ILA LAM Model Solutions
Spring 2024

1. Learning Objectives:

1. The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

Learning Outcomes:

(1a) With respect to stochastic models:
- Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
- Describe and apply the theory and uses of real world versus risk neutral assumptions.
- Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
- Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
- Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
- Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.

Sources:

Stochastic Modeling is on the Rise, Product Matters, Nov 2016

LAM-135-19: Stochastic Modeling, Theory and Reality from and Actuarial Perspective, sections I.A, I.B-I.B.3.a, I.B.4 & I.D-I.D.3

LAM-138-19: A Practitioner's Guide to Generalized Linear Models, 1.1-1.108, 1.118-1.130 & 3.1-3.14

Beware of Stochastic Model Risk!, Stroman, Risk & Rewards, SoA, Aug 2019

Commentary on Question:

The question was generally answered well by the candidates. Both correct statement and supporting points are required to receive full credits.

1. **Continued**

Solution:
(a) Critique the following statements regarding stochastic modeling:

  A. *Stochastic models should only be used when it is explicitly required by a regulatory standard.*

  B. *Real-world scenarios cannot be connected with risk-neutral scenarios due to different usage of expected cash flows and a discount rate.*

  C. *For nested stochastic modeling, real-world scenarios and risk-neutral scenarios should be used for inner-loop and outer-loop, respectively.*

  D. *When using a random number generator, a true random number generator is more efficient than a pseudo random number generator.*

Commentary on Question:
Full credits are given for candidates providing reasonable support for their answers.

A. False
   Stochastic models can also be used to perform tail risk analysis, calculation of diversification effects, risk adjusted strategies.

B. False
   Risk-adjusted cash flows are adjusted in such a way as to ensure that the expected PV of CF's between two are consistent, Two scenarios can be connected via the use of deflators.

C. False
   The fund returns and interest rates driving the fixed and variable account balances of the policy should be generated with real-world outer scenario. The fair-market value of liabilities under market-consistent risk-neutral inner paths must be determined at selected periods.

D. False
   It is more desirable to use Pseudo Random Number Generator (PRNG) than True Randon Number Generator (TRNG) when one is doing stochastic simulations, PRNGs are much more efficient, meaning that the amount of time required for them to generate large sets of numbers is small compared to TRNGs.

1. Continued

(b) A Generalized Linear Model (GLM) is due for an update. The following data is provided regarding claim severity:

	Smoker	Non-smoker
Male	900	500
Female	600	400

The form of the GLM is:

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

Where:

$\beta_1 = $ *Parameter for Male*
$\beta_2 = $ *Parameter for Female*
$\beta_3 = $ *Parameter for Smoking Status*

(i) Solve for parameters β_1, β_2 and β_3 assuming the error term is normally distributed with mean zero and variance σ^2.

(ii) Evaluate the limitations of the use of a normal error structure.

Commentary on Question:
(i)This part is generally not answered well. Full credits were granted to candidate where the formula was shown and calculated correctly. Few candidates showed final answers without description or formula and received partial credits. Partial credits were granted to candidates who described the methodology correctly but did not calculate correctly.
(ii)Most candidates could answer some limitation of using normal error structure to receive partial credits.

(i)
Y: Parameter Value
X_1: Indicator for Male
X_2: Indicator for Female
X_3: Indicator for Smoking Status where 1 for smoker and 0 for non-smoker

$Y_1 = 900 = \beta_1 + 0 + \beta_3 + \varepsilon_1$
$Y_2 = 700 = 0 + \beta_2 + \beta_3 + \varepsilon_2$
$Y_3 = 500 = \beta_1 + 0 + 0 + \varepsilon_3$
$Y_4 = 400 = 0 + \beta_2 + 0 + \varepsilon_4$

1. Continued

Sum of squared errors

$$= \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2$$
$$= (900 - \beta_1 - \beta_3)^2 + (600 - \beta_2 - \beta_3)^2 + (500 - \beta_1)^2 + (400 - \beta_2)^2$$

Differentiate β_1, β_2, and β_3 and solve when setting to zero

Differentiate β_1
$$\beta_1 + \beta_3 + \beta_1 = 1400$$
$$2\beta_1 + \beta_3 = 1400$$

Differentiate β_2
$$\beta_2 + \beta_3 + \beta_2 = 1000$$
$$2\beta_2 + \beta_3 = 1000$$

Differentiate β_3
$$\beta_1 + \beta_3 + \beta_2 + \beta_3 = 1500$$
$$\beta_1 + \beta_2 + 2\beta_3 = 1500$$

Then
$$\beta_1 - \beta_2 = 200$$
$$\beta_2 = \beta_1 - 200$$
$$\beta_3 = 1400 - 2\beta_1$$

$$\beta_1 + \beta_1 - 200 + 2(1400 - 2\beta_1) = 1500$$
$$-2\beta_1 = 1500 - 2600 = -1100$$

Thus,
$$\beta_1 = 550$$
$$\beta_2 = 350$$
$$\beta_3 = 300$$

(ii)
The limitations of the use of a normal error structure:
- It is difficult to assert Normality and constant variance for response variables.
- The values for the response variable may be restricted to be positive. The assumption of Normality violates this restriction.
- The variable is a function of the mean. If the response variable is strictly non-negative then intuitively the variable of Y tends to be zero as the mean of Y tends to be zero.

