Review of Calculator Functions
For The Texas Instruments BA-35©

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This note presents a review of calculator financial functions for the Texas Instruments BA-35 calculator. A number of the examples used as illustrations are from the 3rd edition of the book *Mathematics of Investment and Credit*, by S. Broverman.

A detailed guidebook for the operation of and functions available on the BA-35 can be found at the following internet site:
http://education.ti.com/us/global/guides.html#finance .
It will be assumed that you have available and have reviewed the appropriate guide book for the calculator that you are using.

Financial functions will be reviewed in the order that the related concepts are covered in Chapters 1 to 8 of *Mathematics of Investment and Credit.* Some numerical values will be rounded off to fewer decimals than are actually displayed in the calculator display.

It will be assumed that unless indicated otherwise, each new keystroke sequence starts with clear registers. Calculator registers are cleared with the keystroke $\boxed{\text{AC/ON}}$.

Accumulated and Present Value of a Single Payment Using a Compound Interest Rate

Accumulated values and present values of single payments using annual effective interest rates can be determined using the calculator functions as described below.

Accumulated Value:
We use Example 1.1 to illustrate this function.

A deposit of 1000 made at time 0 grows at annual effective interest rate 9%. The accumulated value at the end of 3 years is $1000(1.09)^3 = 1,295.03$. This can be found using the calculator in two ways:

1. Calculator can be in any MODE.
   Key in 1.09 $\boxed{\text{2nd}}$ $\boxed{y^x}$,
   Key in 3 $\boxed{=}$ $\boxed{\times}$,
   Key in 1000 $\boxed{=}$.
   The screen should display 1,295.029.
   In this function, $y = 1.09$ and $x = 3$.

2. Calculator in "Fin" MODE.
   Key in 1000 $\boxed{\text{PV}}$, Key in 9 $\boxed{\%\text{i}}$, Key in 3 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$.
   The screen should display 1,295.029.

Present Value:
We use Example 1.5(a) to illustrate this function.

The present value of 1,000,000 due in 25 years at effective annual rate .195 is $1{,}000{,}000v^{25} = 1{,}000{,}000(1.195)^{-25} = 11{,}635.96$.
This can be found using the calculator in two ways:

1. Calculator can be in any MODE.
   Key in 1.195 $\boxed{\text{2nd}}$ $\boxed{y^x}$, Key in 25 $\boxed{+/-}$ $\boxed{=}$ $\boxed{\times}$, Key in 1000000. $\boxed{=}$.
   The screen should display 11,635.96

   This keystroke sequence can be replaced by:
   Key in 1.195 $\boxed{\text{2nd}}$ $\boxed{\text{1/x}}$ $\boxed{\text{2nd}}$ $\boxed{y^x}$, Key in 25 $\boxed{=}$ $\boxed{\times}$,
   Key in 1000000 $\boxed{=}$.

2. Calculator in "Fin" MODE.
   Key in 1000000 $\boxed{\text{FV}}$,
   Key in 19.5 $\boxed{\%\text{i}}$,
   Key in 25 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$,

   The screen should display 11,635.96.

As a more general procedure, in the equation $(PV)(1+i)^N = FV$, if any 3 of the 4 variables PV, i, N, FV are entered, then the 4th can be found using the $\boxed{\text{CPT}}$ function.

Unknown Interest Rate:
As an example of solving for the interest rate, we consider Example 1.5(c).

An initial investment of 25,000 at annual effective rate of interest i grows to 1,000,000 in 25 years. Then $25,000(1+i)^{25} = 1,000,000$, from which we get $i = (40)^{1/25} - 1 = .1590$ (15.90%). This can be found using the calculator power function with the following keystrokes:

40 $\boxed{\text{2nd}}$ $\boxed{y^x}$ .04 $\boxed{=}$ $\boxed{-}$ 1 $\boxed{=}$, the screen should display .15899723.

Using financial functions, the keystroke sequence solving for i is
Key in 25,000 $\boxed{\text{PV}}$, Key in 1,000,000 $\boxed{\text{FV}}$,
Key in 25 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\%\text{i}}$.

