_t, \quad t \in [0,\infty)$

(21.69) $dS_t = a(S_t)dt + \sigma(S_t)dW_t, \quad t \in [0,\infty)$

(21.70) $dS_t = a(S_t)dt + \sigma(S_t)dW_t, \quad t \in [0,\infty)$

(21.71) $\mathbb{E}\left[f(S_{t+h})|I_t\right] = \mathbb{E}\left[f(S_{t+h})|S_t\right], \quad h > 0$

(21.90) $\widehat{f}(t,r_t) = \mathbb{E}^{\mathbb{P}}\left[e^{-\int_t^u q(r_s)ds} f(r_u)|r_t\right]$ (This is a correction to the text formula)

(21.91) $\frac{\partial \widehat{f}}{\partial t} = q(r_t)\widehat{f} - A\widehat{f}$ (This is a correction to the text formula)

(21.92) $A\widehat{f} = a_t\frac{\partial \widehat{f}}{\partial r_t} + \frac{1}{2}\sigma_t^2\frac{\partial^2 \widehat{f}}{\partial r_1^2}$

Frequently Asked Questions in Quantative Finance, Wilmott (QFIQ-113-17)

Jensen's Inequality (103-105)

If $f(\cdot)$ is a convex function, $E[f(x)] \geq f(E[x])$.

Girsanov's Theorem (113-115)

$$dS = \mu S dt + \sigma S dW_t$$

$$dS = rSdt + \sigma S d\tilde{W}_t$$

$$\tilde{W}_t = W_t + \int_0^t \gamma_s ds$$

$$d\tilde{W}_t = dW_t + \gamma_t dt$$

Volatility Smile (167-173)

$$\frac{\partial^2 V}{\partial \sigma^2} = S\sqrt{T-t}\frac{d_1 d_2 e^{-D(T-t)}e^{-d_1^2/2}}{\sqrt{2\pi}\sigma}$$

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Chapter 2

(2.35) $d = \frac{100 - P_{bill}(t,T)}{100} \times \frac{360}{n}$

(2.36) $P_{bill}(t,T) = 100 \times \left[1 - \frac{n}{360} \times d\right]$

(2.39) Price floating rate bond with spread $s = 100 + \frac{s}{2} \times \sum_{t=0.5}^{T} Z(0,t)$

(This is a correction to the text formula)

(2.52) $Z(0,T) = e^{-r(0,T)T}$

(2.53) $r(0,T) = \theta_0 + (\theta_1 + \theta_2)\frac{1 - e^{-T/\lambda}}{T/\lambda} - \theta_2 e^{-T/\lambda}$

(2.54) $P_c^{i,NS\ model} = 100 \times \left(\frac{c^i}{2}\sum_{j=1}^{n_i} Z(0,T_j^i) + Z(0,T^1)\right)$

Chapter 3

(3.17) $D = -\frac{1}{P}\frac{dP}{dr}$

(3.19) $MD = -\frac{1}{P}\frac{dP}{dy} = \frac{1}{\left(1 + \frac{y}{2}\right)}\sum_{j=1}^{n} w_j \times T_j$

(3.20) $D^{Mc} = \sum_{j=1}^{n} w_j \times T_j$

(3.21) $Dollar\ duration = D_P^{\$} = -\frac{dP}{dr}$

(3.22) $D_P^{\$} = P \times D_P$

(3.23) $D_W^{\$} = \sum_{i=1}^{n} N_i D_i^{\$}$

(3.26) $Price\ value\ of\ a\ basis\ point = PV01(orPVBP) = -D_P^{\$} \times dr$

(3.44) $D_E = \frac{A}{A-L} \times D_A - \frac{L}{A-L} \times D_L$

Chapter 4

(4.1) $Convexity = C = \frac{1}{P}\frac{d^2P}{dr^2}$

(4.2) $\frac{dP}{P} = -D \times dr + \frac{1}{2} \times C \times dr^2$

(4.6) $Convexity\ of\ portfolio = C_W = \sum_{i=1}^{n} w_i C_i$, where (4.7) $w_i = \frac{N_i \times P_i}{W}$

(4.8) $C = \sum_{i=1}^{n} w_i \times C_{z,i}$, where $C_{z,i} = (T_i - t)^2$ and

(4.9) $w_i = \frac{c/2 \times P_z(t, T_i)}{P_c(t, T)}$ for $i = 1, \ldots, n-1$

(4.10) $w_n = \frac{(1 + c/2) \times P_z(t, T_n)}{P_c(t, T)}$

(4.14) Dollar convexity $= C^{\$} = \frac{d^2 P}{dr^2}$

(4.36) $\max_{a_{11},\ldots,a_{1n}} Var\left(\Delta\phi_i^{PCA}\right) = \sum_{k=1}^{n} \sum_{\ell=1}^{n} a_{1k} a_{1\ell} \sigma_{kl}$ under the restriction

(4.37) $\sum_{j=1}^{n} a_{1j}^2 = 1$

(4.40) $\mathbf{M}\mathbf{v}_i = \lambda_i \mathbf{v}_i$

Chapter 5

(5.5) $F(t, T_1, T_2) = \frac{1}{\left(1 + \frac{f_n(t,T_1,T_2)}{n}\right)^{n \times (T_2 - T_1)}}$

(5.6) $F(t, T_1, T_2) = e^{-f(t,T_1,T_2)(T_2 - T_1)}$

(5.14) $f(0, T, T + \Delta) = r(0, T) + (T + \Delta) \times \frac{r(0, T + \Delta) - r(0, T)}{\Delta}$

(5.21) $r(0, T_n) = \frac{1}{T_n} \sum_{i=1}^{n} f(0, T_{i-1}, T_i) \times \Delta$

(5.25) $\textit{Value of FRA at } t = V^{FRA}(t) = V^{fixed}(t) - V^{floating}(t) = N \times [M \times Z(t, T_2) - Z(t, T_1)]$

(5.40) $V^{swap}(t_i; c, T) = 100 - \left(\frac{c}{2} \times 100 \times \sum_{j=i+1}^{M} Z(T_i, T_j) + Z(T_i, T_M) \times 100\right)$

(5.43) $c = n \times \left(\frac{1 - Z(0, T_M)}{\sum_{j=1}^{M} Z(0, T_j)}\right)$

(5.51) $f_n^s(0, T, T^*) = n \times \frac{1 - F(0, T, T)}{\sum_{j=1}^{m} F(0, T, T_j)}$

Chapter 14

(14.27) $dr_t = \gamma(\bar{r} - r_t)dt$

(14.29) $dr_t = \theta_t dt + \sigma dX_t$

(14.34) $r_t \sim \mathcal{N}(\mu(r_0, t), \sigma^2(t))$ where

(14.35) $\mu(r_0, t) = \bar{r} + (r_0 - \bar{r})e^{-\gamma t}$

(14.36) $\sigma^2(t) = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t})$

(14.37) $dP_t = \frac{1}{2}\left(\frac{d^2 F}{dX^2}\right) dt + \left(\frac{dF}{dX}\right) dX_t$

(14.39) $dP_t = \left\{\left(\frac{\partial F}{\partial t}\right) + \frac{1}{2}\left(\frac{\partial^2 F}{\partial X^2}\right)\right\} dt + \left(\frac{\partial F}{\partial X}\right) dX_t$

Chapter 15

(15.9) $dr_t = \gamma(\bar{r} - r_t)dt + \sigma dX_t$

(15.28) $Z(r,t;T) = e^{A(t;T) - B(t;T)\times r}$

(15.29) $B(t;T) = \frac{1}{\gamma^*}\left(1 - e^{-\gamma^*(T-t)}\right)$

(15.30) $A(t;T) = (B(t;T) - (T-t))\left(\bar{r}^* - \frac{\sigma^2}{2(\gamma^*)^2}\right) - \frac{\sigma^2 B(t;T)^2}{4\gamma^*}$