1. Continued

(c) Your company uses a normal distribution with constant volatility to project equity returns.

    (i) Describe a drawback of using the normal distribution with constant volatility to project returns.

    (ii) Recommend two possible alternatives to using constant volatility. Justify your answer.

Commentary on Question:
(i) The candidate should describe that economic variables have fatter tails and a stronger central peak to receive full credits. Partial credits were granted to candidates to answer fatter tails or stronger central peak only.
(ii) This part is generally done well by the candidates. Most candidates provided two (different) alternative methods with descriptions to receive full credits.

(i)
The actual variability of most economic variables is better characterized by a distribution with both fatter tails and a stronger central peak than the Normal bell-shaped curve.

(ii)
Stochastic Volatility
The Normal distribution is still used, but the volatility parameter is made to follow its own mean-reverting stochastic process over time. When the volatility is lower than average in the scenario, values clump toward the center of the distribution. When the volatility is higher than average, relatively more tail values are generated. Overall, the ultimate distribution has longer tails and a stronger central peak.

Regime Switching
The Normal distribution is still used but the model switches between two regimes, which are characterized by different sets of parameter values for both the volatility and the mean. There is a high-volatility regime and a low-volatility regime, typically with different mean values. Switching between regimes results in an ultimate blended distribution that can have longer tails and a stronger central peak.

Different Underlying Distribution
The Normal distribution is abandoned as a model of variability within each time step. A different distribution that has longer tails is used instead. There are many choices for such a distribution.

2. **Learning Objectives:**
2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

Learning Outcomes:
(2a) Describe, explain, and apply the following Model Efficiency concepts:
- Representative scenarios / Scenario reduction
- Replicating liabilities
- "Cluster Analysis Spatial Approach"

(2b) Describe and evaluate the following actuarial modeling best practices:
- Model risk management
- Model validation techniques and methods
- Best practices for assumptions governance
- Application of Actuarial Standards of Practices
- Reliance on expert judgment in actuarial modelling

(2c) Describe, evaluate, and compare implications on modeling organizations, processes, and best practices because of:
- Use of open code and closed code models
- Centralized vs. De-centralized actuarial modeling function

Sources:
Actuarial Modeling Systems: How Open We WANT Them to be vs. How Closed We NEED Them to be, The Modeling Platform, Nov 2017;

Reviewing, Validating and Auditing Actuarial Models, Rabin, Cantor, and Marco, Aug 2015;

Model Efficiency Study Results, Nov 2011

Commentary on Question:
This question assesses candidates' ability to assess issues and concerns common to actuarial models and their development and management.
In part a and part b, most candidates were able to provide their stances and opinions on the questions asked with some level of justifications. Candidates who provided more thorough justifications were awarded full credits.
In part c, the second statement can be interpreted differently by different candidates. Again, candidates who provided more thorough justifications to their answers received full credits, as compared to candidates who only briefly stated whether they think the statement is correct or incorrect.
In part d, most candidates did well as they are able to list out the steps to conduct the Transfer Scenario Order technique and calculate runtime reduction % correctly.

2. **Continued**

Solution:

(a) Critique the following statements about the modeling system at company AH:

   A. *The robust documentation the vendor provides should provide sufficient details for the company to gain understanding of the actuarial calculation methodology programed in the system.*

   B. *Since the calculations are controlled and locked in, risk of illogical calculation is minimal, and the company can rely heavily on the integrity of the system.*

   C. *Since vendor coders are professional programmers, they have deep expertise in code optimization that results in faster model runs. This gives the company very good control over model efficiency and runtime.*

   D. *Key-person risk is increased with the platform since only a small group of modelers have a detailed understanding of the model and the history of code development. Parameters can sometimes be cryptic, and workarounds incorporated to accommodate rigid aspects of the system.*

   Commentary on Question:
   Part a. of the question tests candidates' knowledge on key aspects of Open vs Closed code modeling platforms. Each statement talks about more than one aspect of the locked-down modeling system provided in the case, and successful candidates were able to identify both aspects and provide reasonable justification for their stances on both aspects. Partial credits were awarded to candidates who provided justifications to only one aspect of their critique. Little credit was awarded to candidates who only expressed agreement/disagreement without much justification.

   A.
   It is true that vendor-provided documentation usually provides insights into calculations of actuarial components and technical details for the system's functionalities, which is robust and comprehensive.
   However, such details may not always be sufficient for users, as it is nearly impossible to document every interaction between variables and calculations for the system. In cases where items that are extremely obscure or rare, it may require direct communication with the vendor for supporting documentation.

2. **Continued**

B.
It is true that risk of illogical calculations for closed systems is low as logics have been tested across industry users, and closed systems usually have built-in functions which will warn users when illogical operations are performed.
On the other hand, given the lack of transparency behind calculations in the closed system, risk of human error may increase if over-reliance is placed on the system.

C.
It is true that closed system vendors will hire professional coders who have deep expertise in code and run-time optimizations. This is an advantage of closed systems. The first statement is accurate.
However, a drawback is that customers will not have much control over the model efficiency outside of what is available in the user interface. The second statement is not accurate.

D.
Closed models would be less prone to key-person risk and are easier to understand due to their standardized features, user interfaces, and modeling approaches across the industry, which makes them easier to maintain and modify by others who have prior knowledge using the modeling platform. The first statement is not accurate.
However, due to the rigid nature of closed models, workarounds may likely be implemented to accommodate companies' specific needs. In which case, parameters can sometimes be cryptic, and this poses additional institutional knowledge risk, and which increases key person risk for closed models.