The screen should display 15.90 (this is the % measure to the nearest .01%).

Unknown Time Period:
As an example of solving for an unknown time period, suppose that an initial investment of 100 at monthly compound rate of interest i grows to 300 in n months at monthly interest rate $i = .75\%$. Then $100(1.0075)^n = 300$, from which we get $n = \frac{\ln 3}{\ln 1.0075} = 147.03$ months. This can be found using the calculator $\boxed{\ln x}$ function.

Using financial functions, the keystroke sequence solving for n is

Key in 100 $\boxed{\text{PV}}$, Key in 300 $\boxed{\text{FV}}$, Key in .75 $\boxed{\%\text{i}}$ $\boxed{\text{CPT}}$ $\boxed{\text{N}}$.
The screen should display 147.03026. Slightly more than 147 months of compounding will be required. The calculator returns a value of n based on compounding including fractional periods, so that the value of 147.03026 means that $100(1.0075)^{147.03026} = 300$.

EQUIVALENT INTEREST AND DISCOUNT RATES

The annual effective rate of discount can be found from the annual effective rate of interest and vice-versa in the following way.

An annual effective interest rate of $i = .10\,(10\%)$ is equivalent to an annual effective rate of discount of $d = .0909\,(9.09\%)$. The simple relationships $d = \frac{i}{1+i}$ and $i = \frac{d}{1-d}$ can be used, or the equivalent rates can be found in the following ways with the calculator in "Fin" MODE.

1. $\boxed{\text{2nd}}\ \boxed{\to \text{EFF}}$,
   Key in 10 $\boxed{\text{2nd}}\ \boxed{\to \text{APR}}$,
   Key in 1 $\boxed{+/-}\ \boxed{=}$.

   The screen should display 9.090909091.

   We have converted the annual effective interest rate of 10% (Key in 10) to the equivalent annual effective discount rate of 9.091%.

2. $\boxed{\text{2nd}}\ \boxed{\to \text{APR}}$,
   Key in 9.091 $\boxed{\text{2nd}}\ \boxed{\to \text{EFF}}$,
   Key in 1 $\boxed{+/-}\ \boxed{=}$,

   The screen should display 10.00011.

   We have converted the annual effective discount rate of 9.091% (Key in 9.091) to the equivalent annual effective interest rate of 10% (10.0001 due to rounding in 9.091).

ACCUMULATED AND PRESENT VALUES USING A COMPOUND DISCOUNT RATE

Accumulated values and present values of single payments using an annual effective rate of discount can be made in the following way.

Accumulated Value:
A deposit of 25 made at time 0 grows at annual effective discount rate 6%.

The accumulated value at the end of 5 years is:

$$25(1-.06)^{-5} = 25(.94)^{-5} = 34.06.$$

This can be found using the calculator in two ways:

1. Calculator can be in any MODE.
   Key in .94 $\boxed{\text{2nd}}$ $\boxed{y^x}$, Key in 5 $\boxed{+/-}$ $\boxed{=}$ $\boxed{\times}$, Key in 25 $\boxed{=}$.
   The screen should display 34.06 (nearest .01).

2. Calculator in "Fin" MODE.
   Key in 25 $\boxed{\text{PV}}$, Key in 6 $\boxed{+/-}$ $\boxed{\%\text{i}}$, Key in 5 $\boxed{+/-}$ $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$,
   The screen should display 34.06. We have used the financial functions to calculate $PV \cdot (1+i)^N$, where $PV = 25$, $i = -.06$ and $N = -5$, to get $25(1-.06)^{-5}$.

We could also find the annual effective interest rate and accumulate.

Present Value:
The present value of 500 due in 8 years at annual effective rate of discount 8% is $500(1-.08)^8 = 500(.92)^8 = 256.61$.

This can be found using the calculator in two ways:

1. Calculator can be in any MODE.
   Key in .92 $\boxed{\text{2nd}}$ $\boxed{y^x}$, Key in 8 $\boxed{=}$ $\boxed{\times}$, Key in 500 $\boxed{=}$.
   The screen should display 256.61.

