Report to the Policyholder Behavior in the Tail Subgroups Project

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Abstract

At the request of the Society of Actuaries' Risk Management Task Force, an analysis was performed on fixed annuity data consisting of sources from Korea and a single U.S. company. The purpose of the study was to explore lapse experience in the tails and see if this required a special model with special parameters. Parameters suitable for the Logit Surrender Rate Model were developed covering not only tail experience but also under "normal" conditions. Parameters were also developed and are linked to variables such as interest spread, duration, unemployment rates, economy growth rates and seasonal effects. The methods used may be applied to any company's experience. For the data tested, the Logit Model closely fit the experience of the data even under extreme financial conditions.

Background

This report was written at the request of the Society of Actuaries' Risk Management Task Force, a group working to develop better estimates of policyholder lapse behavior in the tail of the distribution, where the tail is defined as more than two standard deviations from the expected level (which may vary by duration), under varying economic conditions, and in combination with different policy characteristics.

Research Objectives

The purpose of the research project was to address some or all of the following issues and questions related to unusual policyholder lapse behavior (high or low) for different life insurance products. The SOA provided fixed deferred annuity data for one U.S. company. Korean annuity experience was previously obtained. Therefore, the scope of the research was limited to fixed deferred annuities. The general objectives of the original request for proposals were (depending on data availability):

A. **For universal and interest sensitive life products:** Quantify at what interest crediting rate compared with the market rate (spread to market) lapses are likely to

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increase significantly. Similarly, how does the relative size of the dividends on par policies compared to current market experience (spread) impact lapse rates? Quantify the lapse/spread relationship as the spreads increase. [I was able to address this for annuities.]

B. **For universal and interest sensitive life products:** Quantify how the size and pattern of the surrender charge affects the volatility of lapse rates in combination with other economic factors under extreme conditions. [I was able to address this for annuities.]

C. **For universal and interest sensitive life products:** Quantify how the presence of secondary guarantees impact premium payments and lapse behavior under extreme conditions. [No secondary guarantees were present in any of the products.]

D. **For flexible premium and fixed premium whole life products:** Quantify the impact of economic conditions on the payment of premiums and policy loans under extreme conditions. [This topic was not addressed because I did not have any history on premium payments.]

E. **For any product:** Quantify how the marketing channel, through which the products are sold, affects the potential for high lapse rates under extreme conditions. For example, does the degree of agent control over policyholder decisions impact the potential for large groups of policyholders to lapse? Quantify the lapse rates under extreme conditions. [This topic was not addressed as I was not able to differentiate marketing methods in the data.]

F. **For worksite or salary savings products**: Quantify how the key characteristics of the employer affect lapse rates under extreme conditions. [No data was available on this market.]

G. **For term insurance products with increasing premiums:** Separately for YRT and level followed by YRT, quantify how the change in economic factors and the distribution channel affect lapse rates under extreme conditions. [I only addressed annuities.]

H. **For level premium term products (originally underwritten as standard or**

substandard): Quantify how the policyholder with specific health problems affects the variability of lapse rates under extreme conditions. [I only addressed annuities.]

I. **For various life and annuity products:** Quantify how unusual regulatory changes (changes in COLI—Corporate Owned Life Insurance—taxation or estate taxes) affect lapse behavior under extreme conditions. [I was not able to address this aspect of the request for proposals).

J. **For various life and annuity products:** Quantify how the credit rating or the change in credit rating of a company impacts lapse rates under extreme conditions. [This is incorporated in my report.]

Ultimately, this information will be used to help actuaries and regulators assess the risk in these products and establish an appropriate level of reserves and target surplus.

Specific Components of Research

The task force asked for the following research items:

A. Gather data to understand and quantify the causes of lapse behavior under extreme conditions. Sources of this data might include reinsurers, large U.S. and Canadian insurance writers, Japanese data (to examine economic stress) and companies that have had significant ratings downgrades, restructuring or have been under state supervision. Multiple years of data are needed. This data will need to be adjusted for differences in markets, distribution, culture and economic factors. The Society of Actuaries can work with the companies and the researcher to maintain confidentiality. However, identification of data sources and collection of data will be the responsibility of the researcher. In this regard, I collected data on Korean experience and the SOA supplied me with the data for one U.S. company.

B. Develop, examine and recommend different mathematical models or other assumptions that may be used to project lapse behavior under very adverse conditions. Examples include models that project lapses that vary by spread of credited investment rate and a projected market or indexed rate; models that project lapses varying by changes in premium rates; and models that project lapse rates by product that change with changes in economic or market conditions. Both continuous models and models that may have two or more states (regime-switching) should be considered with the researcher identifying their respective strengths. Identifying breakpoints for these regime-switching models is a priority result. In this regard, I introduced the Logit Model as a powerful way to model lapse rates and show its superiority over other models. The Logit function has the following form;

$$\ln\left(\frac{q_s}{1-q_s}\right) = \beta_0 + \beta_1 V_1 + \dots + \beta_n V_n,$$

where q_s refers to the lapse rate for a particular age and duration, *s*, and the Betas are coefficients to certain key indices, *Vj*, such as, interest rate spread, inflation, unemployment, etc.

C. Review other sources for studies of relevant lapse rates and/or models that relate lapse behavior to economic and policy characteristics and others described in the previous section.

D. Estimate the key parameters of each lapse rate assumption, experience and/or model, such as how different product features and distribution channels affect the parameters. I have shown how this is done for the test data and how actuaries can use the approach for their own data.

Discussion of Data

Korean Data

The(se) type of product(s): Korean Interest Indexed Annuities (Korean IIA), a single premium deferred annuities.

The put options contained in the product(s): Surrender options, partial surrender options, annuitization options.

U.S. Data :

The type of product(s): U.S. single premium deferred annuities (U.S. SPDA)

The put options contained in the product(s): Surrender options, partial surrender options, annuitization options.

Definition of an Extreme/Financial Shock For extreme events/financial rate shocks, define k(t), a risk measure of the financial rate $i(t)$,

$$k(t) = \frac{i(t) - \mu}{\sigma}.$$

Further, define that the financial rate $i(t)$ experiences a financial rate shock at time t if

$$|k(t)| \geq 2,$$

and specify that the financial rate is in a stable status at time t if |k(t)| < 2. The condition of extreme financial conditions exists if financial rates experience financial rate shocks.

For the surrender rate changes of the U.S. SPDA under the assumption that U.S. financial rates experience the financial rate shocks, I made two assumptions on the pattern of k(t), the risk measures of the financial rates.

Assumption 1 (A1): The pattern of k(t) is the same as that of Korean data when the U.S. financial rate measures experience financial shocks. When the rate does not experience any financial shocks, the risk measure k(t) is calculated from U.S. data.

Assumption 2 (A2): k(t) = c, where c is a constant integer such that $|c| \geq 2$ and the financial rate $i(t)$ is changed to $i(t) + c\sigma$. This means I looked at results larger than two standard deviations out.

For details, see the following pages (especially the section entitled, "Surrender Rate Changes Under Financial Rate Shocks.")

Introduction

Modeling appropriate interest rate sensitive surrender/lapse rates is essential in managing assets and liabilities of insurance companies. Even though there are a few research papers on the interest sensitivity of the cash flows, the analysis is usually focused on the asset sides. For example, in Pesando's (1974) paper, the cash flow analysis considers the prepayment rate impacts only. But we have to mention that the interest sensitivity of cash flows through surrender rate fluctuations is a kind of "*dual problem*" to that through prepayment rate fluctuations. So it is important to consider surrender rate impacts on cash flow analysis with proper surrender rate models.

There are many factors affecting surrender/lapse rates such as the difference between reference market rate and policy crediting rate, seasonal effect, age and gender of clients, economy growth rate, foreign exchange rate, inflation rate, policy age since issue date and unemployment rate, among others. The surrender rate level has great influences on the cash flow of assets and liabilities. To reflect the exact impact of surrender rate in asset/liability management (ALM) framework, it is inevitable to consider and devise a proper surrender/lapse rate model.

In this paper, I attempted to define the extreme economic conditions to be considered and quantify their impact on policyholder surrender behaviors. First I gathered data to understand and quantify the causes of lapse behavior under extreme conditions. Sources of this data included one U.S. insurance writer and Korean data (to examine economic stress). I considered surrender rate models reflecting the complicated policyholder surrender behaviors with endogenous and exogenous multi-variables. I decided to use the Logit Model to describe the surrender rate experiences of Korean interest indexed annuities and U.S. single premium deferred annuities. I also worked to model surrender rates with a few explanatory variables and develop better estimates of policyholder surrender/lapse behavior under extreme conditions, where the extreme condition is defined as more than two standard deviations from the expected level (which may vary by duration), under varying economic conditions, and in combination with different policy characteristics.

The Structure of Single Premium Deferred Annuities

Many insurance companies are selling single premium deferred annuities (SPDA). But SPDA are sold with the primary focus on accumulation. Only a few of the policyholders purchase SPDA for the purpose of annuitization. In Korea, the annuity market is still young and growing slowly[1] compared to that of the United States The SPDA crediting interest rates are declared each month/year by the issuing companies. Although that is the predominant structure in Korea, other variants such as multiple-year guarantees and interest-indexed annuities (IIA) are also popular.

[1] The volume of in-force and new contracts of annuities in Korea is not really large compared to that in the United States of America. According to data from American Council of Life Insurers, the reserve value for annuity contracts in the United States is about $1,585,008 million. But, from the Korea Life Insurance Association data, it is about 44,927,906 million Korean wons (US $37,440 million with exchange rate of 1,200 Korean won for U.S. $1) in Korea in year 2001. The number of annuity contracts in force is about 6,406,000 in Korea (it is 66,548,000 in USA) and the number of newly issued annuities is about 822,000 in Korea (it is 7,641,000 in USA) in year 2001.

For Korean IIAs, I considered seven-year interest indexed annuities. The death benefits are the account value plus 10 percent of premium, and another 10 percent of premium in the case of accidental death.

For U.S. SPDA, I considered multiple annuity products with different surrender charge schedules. An example of the products is the seven-year fixed annuity SPDA, and the interest rate may be reset each year at end of each anniversary. After the first policy year, the policy owner may surrender up to 10 percent of total account value each year without a surrender penalty, with excess over 10 percent subject to surrender charges. On full surrenders, the first 10 percent is penalty-free. For my U.S. data, a special provision applied for nursing homes confinement. Upon confinement in nursing home/hospital for at least 60 days, some or all of fund value may be withdrawn, provided it is within 90 days after end of confinement. The death benefits are the full fund value. Annuitizations are permitted starting in the first policy year, with no surrender charges provided the pay-out is for at least five years.

For various characteristics and valuation of SPDA, please refer to Society of Actuaries (1991), Cox, Laporte, Linney, and Lombardi (1992), and Asay, Bouyoucos, and Marciano (1993).