(b)   Identify three areas where additional investigation may be required based on the liability static validation result summary above. Justify your answer.

Commentary on Question:
Part b. of the question addresses the application of static validation techniques on a given case.
Most candidates were able to identify three areas for investigation and speak to their low validation %'s. Successful candidates were able to provide additional context on the implication of potential issues and make hypotheses on what might be the causes.

2. **Continued**

One area of investigation is the policy values for the Fixed Indexed Annuity (FIA). Policy values such as count, and premiums are usually expected to be close to 100% validation. The fact validation %'s are in the low 90% range may signal some policy data or plans are missing. Missing data will have downstream impact to reported values.

Another area of investigation is the reported amount for the FIA. Validation %'s are in the low 80% for stat and tax reserves and only 78.5% for target surplus. While the issue with policy values could explain some of the low validation %'s, it is also worth investigating why validation %'s are even lower than policy values. It is possible there are additional issues with calculations, or a more material plan is being excluded. Reported amount like reserves and target surplus will influence decisions so it is definitely an area worth investigating.

For the Variable Annuity (VA), while validation %'s are close to 100% in tested areas, the fact that tax reserves are higher than stat reserves may suggest some modeling errors, given tax reserve should typically be lower than stat reserves. VA tax reserve methodology should be further investigated to avoid populating misleading post-tax profit metrics.

(c) Critique the following statements regarding static validation:

A. *Desired tolerance % should be determined first and be used consistently across all bases.*

B. *It is reasonable to expect a model to represent the entire block of business well.*

Commentary on Question:
Part c. of the question also addresses the application of static validation techniques more in terms of setting expectations up front.
Candidates generally did well for statement A where they recognize the importance to establish some tolerance thresholds upfront but not necessarily at the same level for every aspect.
For statement B, different candidates had different interpretations of the statement in which answers on both sides were observed. Full credits were awarded to candidates who were able to provide sufficient support on their stance for the statement.

2. Continued

    A. Determining tolerance %s upfront can help to prevent expectation being influenced by the results. Threshold does not necessarily need to be the same across all bases. Tolerance level for certain items such as counts and units should be tight (close to 100%) and more flexible tolerance %s should be used to for larger or more significant plans.

    B. One may expect the model to represent the entire block of business well in the most ideal situation. In reality, there are often edge cases due to complex and obscure product features, special case policies which a subset of the business cannot be modelled accurately and requires manual approximations. It is more practical to have an expectation that some plans might not be well represented by the model in static validation, and that is when different validation threshold across different bases become useful.

(d) Company AH uses a stochastic model to model the asset risk for this annuity block. Management is implementing the Transfer Scenario Order model efficiency technique to reduce runtime on a CTE 70 calculation.

    (i) List the steps needed to use this technique.

    (ii) Calculate the runtime reduction percentage achieved based on the data provided below:

	Policies	Scenarios	Runtime (h)
Original	25,000	1,000	380
New Run 1	1,250	1,000	19
New Run 2	20,000	400	152

Commentary on Question:
Part d of the question tests candidates' knowledge on establishing and measuring model efficiency through the Transfer Scenario Order technique. Candidates generally did well in (i) where they could clearly lay-out the steps of applying such technique.
Most candidates also did well in (ii) which they recognize how much runtime the new Run1 and Run2 cost, how much runtime can be saved compared to the original run, and what's the runtime saving as a percentage of the original runtime. Candidates who only calculated new runtime as a % of the original runtime instead of % reductions were only awarded partial credits. Some candidates recognized that Run2 did not run on all 25,000 policies and scaled-up Run2's runtime proportionally. Full credits were also awarded in this case to candidates who were able to properly compare the new runtime vs original runtime and accurately calculate % runtime savings.

2. **Continued**

i.
New Run 1 is to select potential most adverse scenarios to approximate the CTE 70 calculation.
1. Select a random 5 percent (or 10 precent, or small subset) of policies.
2. Run these policies through the full set of scenarios (1,000 in this case).
3. Order the results, and identify the worst 40% of scenarios (or worst 400 scenarios) The 400 scenarios is a mix of 300 worst scenarios plus small buffer of 100 additional scenarios in case worst scenarios might switch when running full list of policies.

New Run 2 is to approximate the CTE 70 measure with all policies using the worst 400 scenarios identified.
4. Run the full set of policies through the 400 scenarios identified.

5. Order the results and calculate the final CTE70 based on the worst 300 out of the 400 scenarios that were run (this can be viewed as a CTE25 of the new scenario set).

ii.
1 - (Total runtime using new technique) / Original = Runtime Reduction

where Total runtime using new technique = New Run 1 + New Run 2

Runtime reduction % = 1 - (19 + 152) / 380 = 0.55 or 55%

3. **Learning Objectives:**

3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

5. The candidate will understand the role of the Investment Actuary and the Portfolio Management Process in the Life Insurance company context, as well as the common forms of Fixed income securities and their uses, and the methods and processes used for evaluating portfolio performance and asset allocation.