2. Calculator in "Fin" MODE.
   Key in 500 $\boxed{\text{FV}}$, Key in 8 $\boxed{+/-}$ $\boxed{\%\text{i}}$, Key in 8 $\boxed{+/-}$ $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.
   The screen should display 256.61.

CONVERSION BETWEEN EQUIVALENT NOMINAL AND ANNUAL EFFECTIVE RATES

Given a nominal annual interest rate compounded m times per year, the annual effective rate of interest can be determined using the calculator as illustrated below.

A nominal annual interest rate of .24 (24%) compounded monthly is equivalent to an annual effective rate of interest of $i = .2682$ (26.82%). The relationship $i = \left(1 + \frac{i^{(12)}}{12}\right)^{12} - 1$ can be used, or the equivalent rates can be found in the following ways with the calculator in "FIN" MODE.

> $\boxed{\text{2nd}}$ $\boxed{\rightarrow \text{APR}}$, Key in 24 $\boxed{\text{2nd}}$ $\boxed{\rightarrow \text{EFF}}$, Key in 12 $\boxed{=}$.
> The screen should display 26.82. We have converted the nominal annual interest rate of 24% (Key in 24) compounded monthly (Key in 12) to the equivalent annual effective interest rate of 26.82%.

Given an annual effective rate of interest, the nominal annual interest rate compounded m times per year can be determined using the calculator as illustrated below.

> $\boxed{\text{2nd}}$ $\boxed{\rightarrow \text{EFF}}$, Key in 26.82 $\boxed{\text{2nd}}$ $\boxed{\rightarrow \text{APR}}$, Key in 12 $\boxed{=}$. The screen should display 23.9966 (round to 24). We have converted the annual effective interest rate of 26.82% to the equivalent nominal annual interest rate compounded monthly (Key in 12) of 24%.

CONVERSION BETWEEN NOMINAL DISCOUNT AND EFFECTIVE INTEREST RATES

Given a nominal annual discount rate compounded m times per year, the annual effective rate of interest can be determined using the calculator as illustrated below.

A nominal annual discount rate of .09 (9%) compounded quarterly is equivalent to an annual effective rate of interest of $i = .0953$ (9.53%).

The relationship $i = \left(1 - \frac{d^{(4)}}{4}\right)^{-4} - 1$ can be used, or the equivalent rates can be found in the following ways with the calculator in "Fin" MODE.

$\boxed{2nd}$ $\boxed{\rightarrow APR}$, Key in 9 $\boxed{2nd}$ $\boxed{\rightarrow EFF}$, Key in 4 $\boxed{+/-}$ $\boxed{=}$. The screen should display 9.53. We have converted the nominal annual discount rate of 9% compounded quarterly to the equivalent annual effective interest rate of 9.53% .

Given an annual effective rate of interest, the nominal annual discount rate compounded m times per year can be determined using the calculator as illustrated below.

$\boxed{2nd}$ $\boxed{\rightarrow EFF}$, Key in 9.53 $\boxed{2nd}$ $\boxed{\rightarrow APR}$, Key in 4 $\boxed{+/-}$ $\boxed{=}$. The screen should display 9.00. We have converted the annual effective interest rate of 9.53% to the equivalent nominal annual discount rate compounded quarterly of 9%.

LEVEL PAYMENT ANNUITY VALUATION

Annuity Immediate

The accumulated value and present value of a level payment annuity-immediate can be found using calculator functions. Clear calculator registers before starting the keystroke sequence. The calculator should be in "Fin" MODE.

Accumulated Value:

Suppose that a deposit of 1000 is made at the end of each year for 20 years. The deposits earn interest at an annual effective rate of interest of 4%. The accumulate value of the deposits at the time of (and including) the 20^{th} deposit is $1000s_{\overline{20}|.04} = 1000\left[\frac{(1.04)^{20}-1}{.04}\right] = 29,778$. This can be found using the calculator.

Key in 1000 $\boxed{+/-}$ $\boxed{PMT}$, Key in 20 $\boxed{N}$, Key in 4 $\boxed{\%i}$ $\boxed{CPT}$ $\boxed{FV}$.