Crediting Interest Rates

Crediting interest rates may be reset each year at the end of each anniversary for the fixed annuity SPDA. Many contracts guarantee a minimum interest rate below which the renewal crediting interest rates will not fall. For Korean IIA, the crediting interest rates are announced every month based on current market interest rates, current investment gain rates and the expected future portfolio income gain rates. The main factor of the crediting rates is the market interest rates and this is why they call the products interest-indexed annuities.

The majority of contracts in Korea guarantee interest for one-year periods; however, longer guarantees are available, with five years being the most popular. After the initial five-year guarantee, the contract might (a) automatically roll into another five-year guarantee at current rates, (b) automatically switch to annual guarantees or (c) give a choice between the two. The longer guarantees have gained increasing popularity as some purchasers and salesmen have gotten uncomfortable with “trust me” annual interest declarations.

Surrender Charges

Many contracts credit the full premium to the account value and assess surrender charges when the policyholder surrenders. The amount of surrender charges are usually from 7 percent to 10 percent of the account value and decreased to zero over a six- to 10- year period. The range of surrender charges of different companies may be higher or loIr and the penalty periods may run for shorter or longer. For Korean IIA, I considered surrender charges from 7 percent of the account value and decreased to zero over a six-year period. For U.S. SPDA, I considered multiple annuity products with different surrender charge schedules. An example of the surrender charge schedule is 7 percent, 7 percent, 7 percent, 7 percent, 6 percent, 4 percent, 2 percent of the account value in years one through seven, 0 percent thereafter.

Usually the maximum initial surrender charge on an SPDA is about 10 percent and decreased by 1 percent annually. Surrender charges are generally waived for certain withdrawals, which are called free partial withdrawals. On full surrenders, the first portion of the account value, for example 10 percent, is penalty free.

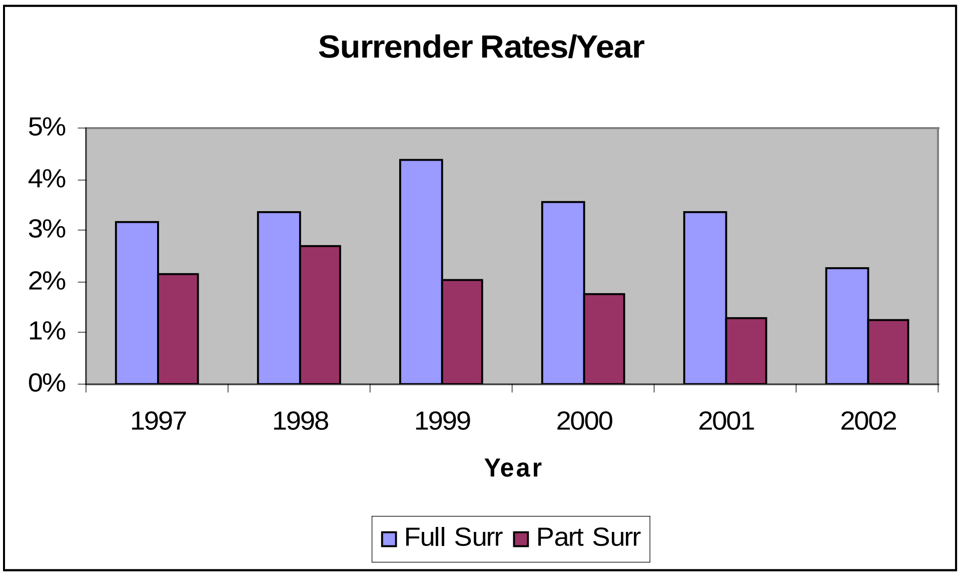
Free Partial Withdrawals

A portion of the account value can be withdrawn at any time without surrender charges to provide liquidity to the contract owner. The maximum level is 90 percent of the account value at the time of partial withdrawal, but a few companies might limit the maximum level much lower than 90 percent of the account value. For example, after the first policy year, the policy-owner may surrender up to 10 percent of total account value each year without a surrender penalty, with excess over 10 percent subject to surrender charges. Often the policyholders can take advantage of this partial withdrawal option several times a year. For example, when the stock markets show signs of an upward jump, the policyholders can draw out their savings from the account without any surrender charges and invest this amount of money in the stock markets. After enjoying the profits from the stock market, they can return to their insurance contracts paying relatively low interest. So this characteristic of high maximum level of partial withdrawal without surrender charges is a source that one might overuse the partial withdrawal option. For some contracts, upon confinement in nursing home/hospital for at least 60days, some or all of fund value may be withdrawn, provided it is within 90days after end of confinement.

Figure 1 shows the full surrender rates and partial surrender rates of U.S. insurance companies from year 1997 to year 2002. The average partial surrender rate is about 1.9 percent each year, relatively high compared to the average of the full surrender rate, 3.4 percent each year.

Moreover the death benefit amount is still guaranteed during the partial withdrawal period.

Figure 1. Full and Partial Surrender Rates of U.S.-SPDA/Year

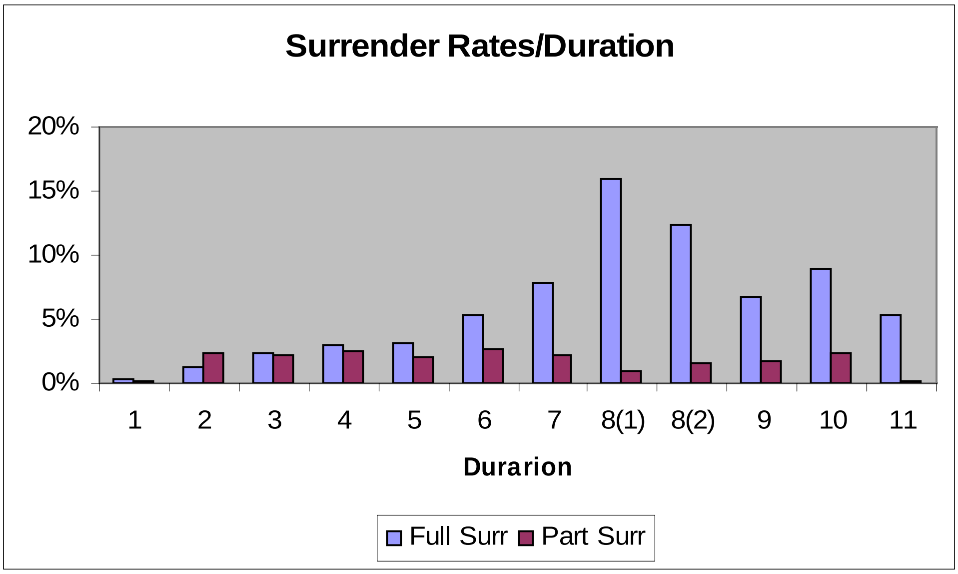


Modeling Surrender Rates for Korean Interest Indexed Annuities and U.S. SPDA

The SPDA/IIA product provides the policyholder with a surrender option that he/she may surrender the contract early with specified surrender charges. As market rates rise, we might think that the SPDA/IIA owners would surrender their contracts and reinvest the surrender cash value in high yielding alternatives. But the surrender option may not be exercised by every policyholder even though the market rates rise. That is, it is not exercised optimally. As we show later, the surrender option is not a function of interest rate only; it depends on policy duration. Surrender experience also reflects the unemployment rate and the economy growth rate. Actually the surrender tendency varies between policyholders. So we have to model the policyholder surrender behavior statistically. The variables considered are (a) the difference between the reference new money rates (or market rates) and the product crediting rates with surrender charges, (b) the policy age since the contract was issued (or the duration), (c) unemployment rates,

(d) economy growth rates and (e) seasonal effects.

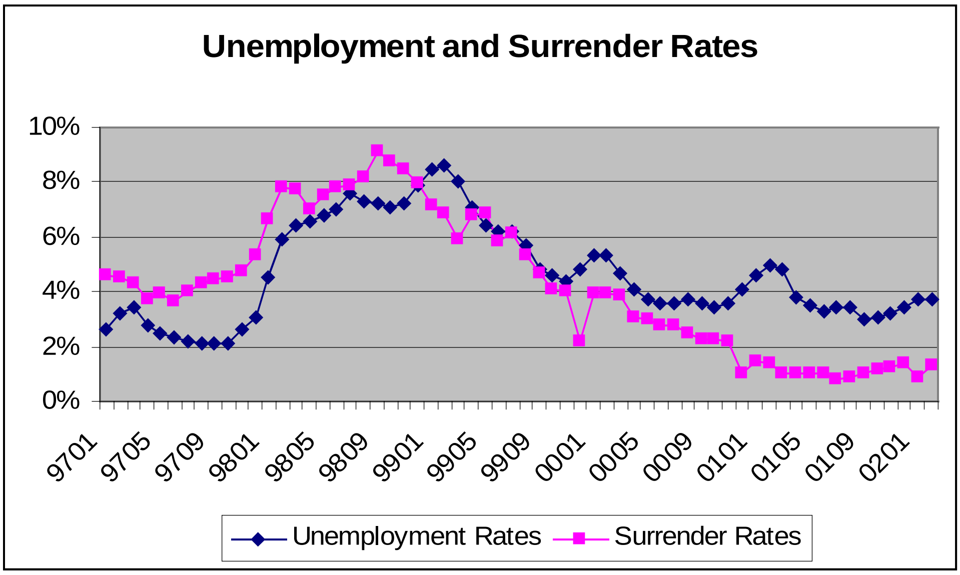
Figure 2. Surrender Rates of U.S.-SPDA/ Duration



Duration is one of the most important factors of surrender rates, primarily due to the surrender charge. Figure 2 shows the surrender rates of U.S. SPDA according to duration. The policy – a seven-year fixed SPDA. For the first five years, the surrender rates are increasing slowly. The surrender rates on the sixth and seventh years are relatively high. Duration 8(1) is the first three months of the eighth contract year, and the surrender rates are almost 16 percent. Duration 8(2) is months four through twelve of the eighth contract year, and the surrender rates are almost 14 percent. We can see that almost 30 percent of the contracts are withdrawn on the eighth contract year, right after the surrender period, seven years.