(15.31) $P_c(r,t;T) = \frac{100 \times c}{2}\sum_{i=1}^{n} Z(r,t;T_i) + 100 \times Z(r,t;T_n)$

(15.34) $Z(r_t, \tau) = Z(r_t, t; T)$

(15.35) $A(\tau) = A(0; T-t)$

(15.36) $B(\tau) = B(0; T-t)$

(15.44) $V(r_0, 0) = Z(r_0, 0; T_B)\mathcal{N}(d_1) - KZ(r_0, 0; T_O)\mathcal{N}(d_2)$ (This is a correction to the text formula)

(15.45) $d_1 = \frac{1}{S_Z(T_O)}\log\left(\frac{Z(r_0,0;T_B)}{KZ(r_0,0;T_O)}\right) + \frac{S_Z(T_O)}{2}$ (This is a correction to the text formula)

(15.46) $d_2 = d_1 - S_Z(T_O)$

(15.47) $S_Z(T_O) = B(T_O; T_B) \times \sqrt{\frac{\sigma^2}{2\gamma^*}(1 - e^{-2\gamma^* T_O})}$

(15.48) $V(r_0, 0) = KZ(r_0, 0; T_O)\mathcal{N}(-d_2) - Z(r_0, 0; T_B)\mathcal{N}(-d_1)$ (This is a correction to the text formula)

(15.66) $dr_t = \gamma(\bar{r} - r_t)dt + \sqrt{\alpha r_t}dX_t$

(15.67) $E[r_t|r_0] = \bar{r} + (r_0 - \bar{r})e^{-\gamma t}$

(15.68) $Var[r_t|r_0] = r_0\frac{\alpha}{\gamma}\left(e^{-\gamma t} - e^{-2\gamma t}\right) + \frac{\bar{r}\alpha}{2\gamma}\left(1 - e^{-\gamma t}\right)^2$

(15.70) $Z(r,t;T) = e^{A(t;T) - B(t;T)\times r}$

(15.71) $B(t;T) = \frac{2\left(e^{\psi_1(T-t)} - 1\right)}{(\gamma^* + \psi_1)\left(e^{\psi_1(T-t)} - 1\right) + 2\psi_1}$

(15.72) $A(t;T) = 2\frac{\bar{r}^*\gamma^*}{\alpha}\log\left(\frac{2\psi_1 e^{(\psi_1 + \gamma^*)\frac{(T-t)}{2}}}{(\gamma^* + \psi_1)\left(e^{\psi_1(T-t)} - 1\right) + 2\psi_1}\right)$, and $\psi_1 = \sqrt{(\gamma^*)^2 + 2\alpha}$

Chapter 16

(16.8) $C_t = Z_1(r_t, t) - \Delta Z_2(r_t, t)$

(16.9) $P_t = \Delta Z_{2,t} + C_t$

(16.10) $dP_t = dZ_{1,t}$

(16.18) $\left(\frac{1}{\Pi}\frac{\partial \Pi}{\partial t}\right) + \frac{1}{2}\left(\frac{1}{\Pi}\frac{\partial^2 \Pi}{\partial r^2}\right)\sigma^2 = r$

Chapter 17

(17.1) $$dr_t = m(r_t, t)dt + s(r_t, t)dX_t$$

(17.2) $$rV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial r} m^*(r,t) + \frac{1}{2}\frac{\partial^2 V}{\partial r^2} s(r,t)^2$$

(17.3) $$V(r_T, T) = g(r_T, T)$$

(17.4) $$R(r)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial r} m^*(r,t) + \frac{1}{2}\frac{\partial^2 V}{\partial r^2} s(r,t)^2$$

(17.5) $$V(r_t, t) = E^*\left[e^{-\int_t^T R(r_u)du} g(r_T, T)|r_t\right]$$

(17.6) $$dr_t = m^*(r_t, t)dt + s(r_t, t)dX_t$$

(17.7) $$V(r_t, t) = E^*\left[e^{-\int_t^T r_u du} g(r_T, T)|r_t\right]$$

(17.17) $$f_n^s(0, T, T^*) = \frac{1 - F(0, T, T^*)}{\Delta \sum_{j=1}^n F(0, T, T_j)}$$

(17.20) $$\textit{Standard error} = \frac{\textit{Standard deviation}}{\sqrt{J}}$$

(17.21) $$\textit{Standard deviation} = \sqrt{\frac{1}{J}\sum_{j=1}^{J}\left(V^j(r_0, 0) - \widehat{V}(r_0, 0)\right)^2}$$

(17.24) $$\frac{\partial V}{\partial r} \approx \frac{\widehat{V}(r_0 + \delta) - \widehat{V}(r_0 - \delta)}{2\delta}$$

(17.26) $$\frac{\partial^2 V}{\partial r^2} \approx \frac{\widehat{V}(r_0 + \delta) + \widehat{V}(r_0 - \delta) - 2 \times \widehat{V}(r_0)}{\delta^2}$$

(17.27) $$rV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial r} m^*(r,t) + \frac{1}{2}\frac{\partial^2 V}{\partial r^2} s^2(r,t)$$ (This is a correction to the text formula)

(17.28) $$\frac{\partial V}{\partial t} \approx \frac{\widehat{V}(r_0, t + \delta) - \widehat{V}(r_0, t)}{\delta}$$

(17.29) $$\frac{\partial V}{\partial t} = rV - \frac{\partial V}{\partial r} m^*(r,t) - \frac{1}{2}\frac{\partial^2 V}{\partial r^2} s^2(r,t)$$ (This is a correction to the text formula)

Chapter 18

(18.7) $$\textit{Risk premium} = E\left[\frac{dZ}{Z}\right]/dt - r = -B(t;T)(\gamma(\bar{r}-r) - \gamma^*(\bar{r}^*-r))$$

(18.8) $$\lambda(r,t) = \frac{1}{\sigma}(\gamma(\bar{r}-r) - \gamma^*(\bar{r}^*-r))$$

(18.9) $$\lambda(r,t) = \lambda_0 + \lambda_1 r$$

(18.13) $$\text{Risk premium} = E\left[\frac{dZ}{Z}\right]/dt - r = \sigma_Z \times \lambda(r,t)$$

(18.16) $$\textit{Risk premium} = E\left[\frac{dZ}{Z}\right]/dt - r = \sigma_Z \lambda(r,t), \text{ where}$$

(18.17) $$\sigma_Z = \frac{1}{Z}\frac{\partial Z}{\partial r} s(r,t) \text{ and}$$

(18.18) $$\lambda(r,t) = \frac{1}{s(r,t)}(m(r,t) - m^*(r,t))$$

(18.26) $$V(r_{t+\delta}) \approx V(r_t) + \frac{\partial V}{\partial r}(r_{t+\delta} - r_t)$$

(page 634) $$\text{Standard deviation of } (V(r_{t+\delta}) - V(r_t)) \approx \frac{\partial V}{\partial r} \times \text{Standard deviation of } (r_{t+\delta} - r_t)$$

(18.28) $$r_\delta - r_0 \sim N(\mu(r_0,\delta), \sigma^2(\delta))$$

(18.29) $$\mu(r_0,\delta) = (r_0 - \bar{r}) \times (e^{-\gamma\delta} - 1);\ \sigma(\delta) = \sqrt{\frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma\delta})}$$