Learning Outcomes:

(3a) With respect to Asset-Liability Models:
- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  o Major product risks for which ALM can be a useful tool for their management.
  o Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
- Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
- Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
- Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
- Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
- Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

(3b) With respect to asset adequacy analysis and cash flow testing, describe and evaluate actuarial practice with respect to:
- Modeling and selecting assets and related assumptions (incl. modeling assets with contingent cash flow risks).
- Handling liability cash flow contingencies and risks.
- Setting up projection model parameters and assumptions.
- Describe how Interest Rate Forwards and Futures and Swaps can be used in ALM, and apply the mathematics in given situations.

3. **Continued**

(5f) Describe and apply methods and processes for evaluating portfolio performance, including performance attribution, sources of earnings analysis on investment income, benchmarks, metrics, and risk adjusted performance appraisals (including total return vs reported earnings).

(5k) Describe the role of LIBOR and SOFR in an insurance company investment management context.

Sources:
LAM-146-19: ALM Management of Financial Institutions Ch 16, Tilman, 2003

LAM-155-23: Secured Overnight Financing Rate (SOFR)

Managing Investment Portfolios, Maginn, John L. and Tuttle, Donald L., 3rd Edition, 2007 - Ch.12: Evaluating PortfoliPerformance (section 4)

Managing Investment Portfolios, Maginn, John L. and Tuttle, Donald L., 3rd Edition, 2007 - Ch. 3: Managing Institutional Investor Portfolios (section 4.1)

LAM-156-23: Managing Liquidity Risk, Industry Practices and Recommendations for CROs, CRO Forum, 2019

Commentary on Question:
This question is to test students are able to identify the embedded options from an insurance product & the risk arising from such options. They should be able to suggest ALM strategy accordingly to manage the risk.
The last part of the question is to assess whether students are able to calculate investment return using different metrics.

Solution:
(a) Explain how a life insurance product is viewed differently by capital market practitioners and actuaries.

Commentary on Question:
Students did quite well on this question. Majority of the students were able to describe how capital market and actuaries view insurance products differently.

Capital markets' practitioners view insurance products as a bundle of options granted by the insurance company to its policyholders in exchange for a fee.

Actuarial approach looks at life insurance products as a series of future benefit payments / cash outflows.

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3. **Continued**

(b)
    (i)   Identify the option(s) granted to policyholders in this annuity product. Justify your answer.

    (ii)   Assess how changes in the interest rate environment can trigger policyholders to exercise the options described in (i).

Commentary on Question:
Most of the students are able to identify how interest rate environment can change policyholders' behaviour to exercise the options. Full mark is only given if the student can correctly identify the option associated with each of the product features as well as when each option will be exercised

    (i)   - The right to deposit additional premium constitutes a call on the value of future annuity payments. Policyholder have the right to purchase future growth on fund value at the guaranteed rate.

        - The withdrawal option can be viewed as a put on the value of the policy. Policyholder have the right to sell the policy back to the insurer at fund value.

    (ii)   - When interest rates are low, policyholders can deposit additional premiums for the purposes of earning a guaranteed return on their premium payment that is higher than the prevailing market interest rates.

        - When interest rates are high, policyholders can surrender their policies for a return of fund value (minus a surrender charge), and reinvest the money in a newer policy that credits a higher rate.

(c)   In response to the volatile interest rate environment, RXZ seeks to better manage their embedded option risk. As a risk mitigant, RXZ decides to change the fixed crediting rate to a floating rate based on SOFR.

    (i)   Describe how LIBOR and SOFR are determined.

    (ii)   Contrast the advantages of using LIBOR and SOFR. Justify your answer.

    (iii)   Propose two changes in product design RXZ can make to mitigate the embedded option risk.

Commentary on Question:
Part (i) and part (iii) are done better than part (ii) in general. Full mark is only given to part (ii) if student can specify the difference between LIBOR and SOFR in terms of maturity and currency associated with each. They also need to mention SOFR is based on actual transaction and is not easily manipulated.

3. **Continued**

    (i)   LIBOR: Determined by calculating the average interest rate at which major banks around the world borrow from one another.
SOFR: Determined based on transactions in the Treasury repurchase market, where investors offer banks overnight loans backed by their bond assets

    (ii)   LIBOR covers 5 currencies where SOFR is US dollar based only
LIBOR covers 7 maturities where SOFR is over-night rates only
SOFR is actual transaction-based with extensive trading data where LIBOR is based on surveying global banks instead of using actual data and can be easily manipulated.

    (iii)   - Adjust / increase surrender charge schedule. This will discourage policyholders from surrendering in exchange for fund value when maker interest rate is higher than the credited rate.

        - Limit the amount of additional deposits each policyholder can make. This will cap XYZ's exposure to unwanted interest rate exposure / committing to crediting a higher than current interest rate.

(d)   Below is a summary of monthly transactions and account values for a newly issued annuity policy:

Transaction Item	Day	Value
Initial deposit	1	100,000
Policy charges / fees	1	150
Interest credited	15	165
Additional deposit (after interest crediting)	15	10,000
Interest credited	30	250
End of period account value	30	110,265

    (i)   Calculate the time-weighted investment return on the account for this month. Show all work.

    (ii)   Estimate the money-weighted investment return on the account for this month. Show all work.