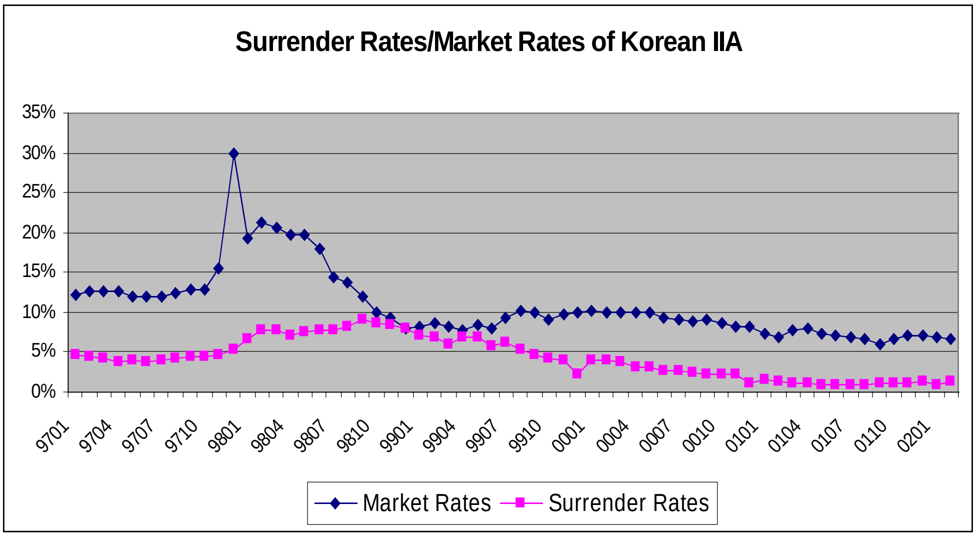
Figure 3 and Figure 4 show the relationships between the unemployment rates, the market interest rates and the surrender rates of Korean IIA. We can easily notice that the unemployment rates, market interest rates and the surrender rates soared up rapidly during the financial crisis, from December 1997 to December 1998. We can assume that the surrender rates are dependent not only on interest rates but also on exogenous factors such as unemployment rates, economy growth rates, seasonal effects, and so on.

Figure 3. Unemployment Rates and Surrender Rates of Korean IIA



Source: Unemployment Rate; Korea National Statistical Office (www.nso.go.kr)

Figure 4. Market Interest Rates and Surrender Rates of Korean IIA



Source: Market Rates; 5-year government bond rates; The Bank of Korea (www.ecos.bok.or.kr)

I prefer to use the logit link function in modeling the surrender rates of Korean IIA and U.S. SPDA. The logit functions is often used for modeling odds and probability functions. There are many examples in which logit functions are used for financial data analysis. Hall (2000) compares logit analysis of data to the results from his prepayment model. Pinder (1996) demonstrates how multinomial Logit Models can be used in a decision analysis framework to estimate expected monetary value. Kolari, Glennon, Shin and Caputo (2002) use the parametric approach of logit analysis to predict large commercial bank failures. See also Johnsen and Melicher (1994), and Lo (1986).

To fit our data to the Logit Model, I used the Generalized Linear Models[2], Procedure GENMOD, Logistic Regression Models and Procedure LOGISTIC, with SAS[3].

The Logit function has the following form,

$$\ln\left(\frac{q_s}{1-q_s}\right) = \beta_0 + \beta_1 V_1 + \ldots + \beta_n V_n, \qquad (1)$$

where q_s is the surrender rate, β_i is the coefficient to be estimated and V_i is the explanatory variable.

For Korean IIA surrender rate models with 3 year duration[4], we use the Logit Model,

$$\ln\left(\frac{q_s(t)}{1-q_s(t)}\right) = \beta_0 + \sum_{j=0,2,4,6,8,10,12} \beta_j *(i_m(t-j) - i_c(t-j))$$

$$+\beta_{UE} * i_{UE}(t) + \beta_{EG} * i_{EG}(t) + \sum_{j=1}^{11} \beta_{month-j} * DV_j, \qquad (2)$$

where DV_j is the seasonal effect dummy-variable.

The parameter estimates are shown in Table 1. It is interesting to note that the parameter β_{UE} for the unemployment rates is very large, 50.6348. It means that the

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[2] I refer the reader to a few books on generalized linear model such as Agresti (1996 and 2002), Harrell (2001), Kutner, Nachtschiem, and Wasserman (1996), McCullagh and Nelder (1989), Firth (1991), and McCulloch and Searle (2000).

[3] In programming with SAS, refer Allison (1999), and SAS Institute Inc. (1999).

[4] We can model surrender rates with the duration (policy age since issue date) as an explanatory variable. For more details see Appendix and Kim(2004a). We notice that almost 30 percent of the contracts are withdrawn on the eighth contract year, right after the accumulation period, seven years, as shown in Figure 2. So duration is one of the main factors of the surrender rates. In this paper, we want to investigate the policyholder surrender behaviors under extreme financial conditions. So we just look at the contracts with the same duration; three years for Korean IIA and five years for U.S. SPDA, this will help us to check the impacts on the surrender rates due to the economic variables.

surrender rates change very greatly according to the unemployment rate movements. But, considering the unemployment rate change ratio is not so radical as that of the reference market rates (new money rates), it is not strange for us to have a large β_{UE}. It seems also reasonable that the parameter β_{EG} for the economy growth rates is a negative number, –5.3360. We can guess that when the economy condition is good the policyholders may not surrender their IIA policies.

Now our final model for the Korean IIA surrender rates, $\{q_s(t)\}$, is given by the following formula,

$$q_s(t) = \frac{1}{1+\exp(-\alpha)}, \tag{3}$$

where

$$\alpha = \beta_0 + \beta_{UE} * i_{UE}(t) + \beta_{EG} * i_{EG}(t) + \sum_{j=0,2,4,6,8,10,12} \beta_j * \{i_m(t-j) - i_c(t-j)\} + \sum_{j=1}^{11} \beta_{month\_j} * DV_j . \tag{4}$$

The graph of the real and predicted (using Logit Model) surrender rates of Korean IIA policies appears below.

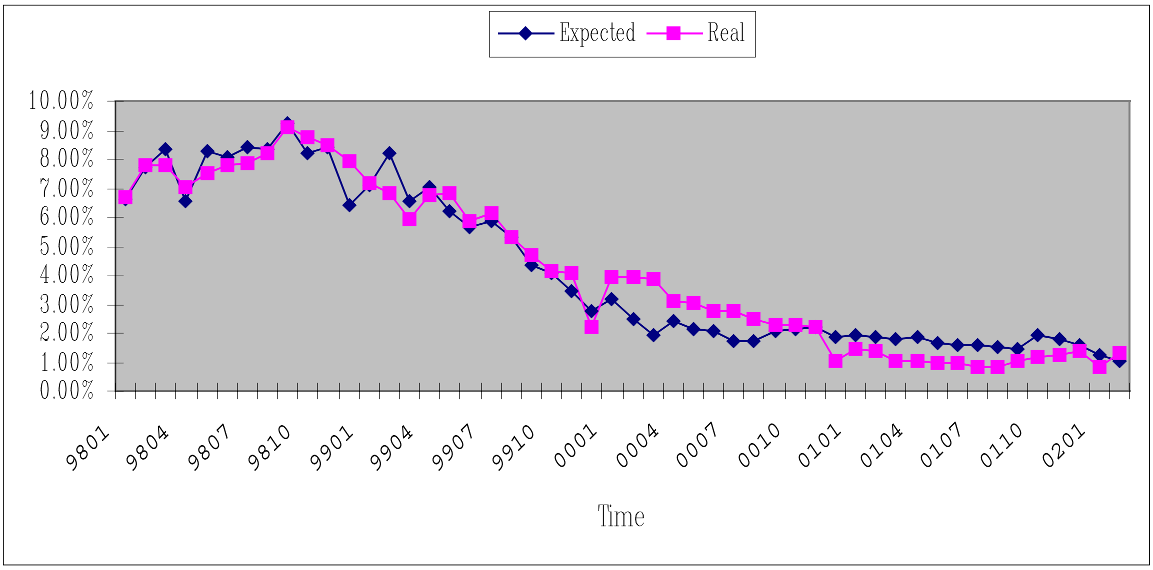
Table 1. Parameter Estimates with Logit Model (IIA)

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-6.0132	0.00617	950275.502	<.0001
DIFFLAG0	1	9.3465	0.0563	27551.1981	<.0001
DIFFLAG2	1	0.9728	0.0412	557.6077	<.0001
DIFFLAG4	1	-6.2020	0.0438	20031.9722	<.0001
DIFFLAG6	1	-2.7553	0.0399	4776.8774	<.0001
DIFFLAG8	1	1.4655	0.0390	1410.1121	<.0001
DIFFLAG10	1	0.5252	0.039	182.5160	<.0001
DIFFLAG12	1	-1.8470	0.0468	1557.8107	<.0001
Unemploy	1	50.6348	0.1640	95314.7985	<.0001
Eco GROWTH	1	-5.3360	0.1723	959.5427	<.0001

MONTH1	1	-0.2111	0.00409	2662.3227	<.0001
MONTH2	1	-0.4199	0.00446	8867.3221	<.0001
MONTH3	1	-0.3629	0.00446	6633.6120	<.0001
MONTH4	1	0.1121	0.00415	728.9672	<.0001
MONTH5	1	0.2443	0.00408	3589.7187	<.0001
MONTH6	1	0.2961	0.00424	4879.2107	<.0001
MONTH7	1	0.2111	0.00429	2421.8388	<.0001
MONTH8	1	0.2082	0.00458	2065.2003	<.0001
MONTH9	1	0.4040	0.00452	7970.0766	<.0001
MONTH10	1	0.4919	0.00469	11024.0567	<.0001
MONTH11	1	0.3720	0.00447	6913.5047	<.0001

Figure 5. Real and Predicted Surrender Rates of Korean IIA



For U.S. SPDA surrender rate models with fiveyear duration, we also use the Logit Model,

$$\ln\left(\frac{q_s(t)}{1-q_s(t)}\right) = \beta_0 + \beta_M * (i_m(\mathrm{t}) - i_c(\mathrm{t})) + \beta_{UE} * i_{UE}(\mathrm{t}) + \beta_{EG} * i_{EG}(\mathrm{t}), \quad (5)$$

where β_M is the parameter for the difference between current reference market rates and policy crediting rates.

The parameter estimates are shown in Table 2. We can notice that the parameter β_{UE} for the unemployment rates is 24.3694 very smaller than that of the Korean IIA unemployment parameter, 50.6348. It means that the U.S. SPDA surrender rates change less sensitively according to the unemployment rate movements. It seems also reasonable that the parameter β_{EG} for the economy growth rates is a negative number, –2.6450.