(18.32) $$Z(t,T) = E_t\left[e^{-\rho(T-t)}\frac{Q_t Y_t^h}{Q_T Y_T^h}\right]$$

(18.33) $$Z(t,T) = E_t\left[e^{-\rho(T-t)-(q_T-q_t)-h(y_T-y_t)}\right]$$

(18.34) $$Z(i,t,T) = e^{A(t;T) - B(t;T)(i+c)}$$

(18.35) $$B(t;T) = \frac{1}{\gamma}\left(1 - e^{-\gamma(T-t)}\right)$$

(18.36) $$A(t;T) = (B(t;T) - (T-t))\left(\bar{r}^* - \frac{\sigma^2}{2\gamma^2}\right) - \frac{\sigma^2 B(t;T)^2}{4\gamma}$$

(18.37) $$c = \left(\rho + hg - \frac{1}{2}h^2\sigma_y^2\right) - h\sigma_y\sigma_q\rho_{qy} - \frac{1}{2}\sigma_q^2$$

(18.38) $$\bar{r}^* = \bar{r} - \frac{1}{\gamma}(h\sigma_i\sigma_y\rho_{yi} + \sigma_i\sigma_y\rho_{iq})$$

(18.39) $$\bar{r} = \bar{i} + c$$

(18.40) $$r_t = i_t + c$$

(18.41) $$\textit{Risk natural (true) dynamics: } dr_t = \gamma(\bar{r} - r_t)dt + \sigma_i dX_i$$

(18.42) $$\textit{Risk neutral dynamics: } dr_t = \gamma(\bar{r}^* - r_t)dt + \sigma_i dX_i$$

(18.43) $$\lambda = \frac{\gamma}{\sigma_i}(\bar{r} - \bar{r}^*) = h\sigma_y\rho_{yi} + \sigma_y\rho_{iq}$$

Chapter 19

(19.7) $dr_t = \theta_t dt + \sigma dX_t$

(19.8) $Z(r,0;T) = e^{A(0;T)-T\times r}$

(19.9) $A(0,T) = -\int_0^T (T-t)\theta_t dt + \frac{T^3}{6}\sigma^2$

(19.13) $\theta_t = \frac{\partial f(0,t)}{\partial t} + \sigma^2 \times t$

(19.14) $\textit{Payoff at } T_O = \max(Z(T_O;T_B) - K, 0)$

(19.15) $V(r_0,0) = Z(r_0,0;T_B)\mathcal{N}(d_1) - KZ(r_0,0;T_O)\mathcal{N}(d_2)$

Note: For this, and many of the following formulas, the text has $Z(0,r_0;\cdot)$ while $Z(r_0,0;\cdot)$ is correct

(19.16) $d_1 = \frac{1}{S_Z(T_O;T_B)} \log\left(\frac{Z(r_0,0;T_B)}{KZ(r_0,0;T_O)}\right) + \frac{S_Z(T_O;T_B)}{2}$

(19.17) $d_2 = d_1 - S_Z(T_O;T_B)$

(19.18) $S_Z(T_O;T_B)^2 = \sigma^2 T_O (T_B - T_O)^2$

(19.19) $\textit{Payoff at } T_O = \max(K - Z(T_O;T_B), 0)$

(19.20) $V(r_0,0) = KZ(r_0,0;T_O)\mathcal{N}(-d_2) - Z(r_0,0;T_B)\mathcal{N}(-d_1)$

(19.25) $Z(r,0;T) = e^{A(0;T)-B(0;T)\times r}$

(19.26) $B(0;T) = \frac{1}{\gamma^*}\left(1 - e^{-\gamma^* T}\right)$

(19.27) $A(0;T) = -\int_0^T B(0;T-t)\theta_t dt + \frac{\sigma^2}{2(\gamma^*)^2}\left(T + \frac{1-e^{-2\gamma^* T}}{2\gamma^*} - 2B(0;T)\right)$

(This is a correction to the text formula)

(19.28) $\theta_t = \frac{\partial f(0,t)}{\partial t} + \gamma^* f(0,t) + \frac{\sigma^2}{2\gamma^*} \times \left(1 - e^{-2\gamma^* t}\right)$

(19.29) $\sigma_t(\tau) = \frac{B(\tau)}{\tau}\sigma$

(19.30) $V(r_0,0) = Z(r_0,0;T_B)\mathcal{N}(d_1) - KZ(r_0,0;T_O)\mathcal{N}(d_2)$

(19.31) $d_1 = \frac{1}{S_Z(T_O;T_B)} \log\left(\frac{Z(r_0,0;T_B)}{KZ(r_0,0;T_O)}\right) + \frac{S_Z(T_O;T_B)}{2}$

(19.32) $d_2 = d_1 - S_Z(T_O;T_B)$

(19.33) $S_Z(T_O;T_B)^2 = B(T_O;T_B)^2 \frac{\sigma^2}{2\gamma^*}\left(1 - e^{-2\gamma^* T_O}\right)$

(19.34) $V(r_0,0) = KZ(r_0,0;T_O)\mathcal{N}(-d_2) - Z(r_0,0;T_B)\mathcal{N}(-d_1)$

(19.36) $\textit{Payoff of call option at } T_O = \max(P_c(r_{T_O}, T_O; T_B) - K, 0)$

(19.37) $\textit{Call } = \sum_{i=1}^{n} c(i)\left(Z(r_0,0;T_i)\mathcal{N}(d_1(i)) - K_i Z(r_0,0;T_O)\mathcal{N}(d_2(i))\right)$

(19.38) $d_1(i) = \frac{1}{S_Z(T_O;T_i)} \log\left(\frac{Z(r_0,0;T_i)}{K_i Z(r_0,0;T_O)}\right) + \frac{S_Z(T_O;T_i)}{2}$

(19.39) $d_2(i) = d_1(i) - S_Z(T_O;T_i)$

(page 664) $Z(r_t,t;T) = e^{A(t;T)-B(t;T)r_t}$

(19.40) $A(t;T) = -\int_t^T B(u;T)\theta_u du + \frac{\sigma^2}{2(\gamma^*)^2}\left((T-t) + \frac{1-e^{-2\gamma^*(T-t)}}{2\gamma^*} - 2B(t;T)\right)$

(This is a correction to the text formula)

(19.41) $A(t;T) = \log\left(\frac{Z(r_0,0;T)}{Z(r_0,0;t)}\right) + B(t;T)f(0,t) - \frac{\sigma^2}{4\gamma^*}B(t;T)^2\left(1 - e^{-2\gamma^* t}\right)$

(19.42) $A(t;T) = \log\left(\frac{Z(r_0,0;T)}{Z(r_0,0;t)}\right) + (T-t)f(0,t) - \frac{\sigma^2}{2}(T-t)^2 t$

(19.44) $V(r_0,0) = M \times (KZ(r_0,0;T-\Delta)\mathcal{N}(-d_2) - Z(r_0,0;T)\mathcal{N}(-d_1))$

(19.45) $d_1 = \frac{1}{S_Z(T-\Delta;T)}\log\left(\frac{Z(r_0,0;T)}{KZ(r_0,0;T-\Delta)}\right) + \frac{S_Z(T-\Delta;T)}{2}$

(19.46) $d_2 = d_1 - S_Z(T-\Delta;T)$

(19.47) $CF(T_j) = \Delta \times N \times \max(r_n(T_{j-1} - T_j) - r_K, 0)$

(19.48) $Cap = \sum_{j=2}^{n} M \times (KZ(r_0,0;T_{j-1})\mathcal{N}(-d_2(j)) - Z(r_0,0;T_j)\mathcal{N}(-d_1(j)))$

(19.49) $d_1(j) = \frac{1}{S_Z(T_{j-1};T_j)}\log\left(\frac{Z(r_0,0;T_j)}{KZ(r_0,0;T_{j-1})}\right) + \frac{S_Z(T_{j-1};T_j)}{2}$