Commentary on Question:
Very few students got full mark on this question. Some were not able to differentiate between time-weighted investment return vs money-weighted investment return. Some students didn't use the right period for calculation and failed to identify there are only 2 periods to be used in part (i)

3.   Continued

(i)

Determine Day 15 fund value: **100,015**   before addt'l deposit

                                                                110,015   after addt'l deposit

Applying formula 12-2, subperiod 1 TWR = [100,015-(100,000-150)]/(100,000-150) = 0.165%

Applying formula 12-2, subperiod 2 TWR = (110,265-110,015)/110,015 = 0.227%

Chain link: TWR for the period = (1+subperiod 1 return)*(1+subperiod 2 return)-1 = 0.393%

(ii)

Apply formula 12-5:

$110{,}265 = (100{,}000-150)*(1+r)^1 + 10{,}000*(1+r)^{(1/2)}$

Determined by trial & error or using IRR function is excel, r is approximately 0.396%.

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- Handling liability cash flow contingencies and risks.
- Setting up projection model parameters and assumptions.
- Describe how Interest Rate Forwards and Futures and Swaps can be used in ALM, and apply the mathematics in given situations.

4. Continued

Sources:
LAM-153-23: Managing your Advisor: A Guide to Getting the Most Out of the Portfolio Management Process

LAM-140-19: Asset Adequacy Analysis Practice Note, 2004 , questions: 3, 5, 10-16, 18-20, 27, 29-31, 39, 42-60, 65-68, 71-82, 85 & 89

LAM-147-19: ALM Management of Financial Institutions Ch 2, Tilman, 2003

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a) Evaluate if the bullet structure is appropriate for managing asset liability mismatch risk. Recommend changes if needed. Justify your answer.

Commentary on Question:
The question tested candidates' understanding of different asset allocation strategies and their ability to recommend an appropriate portfolio structure for a fixed deferred annuity product. Most candidates understood the bullet structure and its limitations. However, many did not fully recognize that fixed deferred annuities have uncertain liability cash flows, making a laddered portfolio structure more appropriate.

ABC is currently using a "bullet" structure. It is a cashflow-matching technique where asset cashflows are concentrated at the point of liability cashflows. A "bullet" structure is only appropriate to support products with fixed benefit payments such as GIC's or payout annuities.

Bullet structure is not appropriate for fixed deferred annuity because the liability cashflows are uncertain. Therefore, cash matching does not eliminate asset liability management mismatch risk.

The "laddered" strategy is better for the fixed deferred annuity because the liability cashflows are uncertain due to the existence of policyholder behavior. Since the liability cashflows are uncertain, cash-matching cannot eliminate asset/liability mismatch risk

(b)   Critique the following proposals from ABC's management:

A.   *Given ABC's high credit rating, the annuities we sell also carry the same high rating. By investing the annuity premiums in junk bonds, we have effectively arbitraged a spread on the transaction.*

B.   *For modeling simplicity, we can use the same credit rating transition to project default losses for both investment-grade and high-yield bonds.*

C.   *We should include defaulted bonds in asset modeling.*

D.   *We can take advantage of falling interest rates by selling older, higher yielding bonds to realize the gains and help boost our reported earnings.*

Commentary on Question:
This question aimed to test candidates understanding of general principles of asset liability management and asset adequacy analysis, requiring them to provide evaluations under different proposals. For Part C, both Yes and No answers will receive full credit as long as correct justification is provided. While candidates generally performed well on Part A and C, but struggled with Part B and D.   Some common mistakes include:

A. *failed to provide a specific definition of arbitrage as risk free profit*
B. *not clearly differentiated the rate transition of default loss for high-quality bonds and junk bonds*
C. *lacked the correct justification for including or excluding default bonds in CFT*
D. *overlooked the constraints on asset sales from accounting considerations*

A) The proposal is incorrect.
Arbitrage is risk free profit. This strategy does not achieve arbitrage. Instead the risk of the short AAA/long junk position is amplified by numerous options offered to holders of the deferred annuities and the option to default on the low grade bond. There is no free lunch and there is no free spread margin. ABC must accept increased risk in the form of variance of returns or otherwise in order to achieve the spread.

B) The proposal is incorrect.
The default loss trend is different between high-quality bonds and junk bonds. For high-quality bonds, this transition effect will increase default losses over time. For low-quality bonds, default loss rates may actually decrease over time for the remaining bonds that survived the higher default rates of the earlier years of a projection.

4. Continued

C) The proposal is correct to include defaulted bonds.
For bonds in default, no coupon payments are typically included in cash flow testing. The market value of bonds in default is indicative of the recovery expected and reflects the expected amount of recovery, as well as the uncertainty in the recovery amount, through the implied discount rate.

The proposal is correct not to include defaulted bonds.
For practical purposes, a defaulted instrument is akin to an equity investment and is subject to the modeling difficulties that are present in equities—factors that are reflected in the volatility of market values for defaulted bonds and their sensitivity to economic conditions. For the same reasons that many actuaries do not include equities in their cashflow testing, they sometimes do not include defaulted bonds.

D) The proposal is incorrect.
Under US statutory accounting, realized capital gains are spread over the remaining life of the original asset via the Interest Maintenance Reserve (IMR). The original intent of the IMR was to prevent insurers from taking advantage of falling interest rates to boost reported profits by selling older, high yielding bonds at a gain and reinvesting at a lower rates, in a way "fronting" the profits of that block of business.

(c) ABC is considering reinsuring a few large blocks of business via a modified coinsurance agreement.

    (i) Assess whether CFT should be performed on the reinsured blocks. Justify your answer.

    (ii) Recommend whether the reserves of the reinsured blocks should be included in the asset adequacy analysis when preparing the Actuarial Opinion and Memorandum Regulation (AOMR) for ABC.