For the U.S. SPDA, the surrender rates, $\{q_s(t)\}$, are estimated using the following formula,

$$q_s(t) = \frac{1}{1+\exp(-\alpha)}\ , \tag{6}$$

where:

$$\alpha = \beta_0 + \beta_M * (i_m(t) - i_c(t)) + \beta_{UE} * i_{UE}(t) + \beta_{EG} * i_{EG}(t). \tag{7}$$

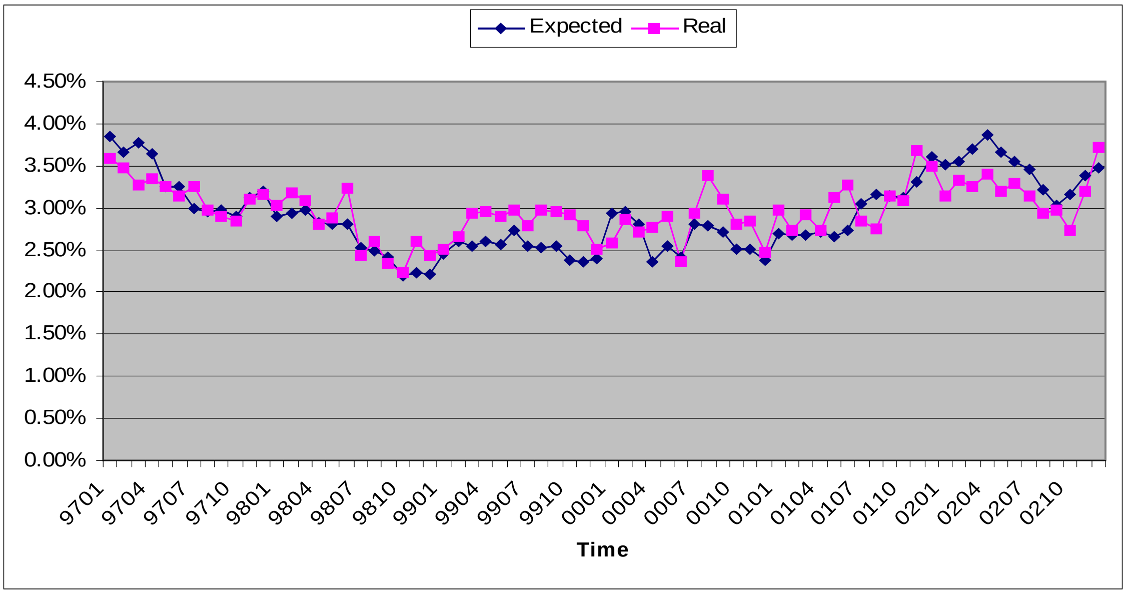
The graph of the real and predicted (using Logit Model) surrender rates of U.S. SPDA policies is shown below. The average of the real surrender rates is 2.97 percent and the average of the expected (predicted) surrender rates using Logit Model is 2.92 percent.

Table 2. Parameter Estimates with Logit Model (U.S. SPDA)

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-4.5452	0.00785	89325.4212	<.0001
DIFFLAG	1	12.7525	0.05831	25413.1981	<.0001
Unemploy	1	24.3694	0.27836	59821.6548	<.0001
Eco GROWTH	1	-2.6450	0.86473	4862.2485	<.0001

Figure 6. Real and Predicted Surrender Rates of U.S. SPDA



Surrender Rate Changes Under Financial Rate Shocks

There are many factors affecting surrender rates such as the difference between reference market rate and policy crediting rate, seasonal effect, age and gender of clients, economy growth rate, foreign exchange rate, inflation rate, policy age since issue date (duration) and unemployment rate, among others. During the stable interest rate period, all of these factors play an important role in determining the surrender rate. But sometimes, if there are any shocks (or sudden changes) on financial rates, such as the unemployment rates, the economy growth rates, or the market interest rates, the surrender rates can be changed much more than expected. For example, during the financial crisis in Korea from December 1997 to December 1998, the surrender rates show a sudden peak.

Figure 4 shows the sudden increase in the market interest rates during the financial crisis and the surrender rates of Korean IIA, and we can see that interest rate fluctuation is really an important factor in determining the surrender rates. Figure 3 shows the unemployment rates and the surrender rates of Korean IIA. We can easily see that the unemployment rates and surrender rates soared up during the financial crisis. Therefore, we can assume that the surrender rates are dependent not only on interest rates but also on exogenous factors such as unemployment rates, economy growth rates, seasonal effects, and so on.

Now we want to investigate the surrender rate changes under the assumption that there are financial rate shocks (or sudden changes). As an example, we first look at the pattern of the financial rate shocks during the Korean financial crisis**.**

Let us denote $i(t)$ to be a financial rate at time *t*. We use the following formula for the financial rate at time t,

$$i(t) = \mu + k(t)\,\sigma, \tag{8}$$

where μ is the average and σ is the standard deviation of the financial rate during a stable state period.

We define k(t) to be a risk measure of the financial rate $i(t)$,

$$k(t) = \frac{i(t) - \mu}{\sigma}. \tag{9}$$

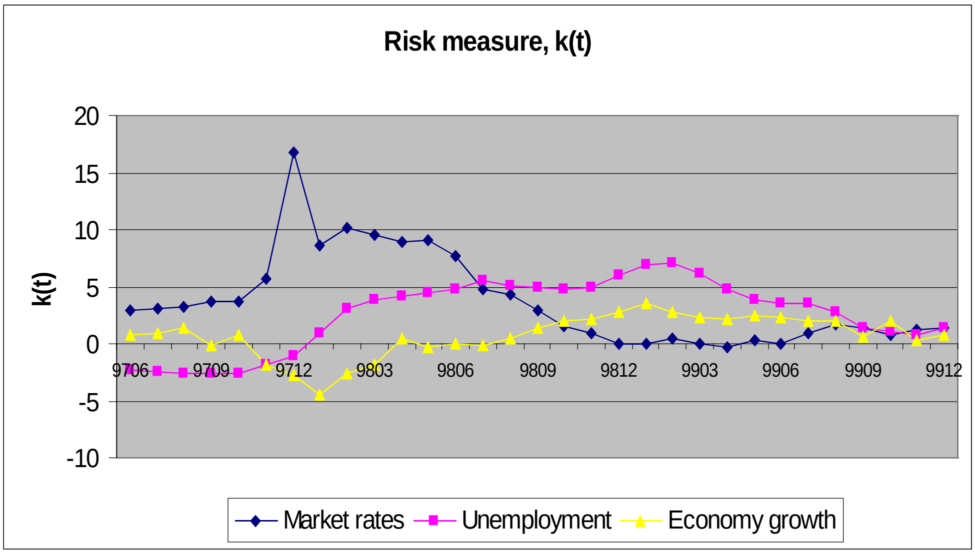
We define that the financial rate $i(t)$ experiences a financial rate shock at time t if

$$|k(t)| \geq 2, \tag{10}$$

and we say that the financial rate is in a stable status at time t if |k(t)| < 2. We also say that we are under extreme financial conditions if the financial rates experience financial rate shocks.

Figure 7 shows the risk measure k(t) of the reference market rates (five-year government bond rates), the unemployment rates and the economy growth rates of Korea around the financial crisis period.

Figure 7. Risk Measure, k(t), of Korean Financial Rates



Source: Market Rates; 5-year government bond rates; The Bank of Korea (www.ecos.bok.or.kr)

Unemployment Rate ; Korea National Statistical Office (www.nso.go.kr)

Economy Growth Rates ; Korean Statistical Information System (www.kosis.nso.go.kr)

From Figure 7, we can see that the market rates experience financial shocks, k(t) > 2, for the period from July of 1997 to September of 1998, for 14 months around the financial crisis. The unemployment rates experience financial shocks, k(t) $\geq$ 2, for the period from February of 1998 to August of 1999, for 19 months around the financial crisis. The economy growth rates experience financial shocks, k(t) $\leq$ -2, for the period from November of 1997 to March of 1998, for five months around the financial crisis.

Figure 5 shows the real and expected surrender rates (using Logit Model) of Korean IIA considering all of the financial rate shocks. The averages of the real and expected (predicted) surrender rates of Korean IIA are 4.2 percent.

Now, we want to consider the surrender rate changes of U.S. SPDA under the assumption that U.S. financial rates experience the financial rate shocks. We made two assumptions on the pattern of k(t), the risk measures of the financial rates. Assumption 1 (A1): the pattern of k(t) is same as that of Korean data when the rate experiences financial shocks. When the rate does not experience any financial shocks, the risk measure k(t) is calculated from U.S. data. Assumption 2 (A2): k(t) = c, where c is a constant integer such that |c| $\geq$ 2 and the financial rate $i(t)$ is changed to $i(t)+c\sigma$.

Figure 8 shows the surrender rate changes of U.S. SPDA under the assumption (A1) that the U.S. market rates (10 year T-bond rates) experience the financial rate shock, k(t) $\geq$

2, as the same pattern of k(t) as that of Korean data.

It shows a very high peak of 12.63 percent at the beginning of the market rate shock period. The average of the expected (predicted) surrender rates is 3.42 percent whereas the average of the real surrender rates is 2.97 percent.

Figure 8. U.S. SPDA Surrender Rate Changes under Market Rate Shock (A1)

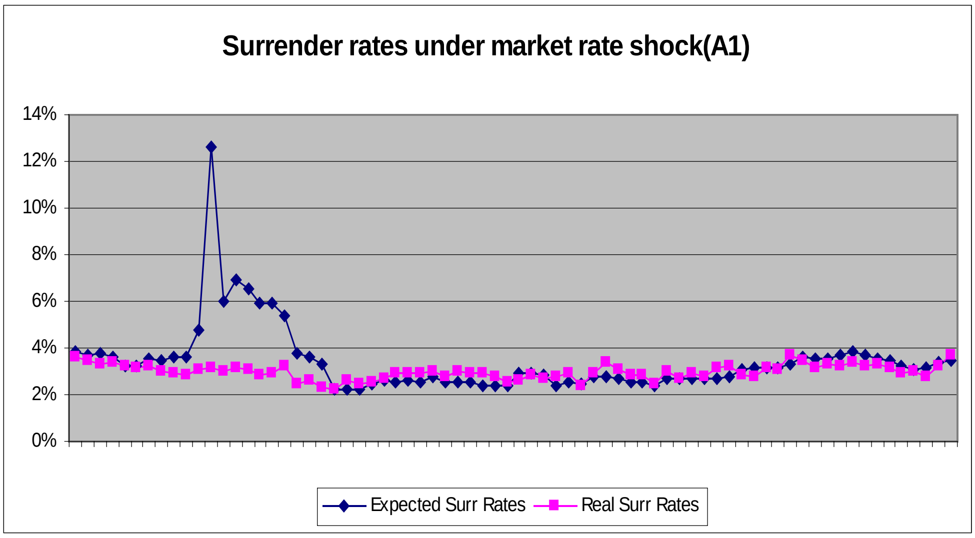


Figure 9. U.S. SPDA Surrender Rate Changes under Market Rate Shock (A2)

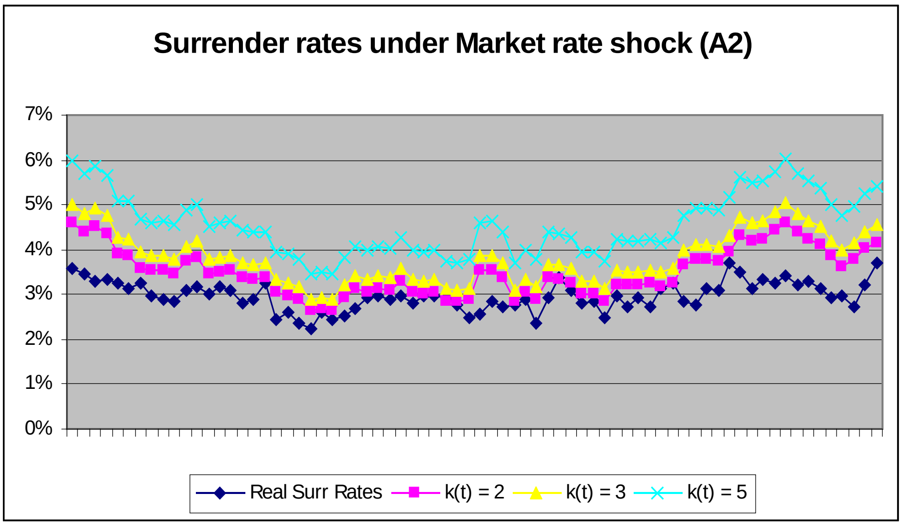


Figure 9 shows the surrender rate changes of U.S. SPDA under the assumption (A2) that the U.S. market rates (10-year T-bond rates) experience the financial rate shock, k(t) = 2, 3, 5 over the whole period.