The screen should display 29,778 (rounded to nearest 1). Note that the payment amount is entered as a negative quantity when finding accumulated value of an annuity (FV).

Present value:

Payments of 50 will be made at the end of each month for 10 years. The monthly compound interest rate is ¾ %. The present value of the annuity one month before the first payment is made is

$$50a_{\overline{120}|.0075} = 50\left[\frac{1-v_{.0075}^{120}}{.0075}\right] = 3{,}947 \text{ (10 years, 12 months per year).}$$

Key in 50 $\boxed{\text{PMT}}$, Key in 120 $\boxed{\text{N}}$, Key in .75 $\boxed{\text{\%i}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.
The screen should display 3,947 (rounded to nearest 1).

In the general equation $PV = PMT \cdot a_{\overline{N}|i}$ if any 3 of the 4 variables PV, PMT, N, i (in %) are given, then the calculator functions can be used to solve for the 4th variable.

The same is true for the equation $PMT \cdot s_{\overline{N}|i} = FV$ (in the FV case, PMT must be entered as negative, and will be returned as negative).

Finding the Payment:
A loan of 1000 is to be repaid with monthly payments for 3 years at a compound monthly interest rate of ½ %. The monthly payment is K where $1000 = Ka_{\overline{36}|.005}$, so that $K = \frac{1000}{a_{\overline{36}|.005}} = 30.42$.

This can be found using the following sequence of keystrokes:

Key in 36 $\boxed{\text{N}}$, Key in .5 $\boxed{\text{\%i}}$, Key in 1000 $\boxed{\text{PV}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PMT}}$.
The screen should display 30.42.

Finding the Unknown Interest Rate:
Suppose that the loan payment is 35 and the interest rate is to be found. Then $1000 = 35a_{\overline{36}|i}$. There is no algebraic solution for i. The following keystrokes give us i.

Key in 36 $\boxed{\text{N}}$, Key in 1000 $\boxed{\text{PV}}$, Key in 35 $\boxed{\text{PMT}}$ $\boxed{\text{CPT}}$ $\boxed{\text{\%i}}$.

The screen should display 1.31 (%).
That is the effective rate of interest per month.

Finding the Unknown Number of Payments:
We will use Example 2.13 to illustrate the calculator function for finding the unknown number of payments. In Example 2.13, Smith wishes to accumulate 1000 by means of semiannual deposits earning interest at nominal annual rate $i^{(2)} = .08$, with interest credited semiannually.

In part (a) of Example 2.13, Smith makes deposits of 50 every six months. We wish to solve for n in the equation $1000 = 50 \cdot s_{\overline{n}|.04}$.
The following keystrokes give us n.

Key in 1000 $\boxed{\text{FV}}$, Key in 50 $\boxed{+/-}$ $\boxed{\text{PMT}}$,
Key in 4 $\boxed{\%\text{i}}$, Key in $\boxed{\text{CPT}}$ $\boxed{\text{N}}$.

The display should read 14.9866 . 14 deposits are not sufficient. The accumulated value 6 months after the 14^{th} deposit is

$$50 \cdot \ddot{s}_{\overline{14}|.04} = 50(1.04) \cdot s_{\overline{14}|.04} = 951.18.$$

The next functions reviewed relate to finding the value of an annuity-due.

The BA II PLUS has functions that find annuity values when the interest period and the payment period do not coincide. The BA-35 Solar calculator does not have such functions, so we would always find the equivalent interest rate for the payment period for the BA-35.

Annuity-Due
The accumulated value and present value of a level payment annuity-due can be found using calculator functions. The same method applies as for annuities-immediate, with the additional requirement that keystrokes $\boxed{\text{2nd}}$ $\boxed{\text{BGN}}$ must be entered. (BGN makes the calculator view payments as being made at the beginning of each period.)

In the equation $PV = PMT \cdot \ddot{a}_{\overline{N}|i}$, if any 3 of PV, PMT, N, i are entered, we can find the 4^{th}.

In the equation $FV = PMT \cdot \ddot{s}_{\overline{N}|i}$ if any 3 of FV, PMT, N, i are entered, we can find the 4^{th} (PMT is entered and returned as a negative number).