(19.50) $d_2(j) = d_1(j) - S_Z(T_{j-1};T_j)$

(19.55) $dy_t = \left(\theta_t + \frac{\partial\sigma_t/\partial t}{\sigma_t}y_t\right)dt + \sigma_t dX_t$

(19.57) $dy_t = (\theta_t - \gamma_t y_t)dt + \sigma_t dX_t$

(19.58) $dr_t = (\theta_t - \gamma_t r_t)dt + \sqrt{\sigma_t^2 + \alpha_t r_t}\,dX_t$

(19.59) $Z(r_t,t;T) = e^{A(t;T) - B(t;T)r_t}$

(19.60) $\frac{\partial B(t;T)}{\partial t} = B(t;T)\gamma_t + \frac{1}{2}B(t;T)^2\alpha_t - 1$

(19.61) $\frac{\partial A(t;T)}{\partial t} = B(t;T)\theta_t - \frac{1}{2}B(t;T)^2\sigma_t^2$

Chapter 20

(20.3) $Caplet(0;T_{i+1}) = N \times \Delta \times Z(0,T_{i+1}) \times [f_n(0,T_i,T_{i+1})\mathcal{N}(d_1) - r_K\mathcal{N}(d_2)]$

(20.4) $d_1 = \frac{1}{\sigma_f\sqrt{T_i}}\log\left(\frac{f_n(0,T_i,T_{i+1})}{r_K}\right) + \frac{1}{2}\sigma_f\sqrt{T_i}$

(20.5) $d_2 = d_1 - \sigma_f\sqrt{T_i}$

(20.6) $Floorlet(0;T_{i+1}) = N \times \Delta \times Z(0,T_{i+1}) \times [r_K\mathcal{N}(-d_2) - f_n(0,T_i,T_{i+1})\mathcal{N}(-d_1)]$

(20.7) $Cap(0;T) = \sum_{i=1}^{n} Caplets(0;T_i)$

(20.17) $V(0,T_O;T_S) = N \times \Delta \times \left[\sum_{i=1}^{n} Z(0;T_i)\right] \times [r_K\mathcal{N}(-d_2) - f_n^s(0,T_O,T_S)\mathcal{N}(-d_1)]$

(20.18) $d_1 = \frac{1}{\sigma_f^s\sqrt{T_O}}\ln\left(\frac{f_n^s(0,T_O,T_S)}{r_K}\right) + \frac{1}{2}\sigma_f^s\sqrt{T_O}; \quad d_2 = d_1 - \sigma_f^s\sqrt{T_O}$

(This is a correction to the text formula)

(20.19) $V(0,T_O;T_S) = N \times \Delta \times \left[\sum_{i=1}^{n} Z(0;T_i)\right] \times [f_n^s(0,T_O,T_S)\mathcal{N}(d_1) - r_K\mathcal{N}(d_2)]$

Chapter 21

(21.2) $V(r,t;T) = E^*\left[e^{-\int_t^T r_u du} g_T\right]$

(21.3) $dr_t = m^*(r_t,t)dt + s(r_t,t)dX_t$

(21.4) $\widetilde{V}(r,t;T) = \dfrac{V(r,t;T)}{Z(r,t;T)}$

(21.5) $0 = \dfrac{\partial \widetilde{V}}{\partial t} + \dfrac{\partial \widetilde{V}}{\partial r}(m^*(r,t) + \sigma_Z(r,t)s(r,t)) + \dfrac{1}{2}\dfrac{\partial^2 \widetilde{V}}{\partial r^2}s(r,t)^2$

(21.6) $\sigma_Z(r,t) = \dfrac{1}{Z}\dfrac{\partial Z}{\partial r}s(r,t)$

(21.7) $\dfrac{dZ}{Z} = \mu_Z(r,t)dt + \sigma_Z(r,t)dX_t$

(21.8) $\widetilde{V}(r,t;T) = E_f^*[g_T]$

(21.9) $dr_t = (m^*(r,t) + \sigma_Z(r,t)s(r,t))dt + s(r,t)dX_t$

(21.10) $V(r,t;T) = Z(r,t;T)E_f^*[g_T]$

(21.11) $E_f^*[\max(g_T - K, 0)] = F(0,T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)$

(21.12) $d_1 = \dfrac{1}{\sigma_T}\log\left(\dfrac{F(0,T)}{K}\right) + \dfrac{1}{2}\sigma_T$

(21.13) $d_2 = d_1 - \sigma_T$

(21.14) $Call = Z(0,T) \times [F(0,T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)]$

(21.15) $Put = Z(0,T) \times [K\mathcal{N}(-d_2) - F(0,T)\mathcal{N}(-d_1)]$

(21.27) $V^{fwd}(0;T) = Z(0,T)N\Delta E_f^*[r_n(\tau,T) - K]$

(21.28) $\dfrac{df_n(t,\tau,T)}{f_n(t,\tau,T)} = \sigma_f(t)dX_t$

(21.29) $r_n(\tau,T) \sim LogN\left(f_n(0,\tau,T), \int_0^\tau \sigma_f(t)^2 dt\right)$

(21.32) $Caplet(0,T_{i+1}) = N\Delta Z(0,T_{i+1})[f_n(0,T_i,T_{i+1})\mathcal{N}(d_1) - r_K\mathcal{N}(d_2)]$

(21.33) $d_1 = \dfrac{1}{\sigma_f\sqrt{T_i}}\log\left(\dfrac{f_n(0,T_i,T_{i+1})}{r_K}\right) + \dfrac{1}{2}\sigma_f\sqrt{T_i}$

(21.34) $d_2 = d_1 - \sigma_f\sqrt{T_i}$

(page 722) $S_1 = \sigma_f^{Fwd}(0.25)$

(page 722) $S_i = \sqrt{\dfrac{1}{\Delta} \times \left(\left(\sigma_f^{Fwd}(T_{i+1})\right)^2 \times T_i - \sum_{j=1}^{i-1} S_j^2 \times \Delta\right)}$ (This is a correction to the text formula)

(21.54) $df(t,T) = m(t,T)dt + \sigma_f(t,T)dX_t$

(21.55) $m(t,T) = \sigma_f(t,T)\int_t^T \sigma_f(t,\tau)d\tau$

(21.59) $f(0,\tau,T) = f^{fut}(0,\tau,T) - \int_0^\tau \dfrac{\sigma_Z(t,T)^2 - \sigma_Z(t,\tau)^2}{2(T-\tau)}dt$ (This is a correction to the text formula)

(21.60) $f(0,\tau,T) = f^{fut}(0,\tau,T) - \dfrac{1}{2}\sigma^2\tau T$

Chapter 22

(22.3) $$dP_t = \left\{ \left(\frac{\partial F}{\partial t}\right) + \left(\frac{\partial F}{\partial \phi_1}\right) m_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) m_{2,t} + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_1^2}\right) s_{1,t}^2 + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_2^2}\right) s_{2,t}^2 \right\} dt + \left(\frac{\partial F}{\partial \phi_1}\right) s_{1,t} dX_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) s_{2,t} dX_{2,t}$$

(22.13) $$R(\phi_1, \phi_2)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \phi_1} m_{1,t}^* + \frac{\partial V}{\partial \phi_2} m_{2,t}^* + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_1^2} s_{1,t}^2 + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_2^2} s_{2,t}^2$$

(22.19) $$Z(\phi_{1,t}, \phi_{2,t}, t; T) = e^{A(t;T) - B_1(t;T)\phi_{1,t} - B_2(t;T)\phi_{2,t}}$$