It shows that the surrender rates are increasing as k(t) goes up, i.e., market rates increase. The average of the expected (predicted) surrender rates is 3.49 percent when k(t) = 2, 3.82 percent when k(t) = 3, and 4.56 percent when k(t) = 5, whereas the average of the real surrender rates is 2.97 percent.

Figure 10 shows the surrender rate changes of U.S. SPDA under the assumption (A1) that the U.S. unemployment rates experience the financial rate shock, k(t) $\geq$ 2, as the same pattern of k(t) as that of Korean data.

It shows a very high peak of 7.56 percent in the middle of the unemployment rate shock period. We also notice the interesting point that the unemployment rate shock period starts later than that of market rate shock, and lasts longer. The average of the expected (predicted) surrender rates is 3.42 percent, whereas the average of the real surrender rates is 2.97 percent.

Figure 10. U.S. SPDA Surrender Rate Changes Under Unemployment Rate Shock (A1)

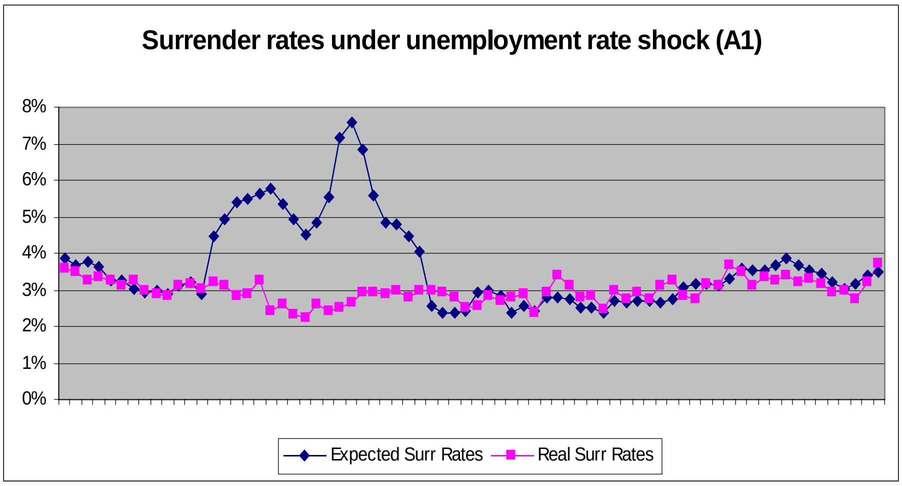


Figure 11 shows the surrender rate changes of U.S. SPDA under the assumption (A2) that the U.S. unemployment rates experience the financial rate shock, k(t) = 2, 3, 5 over the whole period.

It shows that the surrender rates are increasing as k(t) goes up, i.e., unemployment rates increase. The average of the expected (predicted) surrender rates is 3.94 percent when k(t) = 2, 4.57 percent when k(t) = 3, and 6.13 percent when k(t) = 5, whereas the average of the real surrender rates is 2.97 percent.

Figure 11. U.S. SPDA Surrender Rate Changes under Unemployment Rate Shock (A2)

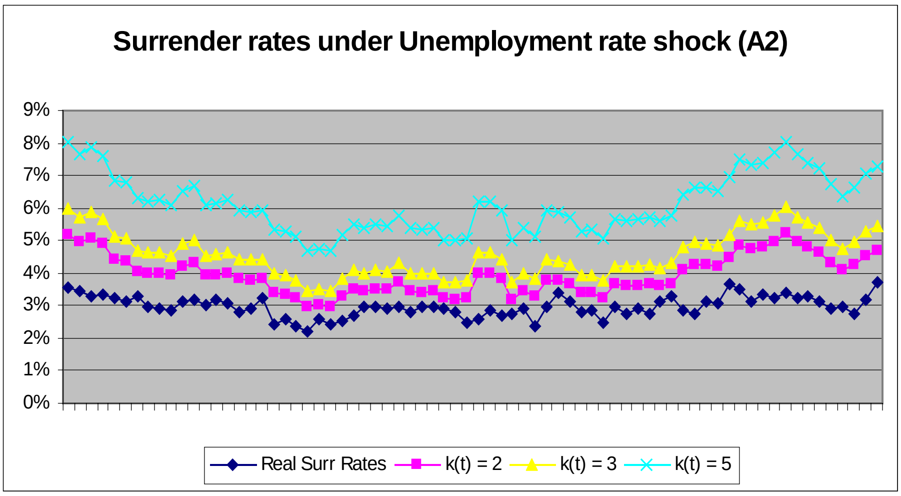


Figure 12 shows the surrender rate changes of U.S. SPDA under the assumption (A1) that the U.S. economy growth (GDP) rates experience the financial rate shock, k(t) $\leq$ -2 , as the same pattern of k(t) as that of Korean data.

It shows a small peak of 3.89 percent in the beginning of the shock period. We can see that the economy growth rate shock period lasts for short periods of five months and the impacts of the economy growth rate shock to surrender rates are relatively small. The average of the expected (predicted) surrender rates is 2.98 percent, whereas the average of the real surrender rates is 2.97 percent.

Figure 12. U.S. SPDA Surrender Rate Changes Under Economy Growth Rate Shock (A1)

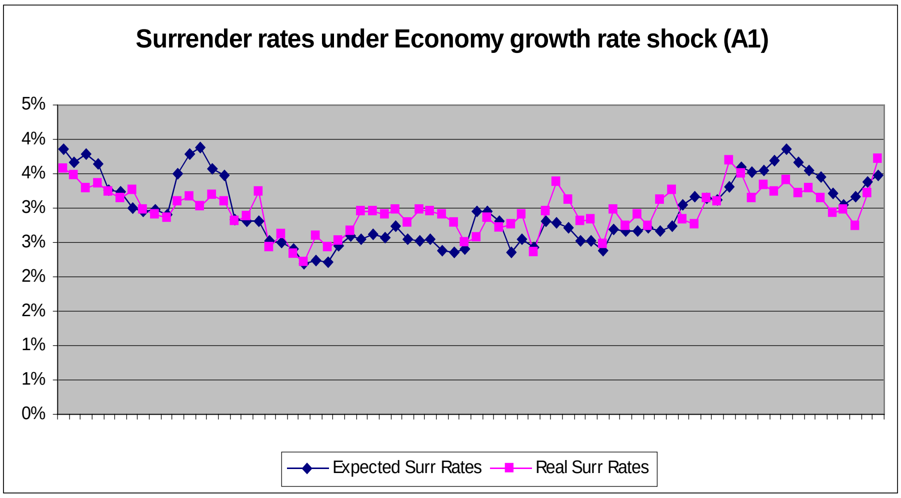


Figure 13 shows the surrender rate changes of U.S. SPDA under the assumption (A2) that the U.S. economy growth (GDP) rates experience the financial rate shock, k(t) = -2, -3, -5 over the whole period.

It shows that the surrender rates are increasing as k(t) goes down, i.e., the economy growth rates decrease. The average of the expected (predicted) surrender rates is 3.27 percent when k(t) = -2, 3.46 percent when k(t) = -3, and 3.87 percent when k(t) = -5, whereas the average of the real surrender rates is 2.97 percent.

Figure 13. U.S. SPDA Surrender Rate Changes Under Economy Growth Rate Shock (A2)

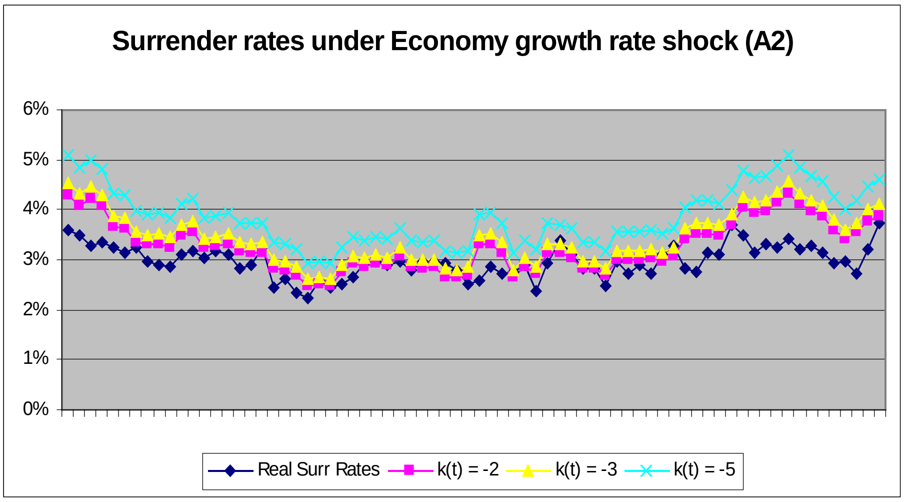


Figure 14. U.S. SPDA Surrender Rate Changes Under Total Rate Shock (A1)

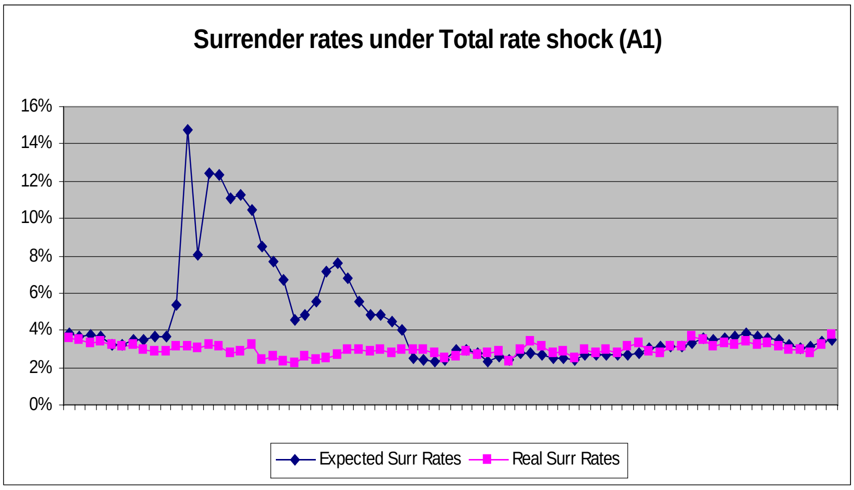


Figure 14 shows the surrender rate changes of U.S. SPDA under the assumption (A1) that the total three U.S. financial rates (market, unemployment and economy growth rates) experience the financial rate shock, $|k(t)| \geq 2$, at the same time, as the same pattern of k(t) as that of Korean data.