Finding the Amount and Time of a Balloon Payment:
We can use the calculator functions to find the balloon payment required to repay a loan which has level payments for as long as necessary with a final balloon payment. In Example 2.15(a) of Chapter 2, a loan of 5000 is being repaid by monthly payments of 100 each, starting one month after the loan is made, for as long as necessary plus an additional fractional payment. At interest rate $i^{(12)} = .09$, we are to find the number of full payments that are required to repay the loan, and the amount of the additional fractional payment required if the additional fractional payment is made at the time of the final regular payment. We find the number of payments needed with the following keystrokes.

Key in 5000 $\boxed{\text{PV}}$, Key in 100 $\boxed{\text{PMT}}$,
Key in .75 $\boxed{\%\text{i}}$, Key in $\boxed{\text{CPT}}$ $\boxed{\text{N}}$.
The display should read 62.9 . This indicates that the 62nd payment is not quite enough to repay the loan. The additional payment needed, say X, at the time of the 62nd regular payment of 100 is found from the relationship

$$X = 5000(1.0075)^{62} - 100 \cdot s_{\overline{62}|.0075} = 89.55.$$

The keystrokes that will produce the value of X are

Key in 5000 $\boxed{\text{PV}}$, Key in 100 $\boxed{\text{PMT}}$,
Key in .75 $\boxed{\%\text{i}}$, Key in 62 $\boxed{\text{N}}$, Key in $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$.
The display should read 89.55.

VALUATION OF INCREASING AND DECREASING ANNUITIES

The values of $(Ia)_{\overline{n}|i}$ (present value) and $(Ds)_{\overline{n}|i}$ (accumulated value) can be found using calculator financial functions. From those values we can then find $(Is)_{\overline{n}|i} = (Ia)_{\overline{n}|i} \cdot (1+i)^n$, and $(Da)_{\overline{n}|i} = (Is)_{\overline{n}|i} \cdot v^n$. The following two examples illustrate the method.

<u>Increasing Annuity</u>:

Suppose that we wish to find $(Ia)_{\overline{20}|.08} = \frac{\ddot{a}_{\overline{20}|.08} - 20v^{20}}{.08}$. We first find the numerator with the following keystrokes.

Key in $\boxed{\text{2nd}}$ $\boxed{\text{BGN}}$, Key in 20 $\boxed{\text{N}}$, Key in 8 $\boxed{\text{\%i}}$,
Key in 1 $\boxed{\text{PMT}}$, 20 $\boxed{+/-}$ $\boxed{\text{FV}}$,
Key in $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.

The display should read 6.3126, which is $\ddot{a}_{\overline{20}|.08} - 20v^{20}$.

In this sequence of keystrokes, we have created a series of 20 payments <u>received</u> of 1 each at the start of each year $\left(\boxed{\text{BGN}}\right)$, combined with a payment of 20 <u>paid out</u> at the end of 20 years $\left(\boxed{\text{FV}}\right)$. The net present value is $\ddot{a}_{\overline{20}|.08} - 20v^{20} = 6.3126$. Then, $(Ia)_{\overline{20}|.80} = \frac{6.3126}{.08} = 78.908$.
Note that instead of entering 1 $\boxed{\text{PMT}}$ we could enter $\frac{1}{.08}$ $\boxed{\text{PMT}}$,
and instead of entering 20 $\boxed{\text{FV}}$ we could enter $\frac{20}{.08}$ $\boxed{\text{FV}}$.
Then $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$ includes division by .08.

Note that we cannot use these calculator functions to find $(Is)_{\overline{n}|i}$, but since the numerator of $(Is)_{\overline{n}|i}$ is $\ddot{s}_{\overline{n}|i} - n$, we can find $\ddot{s}_{\overline{n}|i}$ first, then subtract n, and then divide by i.

<u>Decreasing Annuity</u>:

Suppose that we wish to find $(Ds)_{\overline{35}|.04} = \frac{35(1.04)^{35} - s_{\overline{35}|.04}}{.04}$.

We first find the numerator with the following keystrokes.