(22.20) $$B_i(t;T) = \frac{1}{\gamma_i^*}\left(1 - e^{\gamma_i^*(T-t)}\right)$$

(22.21) $$A(t;T) = (B_1(t;T) - (T-t))\left(\bar{\phi}_1^* - \frac{\sigma_1^2}{2(\gamma_1^*)^2}\right) - \frac{\sigma_1^2}{4\gamma_1^*} B_1(t;T)^2 + (B_2(t;T) - (T-t))\left(\bar{\phi}_2^* - \frac{\sigma_2^2}{2(\gamma_2^*)^2}\right) - \frac{\sigma_2^2}{4\gamma_2^*} B_2(t;T)^2$$

(22.27) $$Z(r_t, r_{\ell,t}, t; T) = e^{A_{\tau_\ell}(\tau) - B_{\tau_\ell,1}(\tau) r_t - C_{\tau_\ell}(\tau) r_{\ell,t}}, \text{ where } \tau = T - t$$

(22.28) $$A_{\tau_\ell}(\tau) = A(\tau) - C(\tau) \times \frac{A(\tau_\ell)}{C(\tau_\ell)}$$

(22.29) $$B_{\tau_\ell,1}(\tau) = B_1(\tau) - C(\tau) \times \frac{B_1(\tau_\ell)}{C(\tau_\ell)}$$

(22.30) $$C_{\tau_\ell}(\tau) = C(\tau) \times \frac{\tau_\ell}{C(\tau_\ell)}$$

(22.31) $$r_t(\tau) = -\frac{A_{\tau_\ell}(\tau)}{\tau} + \frac{B_{\tau_\ell,1}(\tau)}{\tau} r_t + \frac{C_{\tau_\ell}(\tau)}{\tau} r_{\ell,t}$$

(22.34) $$\sigma_{\ell,1} = \sigma_1 \frac{1 - e^{-\gamma_1^* \tau_\ell}}{\tau_\ell}; \quad \sigma_{\ell,2} = \sigma_2 \frac{1 - e^{-\gamma_2^* \tau_\ell}}{\tau_\ell}$$

(22.38) $$\text{Vasicek volatility of } dr_t(\tau) = \sigma_t(\tau) = \frac{\sigma}{\gamma^*}\frac{1 - e^{-\gamma^* \tau}}{\tau}$$

(22.39) $$\text{Volatility of } dr_t(\tau) = \sigma_t(\tau) = \sqrt{\sigma_1^2 \left(\frac{B_1(\tau)}{\tau}\right)^2 + \sigma_2^2 \left(\frac{B_2(\tau)}{\tau}\right)^2}$$

(22.41) $$V(\phi_{1,0}, \phi_{2,0}, 0) = Z(\phi_{1,0}, \phi_{2,0}, 0; T_B)\mathcal{N}(d_1) - KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O)\mathcal{N}(d_2)$$

(22.42) $$d_1 = \frac{1}{S_Z(T_O)} \log\left(\frac{Z(\phi_{1,0}, \phi_{2,0}, 0; T_B)}{KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O)}\right) + \frac{S_Z(T_O)}{2}$$

(22.43) $$d_2 = d_1 - S_Z(T_O)$$

(22.44) $$V(\phi_{1,0}, \phi_{2,0}, 0) = -Z(\phi_{1,0}, \phi_{2,0}, 0; T_B)\mathcal{N}(-d_1) + KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O)\mathcal{N}(-d_2)$$

(22.46) $$d\phi_{1,t} = m_1(\phi_{1,t}, \phi_{2,t}, t)dt + s_1(\phi_{1,t}, \phi_{2,t}, t)dX_{1,t}$$

(22.47) $$d\phi_{2,t} = m_2(\phi_{1,t}, \phi_{2,t}, t)dt + s_2(\phi_{1,t}, \phi_{2,t}, t)dX_{2,t}$$

(22.48) $$dP_t = \left\{ \left(\frac{\partial F}{\partial t}\right) + \left(\frac{\partial F}{\partial \phi_1}\right) m_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) m_{2,t} + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_1^2}\right) s_{1,t}^2 + \frac{1}{2}\left(\frac{\partial^2 F}{\partial \phi_2^2}\right) s_{2,t}^2 + \left(\frac{\partial^2 F}{\partial \phi_1 \phi_2}\right) s_{1,t} s_{2,t} \rho \right\} dt + \left(\frac{\partial F}{\partial \phi_1}\right) s_{1,t} dX_{1,t} + \left(\frac{\partial F}{\partial \phi_2}\right) s_{2,t} dX_{2,t}$$

$$R(\phi_1,\phi_2)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \phi_1}m^*_{1,t} + \frac{\partial V}{\partial \phi_2}m^*_{2,t} + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_1^2}s^2_{1,t} + \frac{1}{2}\frac{\partial^2 V}{\partial \phi_2^2}s^2_{2,t} + \frac{\partial^2 V}{\partial \phi_1 \phi_2}s_{1,t}s_{2,t}\rho \tag{22.49}$$

$$V(\phi_{1,t},\phi_{2,t},t) = E^*\left[e^{-\int_t^T R(\phi_{1,u},\phi_{2,u})du} g_T | \phi_{1,t},\phi_{2,t}\right] \tag{22.61}$$

$$d\phi_{1,t} = m^*_{1,t}dt + s_{1,t}dX_{1,t} \tag{22.62}$$

$$d\phi_{2,t} = m^*_{2,t}dt + s_{2,t}dX_{2,t} \tag{22.63}$$

The Volatility Smile, Derman and Miller

Chapter 3

$$C(S,t) - P(S,t) = S - Ke^{-r(T-t)} \tag{3.3}$$

$$C(S+dS,t+dt) = C(S,t) + \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}dS^2 + \cdots \tag{3.9}$$

$$C(S+dS,t+dt) = C(S,t) + \Theta dt + \Delta dS + \frac{1}{2}\Gamma dS^2 \tag{3.11}$$

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2} = rC \tag{3.16}$$

$$\frac{\partial C}{\partial t} + \frac{1}{2}\Gamma\Sigma^2 S^2 = 0 \tag{3.17}$$

Chapter 4

$$C(S,K,\tau,\sigma,r) = SN(d_1) - Ke^{-r\tau}N(d_2), \quad d_{1,2} = \frac{\ln\left(\frac{S}{K}\right) + \left(r \pm \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{t}}, \qquad N(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{1}{2}y^2}dy \tag{4.1}$$

$$\kappa_\pi = \int_0^\infty \rho(xS)S^2 f(x,\nu,\tau)dx \tag{4.12}$$

$$\frac{1}{T}\int_0^T \sigma_t^2 dt = \frac{2}{T}\left[\int_0^T \frac{1}{S}dS - \ln\left(\frac{S_T}{S_0}\right)\right] \tag{4.32}$$

$$\pi(S_T,S_0,T,T) = \frac{2}{T}\left[\left(\frac{S_T - S_0}{S_0}\right) - \ln\left(\frac{S_T}{S_0}\right)\right] \tag{4.41}$$

$$\sigma(K) = \sigma_F - b\frac{K - S_F}{S_F} \tag{4.43}$$

$$\sigma_K^2 = \sigma_F^2(1 + 3Tb^2) \tag{4.44}$$

Chapter 5

(5.1) $dS = \mu_S S dt + \sigma_S S dZ, \quad dB = Brdt$

(5.2) $$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2 dt$$
$$= \left\{\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu_S S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2\right\}dt + \frac{\partial C}{\partial S}\sigma_S S dZ$$
$$= \mu_C C dt + \sigma_C C dZ$$

(5.3) $$\mu_C = \frac{1}{C}\left\{\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu_S S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(\sigma_S S)^2\right\}, \quad \sigma_C = \frac{S}{C}\frac{\partial C}{\partial S}\sigma_S = \frac{\partial \ln C}{\partial \ln S}\sigma_S$$