Commentary on Question:
The question evaluated candidates' comprehension of how modified coinsurance is treated in asset adequacy analysis and cash flow testing. Overall, candidates struggled with this question. Many failed to recognize that actuaries do not have to perform CFT on ceded modified coinsurance blocks due to minimal risk. However, most candidates did acknowledge the importance of including the reserves of the reinsurance blocks in the asset adequacy analysis.

4. **Continued**

    (i)   Even though ABC still holds the reserve balance and assets on its balance sheet there is no risk (or minimal asset risk) present for the ceded block. Therefore recommend that ABC not perform CFT on the ceded modified coinsurance block. Although the ceding company does not have to perform CFT on ceded modified coinsurance, actuaries should review the CFT work done by the assuming company to confirm that the risk to the ceding company is indeed minimal.

    (ii)   Regardless of whether ABC performs CFT on the ceded modified coinsurance blocks, the reserves should be reported in asset adequacy analysis as part of the AOMR. They might be reported in the opinion as being included in the analysis but as representing minimal asset risk (because the risks have been ceded to another company)

5.

Learning Objectives:

4. The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:

(4a) With respect to Economic Scenario Generators:
- Describe the need for ESGs and explain the structure of ESG models and components.
- Describe and apply basic default free interest rate models, including one-factor continuous time models.
- Assess the propriety of a particular ESG model and related assumptions for particular applications.

(4b) With respect to Equity-Linked models:
- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

Sources:

Economic Scenario Generators: A Practical Guide, 2016, Ch . 1, 2, 4.1, 5, 6, 9, 10, 11.1 & 11.3; LO#4-6 Investment Guarantees Ch 12, Hardy, 2003; LO#4-4 Investment Guarantees Ch 7 (pg 115-125), Hardy, 2003.

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Critique the following statements:

A. *Interest rate modeling is not as complex as stock price modeling as the term structure of interest rates only requires modeling a single variable.*

B. *It is not sufficient to treat the risk-free rate as a fixed parameter. An ESG that is being used for the management of variable annuity risk would need to include model dynamics that capture the path wise features of interest rates.*

C. *Arbitrage-free models are a necessary requirement when one wants to apply an ESG to real-world scenarios. If the scenarios are not arbitrage-free, then one cannot consistently price derivatives.*

D. *Only risk-neutral scenarios are used for hedging variable annuities.*

5. Continued

Commentary on Question:
This question tested the candidate' knowledge of interest rate and stock price modeling under real-world and risk neutral scenarios. Candidates generally performed well across all four parts.

A) Not correct - Interest rate modeling is more complex than stock price modeling. The reason is that the term structure of interest rates requires modeling a curve rather than a single variable, and the no-arbitrage principle constrains the possible outcomes.

B) Correct - The path-wise nature of interest rates and equity market returns are important considerations in hedging risks associated with underwriting investment guarantees.

C) Partially Correct - If ESG models are applied in real-world modeling contexts, then these models should be able to capture return behavior and market dynamics but do not necessarily have to be technically arbitrage-free. Arbitrage-free models are a necessary requirement when one wants to apply an ESG to risk-neutral pricing problems. If the scenarios are not arbitrage-free, then one cannot consistently price derivatives.

D) Not correct - Hedging of variable annuities is a standard example in which real-world and risk-neutral scenarios are applied together. The real-world simulation is used to assess the overall risk of the variable annuity book and to measure the effectiveness of the hedging strategy. Since the liabilities associated with the variable annuity book are complicated, closed-form formulas are not available, and risk-neutral scenarios must be used to price the variable annuity book at each node of the real-world simulation.

(b) Describe the purpose of establishing Stylized Facts prior to the development of the ESG.

Commentary on Question:
This question tested the candidates' knowledge of stylized facts. To receive full credit, candidates were required to demonstrate an understanding of stylized facts and clearly indicate reasons for establishing stylized facts prior to the development of the ESG. A common mistake was to list benefits of stylized facts but neglect to mention how they are used in the development of the ESG.

Stylized facts refer to generalized interpretations of empirical findings that provide a basis for consistent understanding of markets or economic drivers across a wide range of instruments, markets and time periods. Stylized facts are important in guiding the design of an ESG in that they help establish and prioritize the properties that the ESG model must have to be useful for a given application. Detailed knowledge of these dynamics is essential for setting ESG model calibration targets and understanding strengths and weaknesses of various ESG model frameworks.

5. **Continued**

(c)   Management is not convinced about the use of an ESG and recommends using analytical solutions which do not require intensive computation.

Critique management's recommendation.

Commentary on Question:
Candidates were required to specifically address the recommendation and justify their position. Many candidates neglected to directly respond to the recommendation and instead simply listed benefits of using an ESG.

Management's recommendation is not reasonable. While utilizing an analytical solution can save time and resources, they can present many limitations.

Impediments and limitations to analytical solutions:
- An underlying distribution must be identified and this is often unknown or unknowable.
- The problem involves the use of complex joint distributions.
- Discontinuities must be incorporated into the analysis.
- Results require translation or mapping to the desired output.
- Multiperiod projections are required.

Given the complex nature of a variable annuity product and substantial amount of tail risk embedded in potential GMxBs, I recommend that the company continue to utilize an ESG.

(d)   Calculate the risk neutral probability of the account value increasing by 8% in a given year.