Key in 35 $\boxed{\text{N}}$, Key in 4 $\boxed{\text{\%i}}$, Key in 35 $\boxed{\text{PV}}$,
1 $\boxed{\text{PMT}}$, Key in $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$.

The display should read 64.4609, which is $35(1.04)^{35} - s_{\overline{35}|.04}$.

In this sequence of keystrokes we have created an initial payment <u>received</u>

of 35 at time 0 $\left(\boxed{\text{PV}}\right)$, and a series of 35 payments of 1 each paid out at the end of each year. The net accumulated value at the end of 35 years is $35(1.04)^{35} - s_{\overline{35}|.04} = 64.4609$ $\left(\boxed{\text{FV}}\right)$. Then,

$$(Ds)_{\overline{35}|.04} = \frac{64.4609}{.04} = 1{,}611.52.$$

LEVEL PAYMENT LOAN AMORTIZATION

For a loan with level payments, or with level payments plus an additional lump sum payment at the time of the last regular payment, there are calculator functions for finding outstanding balances, interest or principal paid in a single payment, and interest or principal paid in a range of payments.

We use Example 3.3 to illustrate these functions. A homebuyer borrows \$250,000 to be repaid over a 30-year period with level monthly payments beginning one month after the loan is made. The interest rate on the loan is a nominal annual rate of 9% compounded monthly. The loan payment is $K = \frac{250{,}000}{a_{\overline{360}|.0075}} = 2{,}011.56$.

The outstanding balance at the end of the first year (after the 12th monthly payment) $OB_{12} = 2{,}011.56a_{\overline{348}|.0075} = 248{,}292.01$. The principal repaid in the 12th payment is $PR_{12} = Kv^{360-12+1} = 2{,}011.56v^{349} = 148.25$, and the interest paid in the 12^{th} payment is $I_{12} = K(1-v^{360-12+1}) = 1{,}863.30$. The principal repaid in the 2^{nd} year (the 13^{th} through 24^{th} payments inclusive) is

$$\begin{aligned} PR_{13} + PR_{14} + \cdots + PR_{23} + PR_{24} &= K(v^{348}+v^{347}+\cdots+v^{338}+v^{337}) \\ &= 1{,}868.21, \end{aligned}$$

the interest paid in the 2^{nd} year is

$$\begin{aligned} & I_{13} + I_{14} + \cdots + I_{23} + I_{24} \\ &\quad = K(1-v^{348} + 1-v^{347} + \cdots + 1-v^{338} + 1-v^{337}) \\ &\quad = 12K - (PR_{13}+PR_{14}+\cdots+PR_{23}+PR_{24}) = 22{,}270.46. \end{aligned}$$

These calculations can be done using the calculator functions in "Fin" MODE.

Key in 250000 $\boxed{\text{PV}}$, Key in 360 $\boxed{\text{N}}$, Key in .75 $\boxed{\text{\%i}}$,
Key in $\boxed{\text{CPT}}$ $\boxed{\text{PMT}}$ and −2,011.56 is displayed (we must key in the $\boxed{\text{PMT}}$ key in order to continue with the amortization functions).

To find OB_{12} use the following keystrokes:

Key in 12 $\boxed{\text{BAL}}$. The display should read 248,292.01. This is OB_{12}.
To find I_{12} and PR_{12} use the following keystrokes:
Key in 12 $\boxed{\text{I/P}}$. The display should read 1,863.30. This is I_{12}.

Key in $\boxed{x \rightleftarrows y}$. The display should read 148.25. This is PR_{12}.