(5.10) $$\frac{(\mu_C - r)}{\sigma_C} = \frac{(\mu_S - r)}{\sigma_S}$$

(5.12) $$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

(5.13) $$C(S, K, t, T, \sigma, r) = e^{-r(T-t)}[S_F N(d_1) - KN(d_2)], \quad d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}},$$
$$d_2 = \frac{\ln\left(\frac{S_F}{K}\right) - \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

(5.19) $$C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x)[dS_x - S_x r dx]e^{-rx}$$

(5.20) $$C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x)\sigma S_x e^{-rx} dZ_x$$

(5.21) $E[C_0] = E[C_T]e^{-rT}$

(5.22) $\pi(I, R) = V_I - \Delta_R S$

(5.23) $\mathrm{PV}[\mathrm{P\&L}(I, R)] = V(S, \tau, \sigma_R) - V(S, \tau, \Sigma)$

(5.25) $$\Delta_R = e^{-D\tau}N(d_1), \quad d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \frac{1}{2}\sigma_R^2\tau}{\sigma_R\sqrt{\tau}}, \quad S_F = Se^{(r-D)\tau}$$

(5.27) $d\mathrm{P\&L}(I, R) = dV_I - rV_I dt - \Delta_R[dS - (r - D)Sdt]$

(5.28) $d\mathrm{P\&L}(R, R) = 0 = dV_R - V_R r dt - \Delta_R[dS - (r - D)Sdt]$

(5.34) $\mathrm{PV}[\mathrm{P\&L}(I, R)] = e^{rt_0}\left[e^{-rT}\cdot 0 - e^{-rt_0}(V_{I,t} - V_{R,t})\right] = V_{R,t} - V_{I,t}$

(5.38) $$d\mathrm{P\&L}(I, R) = \frac{1}{2}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt + (\Delta_I - \Delta_R)[(\mu - r + D)Sdt + \sigma_R S dZ]$$

(page 100) The upper bound of the P&L is ... ($\mathrm{V}_{R,0} - V_{I,0}$)

(5.41) $$\mathrm{PV}[\pi(I, R)]_L = (V_{R,0} - V_{I,0}) - 2Ke^{-2r\tau}\left[N\left(\frac{1}{2}(\sigma_R - \Sigma)\sqrt{\tau}\right) - \frac{1}{2}\right]$$

(This is a correction to the text formula)

(5.42) $$d\mathrm{P\&L}(I, I) = \frac{1}{2}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt$$

(5.43) $$\mathrm{PV}[\mathrm{P\&L}(I, I)] = \frac{1}{2}\int_{t_0}^T e^{-r(t-t_0)}\Gamma_I S^2(\sigma_R^2 - \Sigma^2)dt$$

(page 103, problem 5-4) $$\mathrm{PV}[\mathrm{P\&L}(I, H)] = V_h - V_I + \frac{1}{2}\int_{t_0}^T e^{-r(t-t_0)}\Gamma_h S^2(\sigma_R^2 - \sigma_h^2)dt$$

Chapter 6

(6.2) $$\pi = C - \frac{\partial C}{\partial S} S$$

(6.6) $$HE \approx \sum_{i=1}^{n} \frac{1}{2} \Gamma_i \sigma_i^2 S_i^2 (Z_i^2 - 1) dt$$

(6.7) $$\sigma_{HE}^2 \approx E \left[\sum_{i=1}^{n} \frac{1}{2} (\Gamma_i S_i^2)^2 (\sigma_i^2 dt)^2 \right]$$

(6.12) $$\sigma_{HE} \approx \sqrt{\frac{\pi}{4}} \frac{\sigma}{\sqrt{n}} \frac{\partial C}{\partial \sigma}$$

(6.14) $$\sigma_{HE} \approx dC \approx \frac{\partial C}{\partial \sigma} d\sigma \approx \frac{\sigma}{\sqrt{n}} \frac{\partial C}{\partial \sigma}$$

(6.18) $$\frac{\sigma_{HE}}{C} \approx \sqrt{\frac{\pi}{4n}} \approx \frac{0.89}{\sqrt{n}}$$

Chapter 7

(7.14) $$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \frac{\partial C}{\partial t} - \sqrt{\frac{2}{\pi dt}} \left| \frac{\partial^2 C}{\partial S^2} \right| \sigma S^2 k = r \left(C - S \frac{\partial C}{\partial S} \right)$$

(7.18) $$\breve{\sigma}^2 = \sigma^2 + 2\sigma k \sqrt{\frac{2}{\pi dt}}$$

(7.19) $$\breve{\sigma} \approx \sigma \pm k \sqrt{\frac{2}{\pi dt}}$$

Chapter 8

(8.3) $$P[\ln(S_T) > \ln(K)] = P \left[Z > \frac{-\ln\left(\frac{S_t}{K}\right) - \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \right] = N(d_2)$$

(8.6) $$\Delta_{\text{ATM}} \approx \frac{1}{2} + \frac{d_1}{\sqrt{2\pi}} \approx \frac{1}{2} + \frac{\sigma\sqrt{\tau}}{2\sqrt{2\pi}}$$

(8.9) $$\Delta \approx \Delta_{\text{ATM}} - \frac{1}{\sqrt{2\pi}} \frac{J}{\nu}$$

Chapter 10

(10.3) $$S = V - B, \ \frac{dS}{S} = \frac{dV}{S} = \frac{V\sigma dZ}{S} = \sigma \frac{S+B}{S} dZ, \ \sigma_S = \sigma \left(1 + \frac{B}{S}\right)$$

(10.4) $$\frac{dS}{S} = \mu(S,t)dt + \sigma S^{\beta-1} dZ$$

(10.5) $$dS = \mu S dt + \sigma S dZ, \ d\sigma = p\sigma dt + q\sigma dW, \ E[dW dZ] = \rho dt$$

(10.10) $$\text{Profit} = \frac{1}{2}\Gamma S^2(\sigma^2 - \Sigma^2)dt = \frac{1}{2}\Gamma(dS)^2 - \frac{1}{2}\Gamma S^2 \Sigma^2 dt$$

(10.15) $$D = -\frac{\partial C_{\text{BSM}}}{\partial K} - \frac{\partial C_{\text{BSM}}}{\partial \Sigma} \frac{\partial \Sigma}{\partial K}$$

Chapter 14

(14.4) $F = Se^{(r-b)dt}$

(14.5) $F = qS_u + (1-q)S_d$

(14.6) $q = \dfrac{F - S_d}{S_u - S_d}$

(14.11) $S_u = Se^{\sigma(S,t)\sqrt{dt}},\ S_d = Se^{-\sigma(S,t)\sqrt{dt}}$

(14.17) $\Sigma(S,K) \approx \sigma_0 + \dfrac{\beta}{2}(S+K)$

(14.18) $\Sigma(S,K) \approx \sigma(S) + \dfrac{\beta}{2}(K-S)$

Chapter 17

(17.6) $d\pi_{\text{BSM}} = d\pi_{\text{loc}} - \varepsilon dS = \dfrac{1}{2}\Gamma_{\text{loc}} S^2 \left[\sigma_R^2 - \sigma_{\text{loc}}^2(S,t)\right] dt - \varepsilon dS$

Chapter 18

(18.6) $\Sigma_{\text{ATM}}(S) = \Sigma(S,S) = \Sigma_0 - \beta(S - S_0)$

(18.7) $\Sigma(S,K) = \Sigma_0 - \beta(K - S)$

(18.11) $\Sigma(S,K,t,T) = \Sigma_0(t,T) - \beta'(t,T)[0.5 - \Delta(S,K,t,T,\Sigma_{\text{ATM}}(S))]$