Commentary on Question:
This question required candidates to demonstrate how to correctly use the risk neutral probability formula. Candidates generally did well on this question with the most common mistake being to confuse the probability of an up-move with the probability of a down move.

- $S(0) = 10000$
- $S(1)u = 10800$
- $S(1)d = 9200$
- $R = 3\%$
- $\text{Prob(Up)} + \text{Prob(Down)} = 1$
- $S(0) * \exp(r) = \text{Prob(Up)} * S(1)u + \text{Prob(Down)} * S(1)d$
- $10000 * \exp(.03) = \text{Prob(Up)} * 10800 + (1 - \text{Prob(Up)}) * 9200$
- $\text{Prob(Up)} = 69.03\%$

5. Continued

(e)   State if the following concepts are illustrated in the model used in part (d). Justify your answer.

    (i)   Replication

    (ii)   No-arbitrage assumption

    (iii)   Risk neutral probability distribution

    (iv)   Dynamic hedging

Commentary on Question:
Candidates were required to identify that all 4 concepts are illustrated in the risk neutral model presented in part (d). Candidates did well on parts (ii) and (iii) but struggled with (i) and (iv)

    (i)   Replication is illustrated in the model. Replication is the process of finding a portfolio that exactly replicates the option payoff. I.e. the market value of the replicating portfolio at maturity exactly matches the option payoff at maturity whatever the outcome of the asset. The model in part (d) replicates an option payoff through a portfolio of risky and risk-free assets.
    (ii)   No-arbitrage is illustrated in the model. The no-arbitrage assumption states that 2 identical cash flows must have the same value. As illustrated by the model in part (d), the portfolio and option have the same payout. By definition, risk neutral models must adhere to an arbitrage free assumption.
    (iii)   Risk neutral probability distribution is illustrated in the model. Under the risk-neutral framework, the model is indifferent between two investment choices that have the same expected payoff. These investment choices will always have the same price as evident in the model in part (d).
    (iv)   Dynamic Hedging is illustrated in the model. Dynamic hedging involves investing and rebalancing in risky and risk-free assets to meet potential GMxB liabilities at future points in time. In the model in part (d) a combination of risky and risk-free assets can be combined to meet the liability of the guarantee.

6. **Learning Objectives:**
5. The candidate will understand the role of the Investment Actuary and the Portfolio Management Process in the Life Insurance company context, as well as the common forms of Fixed income securities and their uses, and the methods and processes used for evaluating portfolio performance and asset allocation.

Learning Outcomes:
(5a) Describe the portfolio management process in an insurance company, and the role of Investment Policy, the Investment Actuary, and external portfolio managers.

(5b) Describe and evaluate how a company's objectives, needs and constraints affect investment strategy and portfolio construction (including capital, funding objectives, risk appetite and risk return tradeoff, tax and accounting, accounting considerations, and constraints such as regulation, rating agency ratings and liquidity.

(5c) Describe and assess the role of and significant considerations related to the design and function of asset allocation strategies.

(5h) Describe and apply conventional yield metrics used in bond performance evaluation.

(5i) Describe the attributes of US Treasuries, Agency Debt Securities, Municipal bonds, Corporate bonds, Private Money Market securities, Floating Rate Agreements, Agency Mortgage Backed securities, Agency Collateralized Mortgage securities, Interest Rate Swaps and Swaptions, Credit Derivatives and High Yield Bonds, and the markets they are traded in.

Sources:
Managing Investment Portfolios, Maginn, John L. and Tuttle, Donald L., 3rd Edition, 2007 - Ch. 3, Ch. 5 and Ch. 6)

Handbook of Fixed Income Securities, Fabozzi, Frank J., 9th Edition, 2021 - Ch. 4, Ch. 7, Ch. 10, Ch. 13, and Ch. 64

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a)
   (i) Describe the return objectives and risk tolerance for the company's investment policy statement.

   (ii) Identify four investment constraints the company should consider when managing their portfolio.

6. **Continued**

Commentary on Question:
For part (i), candidates generally were able to list return objectives and risk tolerances but commonly did not provide any justification and application. Partial credit was giving for just listing return objective and risk tolerance correctly.

For part (ii) many students only received partial credit by only listing general constraints and failing to apply and provide an explanation why the company should consider them.

(i)   Return objectives: 1. Earn a sufficient return to fund all policyholder liabilities and exceed the expected return used in pricing, including the minimum interest guarantee. 2. Increase the value of the company's surplus through capital appreciation to fund future expansion

Risk tolerance: Needs to ensure all policyholder liabilities can be met. Based on recent difficulties senior management is more concerned with solvency than aggressive growth, so risk appetite is fairly low. Consider tough competitive environment, capital considerations, asset ratings, interest rate risk, credit risk

(ii)   Liquidity: All policyholder obligations must be met. This includes death benefit, but also timing of possible lapses, policy loans, or universal life withdrawals
Time horizon: Long term, will establish duration targets for assets in line with liabilities

Regulatory: Comply with all insurance code in governing location. This includes build up of value within universal life contract

Tax: Monitored to provide most favourable after-tax returns

Review schedule: reviewed annually by board of directors

Asset allocation: Asset allocation designed to achieve objectives stated above. Should recognize constraints listed. Need to set asset allocation ranges and ensure these are met at all times

Rebalancing: Moderate rebalancing frequency (to minimize costs while still address market movement) is required.