It shows a high peak of 14.71 percent at the beginning of the shock period. And the surrender rates are quite high with the average of 7.67 percent during the shock period for almost two years. The average of the expected (predicted) surrender rates is 4.48 percent, whereas the average of the real surrender rates is 2.97 percent.

Conclusion

Many insurance companies are selling single premium deferred annuities (SPDA). But SPDA are sold with the primary focus on accumulation. Only a few of the policyholders purchase SPDA for the purpose of annuitization. In Korea, the annuity market is still young and growing slowly compared to that of the United States (U.S.). Interest-indexed annuities (IIA) are one of the most popular SPDA products in Korea. The distinctive features of SPDA are the surrender options and annuitization options. In this paper we consider the surrender behaviors of SPDA /IIA policyholders under extreme economic conditions.

We have considered a model on the policyholder surrender behavior statistically. The variables considered are the difference between reference market rates and product crediting rates with surrender charges, the policy age since the contract was issued, unemployment rates, economy growth rates and seasonal effects. Especially the duration, i.e., the policy age since the contract was issued, is one of the most important factors of surrender rates. We use the Logit Model for the surrender rates.

For extreme events/financial rate shocks, we define k(t), a risk measure of a financial rate $i(t)$,

$$k(t) = \frac{i(t) - \mu}{\sigma}.$$

We define that the financial rate $i(t)$ experiences a financial rate shock at time t if

$$|k(t)| \geq 2,$$

and we say that the financial rate is in a stable status at time t if $|k(t)| < 2$. We also say that we are under extreme financial conditions if the financial rates experience financial rate shocks.

We considered the surrender rate changes of U.S. SPDA under the assumption that U.S. financial rates experience the financial rate shocks. We make two assumptions on the pattern of k(t), the risk measures of the financial rates. Assumption 1 (A1): the pattern of k(t) is the same as that of Korean data when the rate experiences financial shocks. When the rate does not experience any financial shocks, the risk measure k(t) is calculated from U.S. data. Assumption 2 (A2): k(t) = c, where c is a constant integer such that $|c| \geq 2$ and the financial rate $i(t)$ is changed to $i(t) + c\sigma$. We summarized the analysis results in the following table.

Table 3. Surrender Rate Changes under Extreme Conditions

	Market rates		Unemployment rates		Economy Growth rates		Total rates	
Assumption	max	average	max	average	max	average	max	average
A1	12.63%	3.42%	7.56%	3.66%	3.89%	2.96%	14.71%	4.48%
|k(t)| = 2	4.62%	3.49%	5.20%	3.94%	4.32%	3.27%	9.23%	4.37%
|k(t)| = 3	5.04%	3.82%	6.02%	4.57%	4.57%	3.46%	11.57%	6.24%
|k(t)| = 5	6.01%	4.56%	8.04%	6.13%	5.11%	3.87%	15.03%	10.85%

From Table 3, we can see that the surrender rates change very much under extreme conditions. We see a high peak of 14.71 percent when all of the three variables experience financial rate shocks under the assumption 1. And the surrender rates are quite high with the average of 7.67 percent during the shock period for almost two years. The average of the expected (predicted) surrender rates is 4.48 percent, whereas the average of the real surrender rates (without extreme condition assumptions) is 2.97 percent.

It may be a consideration in risk management of insurance business to predict sudden increase of surrender rates and prepare appropriate hedging strategies.

Appendix Modeling Surrender Rates of Korean Annuities

We wanted to show a method to model surrender rates with economic variables and durations (policy age since issue date) for Korean annuities. Therefore, we e showed how to choose the explanatory variables. We also showed how to compare the surrender rate models and choose a better model for Korean annuities. This method can be applied to other insurance policies.

Variables and Assumptions

Following is a summary of the explanatory variables and the assumptions used in modeling the surrender rates of Korean annuities.

Table A1. Explanatory Variables Considered

Variable	Contents	Memo
BASEYM	Year, Month of data	
DIFFLAG0	Difference of rates	=market rate-crediting rate at current time
DIFFLAG2	Difference of rates	=market rate-crediting rate 2 months ago
DIFFLAG4	〃	=market rate-crediting rate 4 months ago
DIFFLAG6	〃	=market rate-crediting rate 6 months ago
DIFFLAG8	〃	=market rate-crediting rate 8 months ago
DIFFLAG10	〃	=market rate-crediting rate 10 months ago
DIFFLAG12	〃	=market rate-crediting rate 12 months ago
POL-AGE	Policy age	Average policy age since issue
LOST	Unemployment rates	
GROWTH	Economy growth rates	
IMF	Financial crisis period under IMF control	Period from 1997.12 to 1998.12 Dummy variable = 1 during the period
MONTH1	January	Dummy variable = 1 on current month
MONTH2	February	〃
MONTH3	March	〃
MONTH4	April	〃
MONTH5	May	〃
MONTH6	Jun	〃
MONTH7	July	〃
MONTH8	August	〃
MONTH9	September	〃
MONTH10	October	〃
MONTH11	November	〃
SUR_RATE	Real surrender rate	Dependent variables

For seasonal effects, we investigated the surrender rates from January to November. We considered the financial crisis period since the surrender rates skyrocketed during this period. We used dummy variable 1 during the financial crisis period from Dec. 1997 to Dec. 1998 and 0 elsewhere.

The dependent variable SUR_RATE denotes the real surrender rates, and it is the face amount of surrendered policies divided by the face amount of initial policies. We consider Korean annuities with more than 1,000,000 policyholders.[5]

Surrender Rate Models

We used Logit link function and Complementary Log-Log (CLL) link function. As modeling programs, we use the Generalized Linear Models, Procedure GENMOD, Logistic Regression Models, and Procedure LOGISTIC, with SAS. The Logit function has the following form,

$$\ln\left(\frac{q_s}{1-q_s}\right) = \beta_0 + \beta_1 V_1 + \ldots + \beta_n V_n, \tag{A.1}$$

and the Complementary Log-Log (CLL) function is of the form,

$$\log(-\log(1-q_s)) = \beta_0 + \beta_1 V_1 + \ldots + \beta_n V_n, \tag{A.2}$$

where q_s is the surrender rate, β_i is the coefficient to be estimated and V_i is the explanatory variable.

Significance of Each Explanatory Variable

We checked the significance of each explanatory variable. There are many factors which affect the surrender rate fluctuations, such as the difference between reference market rates and crediting rates, policy age since issue, unemployment rates, economy growth rates, financial crisis and seasonal effects. According to each explanatory variable, we analyzed the significance as a whole with logistic regression analysis. The specific analysis for the variable selection and the reduced models will be done next.

In Table A2, (*) means the p-value for the test statistic χ^2 is less than 0.0001. Since the p-value is less than 1 percent or 5 percent, each variable has its own significance for surrender rates.

The difference between reference market rates and crediting rates are considered important for surrender rate modeling. We summarized the points below.

(i) The estimated parameters are all positive numbers. So the surrender rate goes up as the difference between the reference market rates and crediting rates becomes large.

(ii) From Table A2, we see that each interest rate difference variable has its own effects on surrender rates. Especially the difference of interest rates two months ago is really significant, noting the relatively large parameter estimate of 6.9440 (LOGIT) and 6.8517 (CLL).

Table A2. Significance of Explanatory Variables

Variables	LOGIT LINK FUNCTION			CLL LINK FUNCTION		
	Parameter	Std error	Chi-square	parameter	Std error	Chi-square
DIFFLAG0	5.6600	0.0037	2380693(*)	5.5888	0.0036	2391882(*)
DIFFLAG2	6.9440	0.0036	3693589(*)	6.8517	0.0036	3716484(*)
DIFFLAG4	6.6217	0.0037	3221988(*)	6.5291	0.0036	3237336(*)
DIFFLAG6	6.5901	0.0037	3121563(*)	6.4988	0.0037	3136179(*)
DIFFLAG8	5.7086	0.0038	2226700(*)	5.6308	0.0038	2234024(*)
DIFFLAG10	4.3584	0.0040	1205621(*)	4.2986	0.0039	1206910(*)
DIFFLAG12	3.0710	0.0041	552153(*)	3.0297	0.0041	551929(*)
POL-AGE	-0.1076	0.0001	3192912(*)	-0.1066	0.0001	3196557(*)
LOST	13.4398	0.0086	2438932(*)	13.3027	0.0085	2440284(*)
GROWTH	-5.9912	0.0139	186882(*)	-5.9436	0.0137	187356(*)
IMF	0.7662	0.0003	5112065(*)	0.7578	0.0003	5122041(*)
MONTH1	0.1296	0.0006	51236.9(*)	0.1283	0.0006	51292.5(*)
MONTH2	0.1193	0.0006	43069.9(*)	0.1181	0.0006	43112.8(*)
MONTH3	0.1235	0.0006	46335.6(*)	0.1223	0.0006	46383.5(*)
MONTH4	-0.0414	0.0006	4574.70(*)	-0.0410	0.0006	4573.22(*)
MONTH5	-0.0356	0.0006	3386.19(*)	-0.0352	0.0006	3385.25(*)
MONTH6	0.0108	0.0006	326.21(*)	0.0107	0.0006	326.24(*)
MONTH7	-0.0507	0.0006	6811.54(*)	-0.0503	0.0006	6808.86(*)
MONTH8	-0.0667	0.0006	11621.2(*)	-0.0661	0.0006	11615.2(*)
MONTH9	-0.0652	0.0006	11127.1(*)	-0.0646	0.0006	11121.5(*)
MONTH10	-0.1009	0.0006	25883.8(*)	-0.1000	0.0006	25863.9(*)
MONTH11	-0.1191	0.0006	35476.5(*)	-0.1180	0.0006	35444.7(*)

So we may assume that the two-month-ago interest rates strongly influencethe surrender behaviors of the policyholders. Also, the interest rate differences from two months ago to six months affect the surrender rate fluctuations. *So the policyholders observe the interest rate movements for two to six months and decide to surrender their policies.*

The estimated parameter for policy-age since issue is negative. So the surrender rates decrease as the policy age increases.

The positive parameter for unemployment rates indicates that surrender rates go up when the unemployment rates increase. It is natural and it is really significant to take the unemployment rates into account as an explanatory variable in modeling surrender rates considering the relatively high parameter estimate of 13.4398 (LOGIT) and 13.3027 (CLL).

The parameter for economy growth rates is negative and we may think that the surrender rates go down under good economy conditions.

The positive parameter for the dummy variable, financial crisis under IMF control, means that the surrender rates can increase when unexpected economy/finance shock happens.

It is interesting to note that the parameters for January, February, March, and June are positive and the others are negative, but all are small. Thus the season has a small effect on surrender behavior.

Reduced Models

We may not need all of the variables in modeling surrender rates, i.e., the full information model. In this section, we found appropriate reduced models with least number of explanatory variables for Korean annuities. We worked to keep the same fit with the reduced model as that of the full information model. We know that there are a few methods, such as forward selection method, backward elimination method, stepwise regression, and all possible regressions, to select the most significant variables.