(18.12) $\Sigma(S,K,\tau) = \Sigma_0 - \beta'(0.5 - \Delta(S,K,\tau,\Sigma_{\text{ATM}}))$

Chapter 19

(19.8) $\dfrac{dV}{V} = \alpha dt + \xi dW$ where $V = \sigma^2$

(19.9) $dY = \alpha(m - Y)dt + \beta dW$

(19.15) $Y_t = m + (Y_0 - m)e^{-\alpha t} + \beta \displaystyle\int_0^t e^{-\alpha(t-s)} dW_s$

(19.21) $\text{Var}[Y_t] = \dfrac{\beta^2}{2\alpha}\left(1 - e^{-2\alpha t}\right)$

(19.24) $d\sigma = \alpha(m - \sigma)dt + \beta dW$

(19.25) $dV = \alpha(m - V)dt + \beta dW$

(19.26) $dV = \alpha(m - V)dt + \beta V dW$

(19.27) $dV = \alpha(m - V)dt + \beta\sqrt{V} dW$

Pricing and Hedging Financial Derivatives, Marroni and Perdomo

Chapter 6

(page 124) $\text{Delta(call)} = N(d_1)$

(page 124) $\text{Delta(put)} = N(d_1) - 1$

(page 127) $\text{Gamma} = \dfrac{N'(d_1)}{S\sigma\sqrt{T-t}}$

(page 128) $d_1 = \dfrac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$

(page 128) $N'(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$

(page 133) $\text{Vega} = S\sqrt{T-t}\, N'(d_1)$

(page 137) $\text{Theta(call)} = -\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$

(page 137) $d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$

(page 137) $d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$

(page 137) $\text{Theta(put)} = -\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}N(-d_2)$

(page 138) $\text{Theta } \Delta t + \frac{1}{2}\text{Gamma } \Delta S^2 \cong 0$

(page 143) $\text{Rho(call) } = K(T-t)e^{-r(T-t)}N(d_2)$

(page 143) $\text{Rho(put) } = K(T-t)e^{-r(T-t)}N(-d_2)$

Chapter 7

(page 180) $S(k) = S(k-1) * [\text{yield}(t) * dt + \text{vol}(t, S) * W * \text{sqrt}(dt)]$

(page 181) $\text{vol}^2 * (t-s) = \text{vol\_mkt}^2(t) * t - \text{vol\_mkt}^2(s) * s$

Analysis of Financial Time Series, third edition, R. Tsay
Chapter 3

(3.2) $\mu_t = E(r_t|F_{t-1}), \quad \sigma_t^2 = Var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}]$

(3.3) $r_t = \mu_t + a_t, \quad \mu_t = \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{i=1}^{q} \theta_i a_{t-i}, \quad y_t = r_t - \phi_0 - \sum_{i=1}^{k} \beta_i x_{it}$

(3.4) $\sigma_t^2 = Var(r_t|F_{t-1}) = Var(a_t|F_{t-1})$

(p114) $a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2 + e_t, \quad t = m+1, \ldots, T$

(p114) $F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T - 2m - 1)}$

(3.5) $a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2$

(p117) $a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$

(p118) $\text{Var}(a_t) = E(a_t^2) = E[E(a_t^2|F_{t-1})] = E(\alpha_0 + \alpha_1 a_{t-1}^2) = \alpha_0 + \alpha_1 E(a_{t-1}^2)$

(p118) $m_4 = E(a_t^4) = \frac{3\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)}$

(p119) $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2$

(p120) $a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2 + \eta_t$

(p120) $$f(a_1, \ldots, a_T|\alpha) = f(a_T|F_{T-1})f(a_{T-1}|F_{T-2}) \cdots f(a_{m+1}|F_m)f(a_1, \ldots, a_m|\alpha)$$
$$= \prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{a_t^2}{2\sigma_t^2}\right) \times f(a_1, \ldots, a_m|\alpha)$$

(p121) $\ell(a_{m+1}, \ldots, a_T|\alpha, \alpha_1, \ldots, \alpha_m) = -\sum_{t=m+1}^{T} \left[\frac{1}{2}\ln(\sigma_t^2) + \frac{1}{2}\frac{a_t^2}{\sigma_t^2}\right]$

(3.7) $$f(\epsilon_t|v) = \frac{\Gamma[(v+1)/2]}{\Gamma(v/2)\sqrt{(v-2)\pi}}\left(1+\frac{\epsilon_t^2}{v-2}\right)^{-(v+1)/2}, \ v>2$$

(p121) $$\Gamma(x) = \int_0^\infty y^{x+1}e^{-y}dy$$

(p121) $$a_t = \sigma_t\epsilon_t$$

(p121) $$f(a_{m+1},\ldots a_T|\alpha, A_m) = \prod_{t=m+1}^{T}\frac{\Gamma[(v+1)/2]}{\Gamma(v-2)\sqrt{(v-2)\pi}}\frac{1}{\sigma_t}\left[1+\frac{a_t^2}{(v-2)\sigma_t^2}\right]^{-(v+1)/2}$$

(3.8) $$\ell(a_{m+1},\ldots,a_T|\alpha, A_m) = -\sum_{t=m+1}^{T}\left[\frac{v+1}{2}\ln\left(1+\frac{a_t^2}{(v-2)\sigma_t^2}\right)+\frac{1}{2}\ln(\sigma_t^2)\right]$$

(p121) $$\ell(a_{m+1},\ldots,a_T|\alpha,\upsilon,A_m) = (T-m)\left\{\ln\left[\Gamma\left(\frac{\upsilon+1}{2}\right)\right]-\ln\left[\Gamma\left(\frac{\upsilon}{2}\right)\right]\right.$$
$$\left.-0.5\ln[(\upsilon-2)\pi]\right\}+\ell(a_{m+1},\ldots,a_T|\alpha,A_m)$$

(3.9) $$g(\epsilon_t|\xi,v) = \begin{cases} \frac{2}{\xi+\frac{1}{\xi}}\varrho f[\xi(\varrho\epsilon_t+\bar{\omega})|v] & \text{if } \epsilon_t < -\bar{\omega}/\varrho \\ \frac{2}{\xi+\frac{1}{\xi}}\varrho f[(\varrho\epsilon_t+\bar{\omega})/\xi|v] & \text{if } \epsilon_t \geq -\bar{\omega}/\varrho \end{cases}$$

(p122) $$\varpi = \frac{\Gamma[(\upsilon-1)/2]\sqrt{\upsilon-2}}{\sqrt{\pi}\Gamma(\upsilon/2)}\left(\xi-\frac{1}{\xi}\right), \quad \varrho^2 = \left(\xi^2+\frac{1}{\xi^2}-1\right)-\varpi^2$$

(3.10) $$f(x) = \frac{v\exp\left(-\frac{1}{2}|x/\lambda|^v\right)}{\lambda 2^{(1+1/v)}\Gamma(1/v)}, \ -\infty < x < \infty, \ 0 < v \leq \infty$$

(3.11) $$\sigma_h^2(\ell) = \alpha_0 + \sum_{i=1}^{m}\alpha_i\sigma_h^2(\ell-i). \ \text{ where } \ \sigma_h^2(\ell-i) = a_{h+\ell-i}^2 \text{ if } \ell-i\leq 0$$

(3.14) $$GARCH(m,s): a_t = \sigma_t\epsilon_t, \ \sigma_t^2 = \alpha_0+\sum_{i=1}^{m}\alpha_i a_{t-i}^2+\sum_{j=1}^{s}\beta_j\sigma_{t-j}^2$$

(3.15) $$GARCH(m,s): a_t^2 = \alpha_0 + \sum_{i=1}^{\max(m,s)}(\alpha_i+\beta_i)a_{t-i}^2+\eta_t-\sum_{j=1}^{s}\beta_j\eta_{t-j}$$