6. Continued

(b)   The following is the company's current asset allocation strategy:

Asset Class	Target Allocation	Permissible Range
Treasury bonds	25%	20% - 40%
Bonds (municipal and corporate)	25%	20% - 40%
Equities (domestic, international, and mortgages)	20%	10% - 30%
Commercial paper	15%	5% - 25%
Cash	15%	5% - 25%

Your manager believes the asset allocations should be adjusted over time as the market shifts.

Assess the asset allocation strategy.

Commentary on Question:
Candidates generally performed well on this section. Partial marks were awarded for commenting on overall portfolio allocation, and full marks for those commenting on different assets as well.

This allocation breaks one of the key rules of asset allocation, which is that assets within a class should be homogenous. Need to set separate allocation for each unique asset, grouping them reduces effectiveness.
An ideal asset allocation should provide sufficient liquidity to ensure that the policyholder obligations can be met.
Treasury bonds are long term and with disintermediation risk, as their value drops with rising interest rates. This poses a problem when used to back long-term liabilities.
Although commercial paper is generally short-term, it is not a reliable source of funding as it is unsecured and lack of secondary market. 15% allocation of CP in the portfolio is too high for the risks they pose.
While cash is very liquid, holding large amount of cash will jeopardize the goal of earning more than the expected return as it provides no earning potential
While the overall asset allocation should be reviewed periodically, the goal is not to constantly refine the allocations based on short-term market movements. The portfolio must be rebalanced if an asset class weight moves outside the permissible range. Setting permissible ranges mitigate against systemic risk. By allowing them to move frequently it increases the risk faced by the company.

6. Continued

(c) Critique each of the following statements for structuring a portfolio:

A. *You can evaluate and identify assets with misaligned credit ratings to build a portfolio that will outperform a benchmark.*

B. *The assets purchased should be matched to the average duration of the liabilities. By doing this, it will protect against all changes in the market yield curve.*

C. *A portfolio should be built to track a benchmark index. This can reduce fees compared to investing in the assets that are part of the index directly. Given it is unlikely the portfolio will perfectly track the index, the focus for selecting assets should be on expected returns.*

D. *Interest rate swaps can help reduce interest rate risk and improve returns. You can also consider buying forwards for specific points in the future as an alternative.*

Commentary on Question:
Candidates generally performed well on this section. To received full credit, candidates needed to explain why the statement was correct or incorrect.

A. It's true that you can perform your own credit analysis and potentially beat market expectations this way. However, leveraging this strategy requires actuary with experiences and could be costly to set up without the appropriate expertise

B. It has been shown that matching the duration of the portfolio to the average duration of liabilities is not sufficient to immunize multiple liabilities. Each liability should be separately immunized. This strategy, even if employed correctly, only mitigates against parallel shifts in the yield curve. This will not mitigate the risk of a non-parallel shift (twist) of the yield curve

C. Correct, full replication of an index is costly, so by putting together a simpler portfolio it will save on costs

D. Earnings from an interest rate swap can only happen if you are betting on the direction of the interest rate market. Given the investment actuary believes rates will rise this could be leveraged to sell swaps for a gain, but only if that information is not already known in the market. It could also be costly with transactions fees compared to other approaches. Purchasing forwards is a way to replicate an interest rate swap but come with much higher costs and is not a good strategy

6. Continued

(d)   The company is evaluating bonds to add to the portfolio that backs a 10-year liability. Prevailing market interest rates are increasing and expected to continue to rise. You are given the following information:

Bond	Coupon Rate	Coupon Frequency	Term (years)	Yield to Maturity	Maturity Value
Bond 1	11%	Semi-annual	10	5.0%	$1,000
Bond 2	7%	Semi-annual	20	6.1%	$1,000
Bond 3	0%	N/A	15	2.2%	$1,000
Bond 4	6%	Semi-annual	5	8.4%	$1,000

(i)   Calculate the price of each bond.

(ii) Recommend a bond to add to the portfolio from the options above. Justify your answer.

Commentary on Question:
Most students did well on the calculation. The most common mistake was not correctly converting the given annual rates into semi-annual rates for both the coupon and yield to maturity.

(i)

Bond	Coupon Rate	Coupon frequency	Term (years)	Yield to Maturity (annual)	Maturity Value		Price
Bond 1	11%	Semi-annual	10	5.0%	1000		$1,467.67
Bond 2	7%	Semi-annual	20	6.1%	1000		$1,103.18
Bond 3	0%	N/A	15	2.2%	1000		$720.22
Bond 4	6%	Semi-annual	5	8.4%	1000		$903.63

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$$Price = \sum_{t=1}^{Term*2} \frac{Coupon\ Rate * .5 * Maturity\ Value}{(1 + YTM * .5)^t} + \frac{Maturity\ Value}{(1 + YTM * .5)^{Term*2}}$$

Calculation for bond 1: $\sum_{t=1}^{20} \frac{0.055*1000}{(1.025)^t} + \frac{1000}{(1.025)^{20}} = 1,467.67$

Repeat for bonds 2 through 4.

6. **Continued**

(ii)
Normally, the higher the coupon rate and the higher the YTM, the higher the reinvestment risk. However, these are less significant in a rising interest rates environment. Since the yield is expected to rise, it is the most beneficial to invest in the short-term asset and reinvest later at a higher rate.
Based on above, make recommendation on Bond 4