We followed three steps to find the most appropriate reduced models. The first step was to select a few significant explanatory variables with the backward elimination method. The second step was to set up reduced models with the selected variables. The third step was to transform the policy age (or duration). The reason we transformed the policy age is that there is a possibility that the fit may become worse if we use the real policy age without transformation.

Also, we compared the three models, Arctangent Model, Logit Model and CLL model, and chose the most appropriate one for Korean annuities.[6]

Step 1. Selecting Explanatory Variables

We wanted to delete the variables one by one from the least significant one until we get a reduced model. It is a kind of backward elimination method.[7] As criterion for the selection of variables we use -2*Log Likelihood function (-2*Log L), Akaike information criterion (AIC), and Bayesian information criterion (BIC). We also showed Schwartz criterion (SC).[8]

Comparing the variables in Table A3, we rank the relative contributions of each variable to the model; *interest rate differences, policy age since issue date, unemployment rate (lost), financial crisis (IMF), seasonal effects and economy growth rates*.

And we noticed that the financial crisis (IMF) and economy growth rates have little influence on surrender rates. But unemployment rates are really affecting the surrender rate behaviors of the policyholders.

The numbers in Table A3 show the increased model fit statistics (AIC, BIC, -2*Log L) as we delete the variables one by one in each step. The increased amounts indicate the relative significance. That is, the deleted variables make contributions to the fit of the reduced model compared with the full information model as much as the changed amount does.

For seasonal effects and interest difference effects, we averaged the increased amount divided by the number of variables. Figure A1 shows the decreased fit according to the reduced model steps. After reducing step three, we can notice that the fit is reduced significantly. So we stop at step three and decide to delete the first three variables that have less significant contributions.

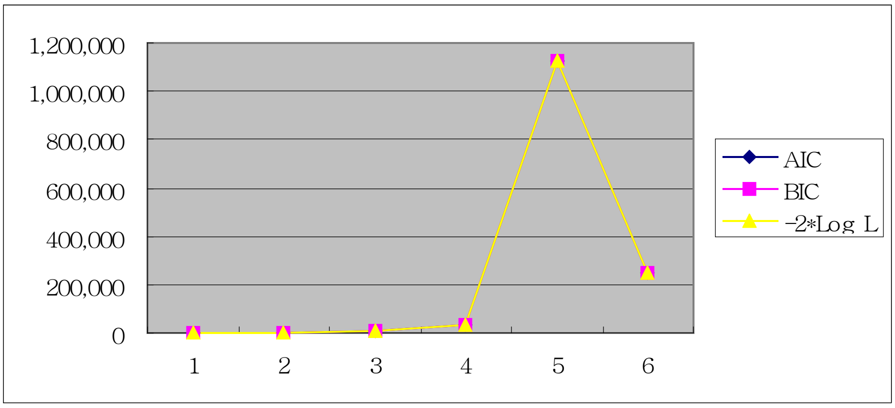
We discovered that the interest rate differences and policy age since issue are the most important factors and the unemployment rates are also important in modeling surrender rates. Modeling with these three explanatory variables, we have p-value less than 0.0001 and we conclude that it is reasonable to estimate the three parameters.

Table A3. Model Fit Statistics Changed

Reducing	Reduced	AIC	BIC	-2*Log L

step	variable			
1	Economy growth	45	27	47
2	Seasonal	2,039	2,021	2,041
3	IMF	9,015	8,998	9,017
4	Unemployment	29,162	29,144	29,164
5	Policy age	1,127,643	1,127,625	1,127,645
6	Interest diff.	251,787	251,769	251,789

Figure A1. Model Fit Statistics Changed



For Korean annuity, we selected policy age, interest rate differences and unemployment rates as the explanatory variables.

Step 2. Reduced Models

The second step included set up of reduced models with the selected variables from step 1. We present three tables. The first and second tables show the estimated parameters for the selected variables from Logit and CLL model. The third table shows the estimated errors for the three models, Arctangent Model, Logit Model and CLL Model, and also compares the models by the differences of the estimated errors between Arctangent Model, Logit Model, Arctangent Model and CLL Model according to the policy age since issue[9.] For comparison purposes, we define RMSE and MAPE as follows,

$$\text{RMSE} = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n}}, \qquad \text{(A.3)}$$

and

$$\text{MAPE} = \frac{1}{n}\sum\frac{|y_i - \hat{y}_i|}{y_i}, \qquad \text{(A.4)}$$

where, y_i is the i-th real value, $\hat{y}_i$ is the i-th predicted value, and n is the sample size.

We defined the terminologies used in the third table as follows. RMSE1 is RMSE of Arctangent Model, RMSE2 is RMSE of Logit Model, and RMSE3 is RMSE of CLL Model. MAPE1, MAPE2 and MAPE3 represent the MAPE of Arctangent Model, Logit Model and CLL Model respectively.

RMSEGAP1 denotes RMSE1-RMSE2, so Logit Model is better than Arctangent Model if RMSEGAP1 is positive. RMSEGAP2 is RMSE1-RMSE3, so CLL Model is better than Arctangent Model if RMSEGAP2 is positive. MAPEGAP1 is MAPE1-MAPE2 and Logit Model is better than Arctangent Model if MAPEGAP1 is positive. MAPEGAP2 is MAPE1-MAPE3 and CLL Model is better than Arctangent Model if MAPEGAP2 is positive.

We have shown the parameter estimates from Logit Model and CLL Model for Korean annuities below. We also showed the estimated errors and comparison of models in the following tables.

Table A4. Parameter Estimates with Logit Model

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-4.0378	0.00103	15373699.0	<.0001
DIFFLAG0	1	5.0670	0.0151	113287.687	<.0001
DIFFLAG2	1	2.7905	0.0147	36030.6698	<.0001
DIFFLAG4	1	-0.2736	0.0133	425.5740	<.0001
DIFFLAG6	1	2.3560	0.0131	32358.8416	<.0001
DIFFLAG8	1	2.5700	0.0131	38450.9595	<.0001
DIFFLAG10	1	1.7711	0.0136	17042.5285	<.0001
DIFFLAG12	1	2.9900	0.0119	63226.0859	<.0001
POLICY AGE	1	-0.1369	0.000132	1078276.82	<.0001

Table A5. Parameter Estimates with CLL Model

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-4.0471	0.00102	15755515.4	<.0001
DIFFLAG0	1	4.9841	0.0147	114220.414	<.0001
DIFFLAG2	1	2.7394	0.0144	36268.0157	<.0001
DIFFLAG4	1	-0.2493	0.0130	367.7235	<.0001
DIFFLAG6	1	2.3170	0.0128	32612.5292	<.0001
DIFFLAG8	1	2.5327	0.0128	38929.2327	<.0001
DIFFLAG10	1	1.7545	0.0133	17367.0213	<.0001
DIFFLAG12	1	2.9481	0.0117	63544.4834	<.0001
POLICY AGE	1	-0.1351	0.000130	1081687.16	<.0001

Table A6. Estimated Errors and Comparison of Models

time	RMSE1	RMSE2	RMSE3	MAPE1	MAPE2	MAPE3	RMSEGAP1	RMSEGAP2	MAPEGAP1	MAPEGAP2
0.5	0.02707	0.02083	0.02078	0.3268	0.29949	0.29887	0.00624	0.00629	0.02731	0.02792
1.5	0.00773	0.00858	0.00861	0.19563	0.22432	0.22502	-0.00084	-0.00088	-0.02869	-0.02939
2.5	0.00569	0.01358	0.01357	0.24269	0.95731	0.95601	-0.00789	-0.00788	-0.71462	-0.71332
3.5	0.00607	0.00903	0.00901	0.30749	0.76591	0.76488	-0.00296	-0.00294	-0.45841	-0.45739
4.5	0.007	0.00477	0.00475	0.24598	0.32145	0.32079	0.00223	0.00225	-0.07547	-0.07481
5.5	0.00734	0.00297	0.00301	0.20046	0.09406	0.09451	0.00437	0.00433	0.1064	0.10595
6.5	0.00869	0.00299	0.00304	0.7049	0.1216	0.1229	0.00569	0.00565	0.5833	0.582
7.5	0.00849	0.00386	0.0039	0.34024	0.20862	0.21083	0.00463	0.00459	0.13162	0.12941
8.5	0.00842	0.0039	0.00393	0.31219	0.20492	0.20738	0.00452	0.00449	0.10727	0.10481
9.5	0.00817	0.00452	0.00454	0.30694	0.20528	0.20597	0.00365	0.00363	0.10166	0.10097

For an annuity plan, we may not be able to conclude that Logit or CLL Model is better than Arctangent Model. Even when we added unemployment rates and IMF effects to the Logit and CLL Models, we did not have enough evidence that one model was better than the other ones. Also the sign of DIFFLAG4 is negative and it seems to be unexplainable.

Step 3. Transformation of Duration

The third step is to transform the policy age (duration) since issue. The reason we transformed the policy age is that the surrender rates are dependent on durations and there is the possibility that the fit may be decreased if we use the real policy age without transformation. We tried three formulas that are usually used in transformation[10,]

$$\sqrt[n]{x}\,,\ \log x\,,\ \text{and}\ -\frac{1}{x}\,. \qquad \text{(A.5)}$$

The policy age may be transformed to

$$(policy\ age)^{\frac{1}{n}}\,,\ \log(\text{policy age}),\ \text{or}\ -\frac{1}{policy\ age} \qquad \text{(A.6)}$$

We chose the best transformation formula using the model fit statistics –2Log L. We compared Arctangent Model and Logit Model and Arctangent Model and CLL Model, and concluded which model is the best one.

Table A7. Model Fit Statistics According to Transformed Policy Age

$(policy\ age)^{\frac{1}{n}}$	-2*Log L
n=1	72208277
n=2	71946654
n=3	71863316
n=4	71825035
n=5	71803493
n=6	71789797
n=7	71780359
n=8	71773477
n=9	71768243
n=10	71764133

Formula	-2*Log L
Log(policy age)	71730319
-1/(policy age)	71680566

Figure A2. Model Fit Statistics According to Transformed Policy Age

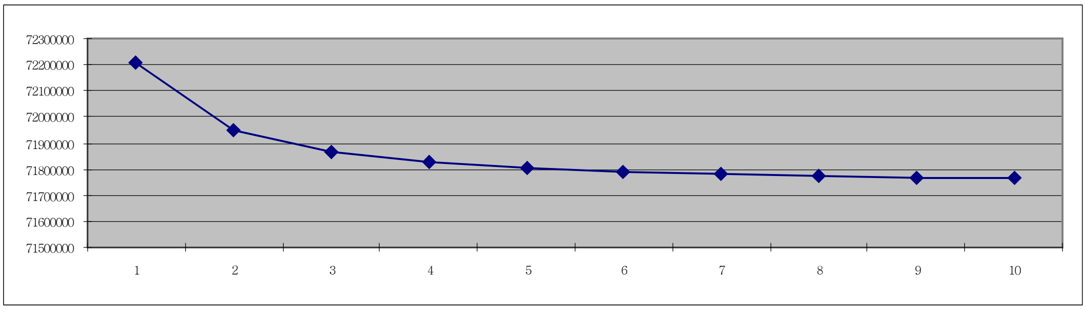


Table A7 and Figure A2 show the model fit statistics (-2*Log L) according to the policy age. We noticed that the model fits well when we transform the policy age. Comparing the model fit statistics, we concluded that the best transformation formula is

$$-\frac{1}{policy\,age}\ .$$

We have shown the analysis results in the following tables.