(p132) $$\frac{E(a_t^4)}{[E(a_t^2)]^2} = \frac{3[1-(\alpha_1+\beta_1)^2]}{1-(\alpha_1+\beta_1)^2-2\alpha_1^2} > 3$$

(3.17) $$\sigma_h^2(\ell) = \alpha_0+(\alpha_1+\beta_1)\sigma_h^2(\ell-1), \quad \ell > 1$$

(p133) $$\sigma_h^2(\ell) = \frac{\alpha_0[1-(\alpha_1+\beta_1)^{\ell-1}]}{1-\alpha_1-\beta_1}+(\alpha_1+\beta_1)^{\ell-1}\sigma_h^2(1)$$

(p141) $$IGARCH(1,1): a_t = \sigma_t\epsilon_t, \quad \sigma_t^2 = \alpha_0+\beta_1\sigma_{t-1}^2+(1-\beta_1)a_{t-1}^2$$

(3.22) $$\sigma_h^2(\ell) = \sigma_h^2(1)+(\ell-1)\alpha_0, \quad \ell\geq 1$$

(p141) $$\sigma_t^2 = (1-\beta_1)(a_{t-1}^2+\beta_1 a_{t-2}^2+\beta_1^2 a_{t-3}^3+\cdots)$$

(3.23) $$GARCH(1,1)-M: \ r_t = \mu + c\sigma_t^2 + a_t, \ a_t = \sigma_t\epsilon_t, \ \sigma_t^2 = \alpha_0+\alpha_1 a_{t-1}^2+\beta_1\sigma_{t-1}^2$$

(3.24) $$g(\epsilon_t) = \theta\epsilon_t+\gamma[|\epsilon_t|-E(|\epsilon_t|)]$$

(p143) $$g(\epsilon_t) = \begin{cases} (\theta+\gamma)\epsilon_t-\gamma E(|\epsilon_t|) & \text{if } \epsilon_t\geq 0 \\ (\theta-\gamma)\epsilon_t-\gamma E(|\epsilon_t|) & \text{if } \epsilon_t < 0 \end{cases}$$

(p143) $$E(|\epsilon_t|) = \frac{2\sqrt{\upsilon-2}\Gamma[(\upsilon+1)/2]}{(\upsilon-1)\Gamma(\upsilon/2)\sqrt{\pi}}$$

(3.25) $$EGARCH(m,s): a_t = \sigma_t \epsilon_t, \quad \ln(\sigma_t^2) = \alpha_0 + \frac{1+\beta_1 B + \cdots + \beta_{s-1}B^{s-1}}{1-\alpha_1 B - \cdots - \alpha_m B^m} g(\epsilon_{t-1})$$

(3.26) $$a_t = \sigma_t \epsilon_t, \quad (1-\alpha B)\ln(\sigma_t^2) = (1-\alpha)\alpha_0 + g(\epsilon_{t-1})$$

(3.27) $$(1-\alpha B)\ln(\sigma_t^2) = \begin{cases} \alpha_* + (\gamma+\theta)\epsilon_{t-1} & \text{if } \epsilon_{t-1} \geq 0 \\ \alpha_* + (\gamma-\theta)(-\epsilon_{t-1}) & \text{if } \epsilon_{t-1} < 0 \end{cases}$$

(p144) $$\sigma_t^2 = \sigma_{t-1}^{2\alpha} \exp(\alpha_*) \begin{cases} \exp\left[(\gamma+\theta)\frac{a_{t-1}}{\sigma_{t-1}}\right] & \text{if } a_{t-1} \geq 0 \\ \exp\left[(\gamma-\theta)\frac{|a_{t-1}|}{\sigma_{t-1}}\right] & \text{if } a_{t-1} < 0 \end{cases}$$

(3.28) $$\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{s} \alpha_i \frac{|a_{t-i}| + \gamma_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j \ln(\sigma_{t-j}^2)$$

(3.33) $$\sigma_t^2 = \sigma_{t-1}^{2\alpha_1} \exp[(1-\alpha_1)\alpha_0] \exp[g(\epsilon_{t-1})], \quad g(\epsilon_{t-1}) = \theta\epsilon_{t-1} + \gamma(|\epsilon_{t-1}| - \sqrt{2/\pi})$$

(p148) $$\sigma_{h+1}^2 = \sigma_h^{2\alpha_1} \exp[(1-\alpha_1)\alpha_0] \exp[g(\epsilon_h)]$$

(p148) $$\sigma_{h+2}^2 = \sigma_{h+1}^{2\alpha_1} \exp[(1-\alpha_1)\alpha_0] \exp[g(\epsilon_{h+1})]$$

(p148) $$\hat{\sigma}_h^2 = \hat{\sigma}_h^{2\alpha_1}(1) \exp[(1-\alpha_1)\alpha_0] E_h\{\exp[g(\epsilon_{h+1})]\}$$

(p148) $$E\{\exp[g(\epsilon)]\} = \exp\left(-\gamma\sqrt{2/\pi}\right)\left[e^{(\theta+\gamma)^2/2}\Phi(\theta+\gamma) + e^{(\theta-\gamma)^2/2}\Phi(\gamma-\theta)\right]$$

(p148) $$\hat{\sigma}_h^2(j) = \widehat{\sigma_h^{2\alpha_1}}(j-1)\exp(\omega)\left\{\exp[(\theta+\gamma)^2/2]\Phi(\theta+\gamma) + \exp[(\theta-\gamma)^2/2]\Phi(\gamma-\theta)\right\}$$

Risk Management and Financial Institutions, second edition, Hull (QFIQ-109-15)
Chapter 9 Volatility

(page 179) $$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} u_i\right)^2}$$

(9.1) $$\text{Prob}(\upsilon > x) = Kx^{-\alpha}$$

(9.4) $$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2$$

(9.5) $$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i u_{n-i}^2$$

(page 186) $$\sum_{i=1}^{m} \alpha_i = 1$$

(9.6) $$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i u_{n-i}^2$$

(9.7) $$\sigma_n^2 = \omega + \sum_{i=1}^{m} \alpha_i u_{n-i}^2$$

(9.8) $$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

(9.9) GARCH(1,1): $\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$

(9.10) $\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$

(9.12) $\sum_{i=1}^{m}\left[-\ln v - \frac{u_i^2}{v}\right]$

(9.13) $\sum_{i=1}^{m}\left[-\ln(v_i) - \frac{u_i^2}{v_i}\right]$

(9.14) $E[\sigma_{n+t}^2] = V_L + (\alpha+\beta)^t(\sigma_n^2 - V_L)$

(9.15) $\sigma(T)^2 = 252\left\{V_L + \frac{1-e^{-aT}}{aT}[V(0) - V_L]\right\}$

(9.16) $\frac{\Delta\sigma(T)}{\Delta\sigma(0)} \approx \frac{1-e^{-aT}}{aT}\frac{\sigma(0)}{\sigma(T)}$

Market Models: A Guide for Financial Data Analysis, Alexander (QFIQ-119-19) Chapter 6 - Principal Component Analysis

(6.3) $\sigma_K - \sigma_{\text{ATM}} = -b(K - S)$

Economic Scenario Generators: A Practical Guide, SOA

(page 100) $\text{Price}(\{C_u\}) = \text{E}^*\left[\int_0^T \frac{1}{B_u} dC_u\right]$

(page 100) $\text{Price}(\{C_u\}) = \text{E}\left[\int_0^T \rho_u dC_u\right]$

(page 107) $X_0 = e^{-rT}\frac{1}{M}\sum_{i=1}^{M} X_T(i)$