To find $I_{13} + I_{14} + \cdots + I_{23} + I_{24}$ and $PR_{13} + PR_{14} + \cdots + PR_{23} + PR_{24}$ use the following keystrokes:

Key in 13 $\boxed{P_1/P_2}$, Key in 24 $\boxed{\text{I/P}}$. The display should read 22,270.46.
This is $I_{13} + I_{14} + \cdots + I_{23} + I_{24}$

Key in $\boxed{x \rightleftarrows y}$. The display should read 1,868.21.
This is $PR_{13} + PR_{14} + \cdots + PR_{23} + PR_{24}$

Note that $PR_{13} + PR_{14} + \cdots + PR_{23} + PR_{24} = OB_{12} - OB_{24}$ could be found from OB_{12} and OB_{24} and $I_{13} + I_{14} + \cdots + I_{23} + I_{24}$ can be derived from the relationship

$$I_{13} + I_{14} + \cdots + I_{23} + I_{24} = 12K - (PR_{13} + \cdots + PR_{24}) = 12K - (OB_{12} - OB_{24})$$

BOND VALUATION AND AMORTIZATION

It is possible to calculate the price or yield to maturity of a bond using the calculator. We use Example 4.1(a) to illustrate the functions.

Finding Bond Price on a Coupon Date:
A 10% bond with semiannual coupons has a face amount (par value) of 100 and is issued on June 18, 1990. The bond has a maturity date of June 18, 2010. We wish to find the price of the bond on its issue date using a nominal annual yield rate of 5% convertible semi-annually.

The bond price is $100v_{.025}^{40} + 100(.05)\cdot a_{\overline{40}|.025} = 162.76$ (nearest .01). If the bond has maturity value 110. then the price is

$$110v_{.025}^{40} + 110(.05)\cdot a_{\overline{40}|.025} = 166.48.$$

These prices can be found using the calculator in the following way:

There are 40 coupons to maturity, and the yield rate is 2.5% per coupon period.

Key in 100 $\boxed{\text{FV}}$, Key in 5 $\boxed{\text{PMT}}$,
Key in 2.5 $\boxed{\text{\%i}}$, Key in 40 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.

The display should read 162.76.
Note that the coupon payment (PMT) is 5.

Key in 110 $\boxed{\text{FV}}$, Key in 5 $\boxed{\text{PMT}}$,
Key in 2.5 $\boxed{\text{\%i}}$, Key in 40 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.

The display should read 166.48.
Note that the coupon payment is 5, and maturity (FV) is 110.

Finding Bond Yield on a Coupon Date:
We can use the worksheet to find the yield rate from the price. Suppose that the bond above with face amount 100 has a price of 150. There is no algebraic solution for the yield-to-maturity $i^{(2)} = 2j$, where j, the 6-month yield rate is the solution of the equation $150 = 100v_j^{40} + 5a_{\overline{40}|j}$.

The yield-to-maturity can be found using the following sequence of keystrokes.

Key in 100 $\boxed{\text{FV}}$, Key in 5 $\boxed{\text{PMT}}$,

Key in 150 $\boxed{\text{PV}}$, Key in 40 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{\%i}}$.

The display should read 2.88 (YTM is 2.88% per coupon period).
This would be expressed as a nominal annual interest rate of 5.76% compound semi-annually.

Bond Amortization:
The bond amortization components can be found using the calculator in much the same way they are found for loan amortization.

A bond has face amount 1000, coupon rate 5% per coupon period, maturity value 1000, 20 coupon periods until maturity and yield-to-maturity 6% (per coupon period). The bond's amortized value just after the 5^{th} coupon is

$$BV_5 = 1000v_{.06}^{15} + 1000(.05) \cdot a_{\overline{15}|.06} = 902.88.$$

This can be found using the following keystrokes:

Key in 1000 $\boxed{\text{FV}}$,

Key in 50 $\boxed{\text{PMT}}$,

Key in 6 $\boxed{\text{\%i}}$,

Key in 20 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$. The display should read 885.30. Then

Key in 5 $\boxed{\text{BAL}}$.

The display should read 902.88.

As an alternative, the following keystrokes will also give BV_5.

Key in 1000 $\boxed{\text{FV}}$,

Key in 50 $\boxed{\text{PMT}}$,

Key in 6 $\boxed{\text{\%i}}$,

Key in 15 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.

The display should read 902.88.

We have found the price of the bond with 15 coupons remaining to maturity, using the original coupon rate and yield to maturity.

The BA II PLUS calculator has a function than can find the price of a bond on any date. The BA-35 does not have this function. Also, the BA II PLUS has NPV and IRR functions that are not available on the BA-35.