Table A8. Parameter Estimates with Logit Model under Transformation

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-5.4203	0.00255	4506516.79	<.0001
DIFFLAG0	1	6.0594	0.0163	137622.131	<.0001
DIFFLAG2	1	1.9039	0.0157	14768.9749	<.0001
DIFFLAG4	1	-1.0608	0.0141	5678.8636	<.0001
DIFFLAG6	1	1.6583	0.0137	14577.7492	<.0001
DIFFLAG8	1	2.3668	0.0131	32620.9627	<.0001
DIFFLAG10	1	1.4915	0.0137	11839.3940	<.0001
DIFFLAG12	1	0.7086	0.0181	1528.7608	<.0001
POLICY AGE	1	-0.6715	0.000478	1971550.62	<.0001
LOST	1	10.5238	0.0617	29044.8354	<.0001

Table A9. Parameter Estimates with CLL Model under Transformation

Analysis of Maximum Likelihood Estimates

Parameter	DF	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	1	-5.4089	0.00252	4597008.35	<.0001
DIFFLAG0	1	5.9574	0.0160	138442.495	<.0001
DIFFLAG2	1	1.8506	0.0153	14580.5856	<.0001
DIFFLAG4	1	-1.0303	0.0138	5583.9827	<.0001

DIFFLAG6	1	1.6172	0.0134	14475.6309	<.0001
DIFFLAG8	1	2.3250	0.0128	32949.0219	<.0001
DIFFLAG10	1	1.4808	0.0134	12166.9626	<.0001
DIFFLAG12	1	0.6886	0.0178	1491.4247	<.0001
POLICY AGE	1	-0.6561	0.000465	1991007.04	<.0001
LOST	1	10.4363	0.0610	29316.3498	<.0001

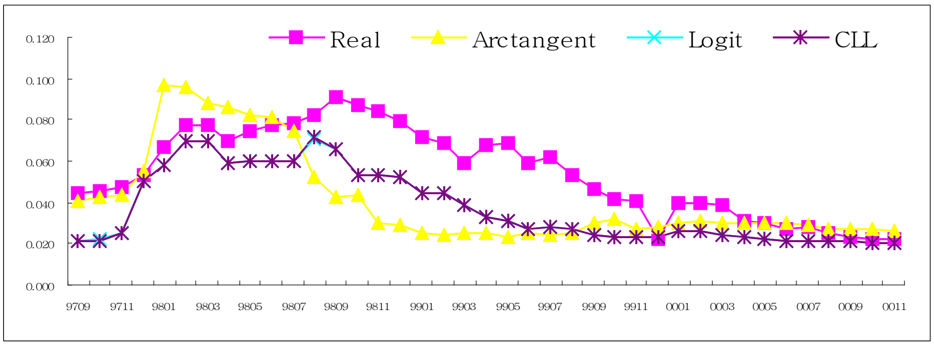
Table A10. Errors and Comparison of Models under Transformation

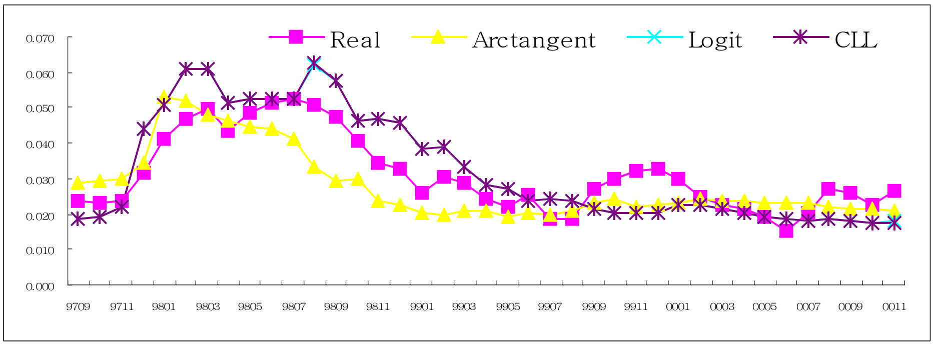
time	RMSE1	RMSE2	RMSE3	MAPE1	MAPE2	MAPE3	RMSEGAP1	RMSEGAP2	MAPEGAP1	MAPEGAP2
0.5	0.02707	0.01256	0.01274	0.3268	0.19366	0.19315	0.01451	0.01433	0.13314	0.13364
1.5	0.00773	0.00951	0.00954	0.19563	0.26445	0.26563	-0.00177	-0.00181	-0.06882	-0.06999
2.5	0.00569	0.00474	0.00474	0.24269	0.36096	0.36226	0.00095	0.00096	-0.11827	-0.11957
3.5	0.00607	0.00364	0.00368	0.30749	0.34218	0.34641	0.00243	0.00239	-0.03468	-0.03892
4.5	0.007	0.00258	0.00263	0.24598	0.13063	0.13339	0.00443	0.00437	0.11535	0.11259
5.5	0.00734	0.00463	0.00465	0.20046	0.16366	0.16235	0.00272	0.0027	0.0368	0.0381
6.5	0.00869	0.0029	0.00293	0.7049	0.11537	0.11695	0.00579	0.00576	0.58953	0.58795
7.5	0.00849	0.00281	0.00284	0.34024	0.24663	0.25122	0.00568	0.00565	0.09361	0.08902
8.5	0.00842	0.00305	0.00307	0.31219	0.30407	0.30834	0.00537	0.00535	0.00812	0.00385
9.5	0.00817	0.00351	0.00352	0.30694	0.29396	0.29819	0.00466	0.00465	0.01297	0.00875

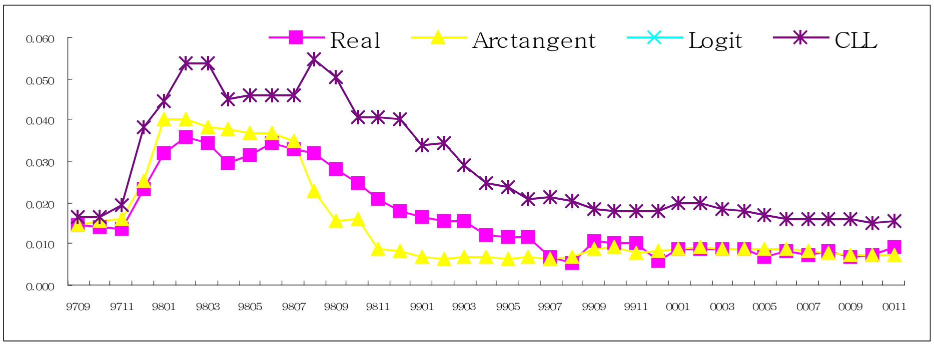
When we did not transform the policy ages, we did not have enough evidence to suggest that one model was better than the other ones. *But Logit and CLL Models are better than the Arctangent Model on many policy ages after we transform the policy ages.*

In conclusion, we show the 10 graphs of real and estimated surrender rates for each model according to the policy age from duration one to duration 10 for Korean annuity. Note that the Logit Model and the CLL Model produce almost the same results and the two graphs are overlapping.

Figure A3. Surrender Rates According to the Policy Age (Duration)







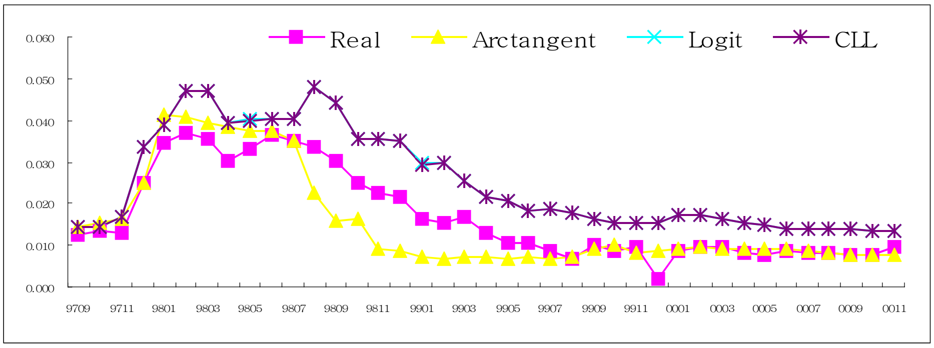
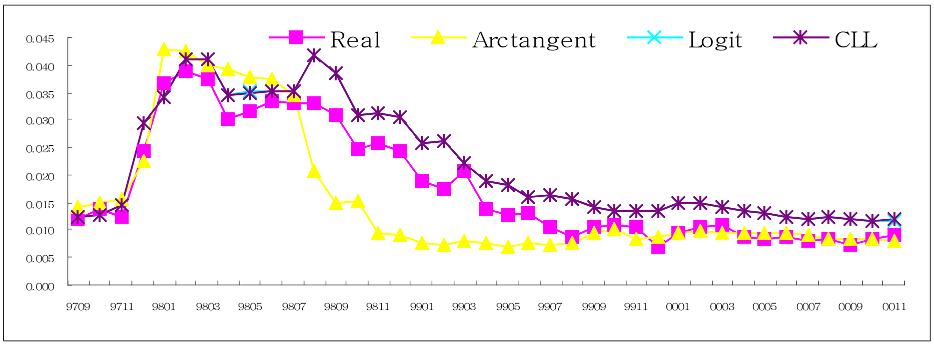
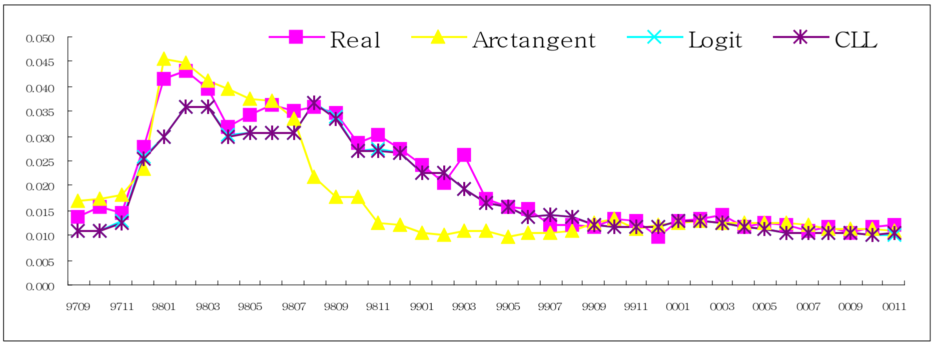
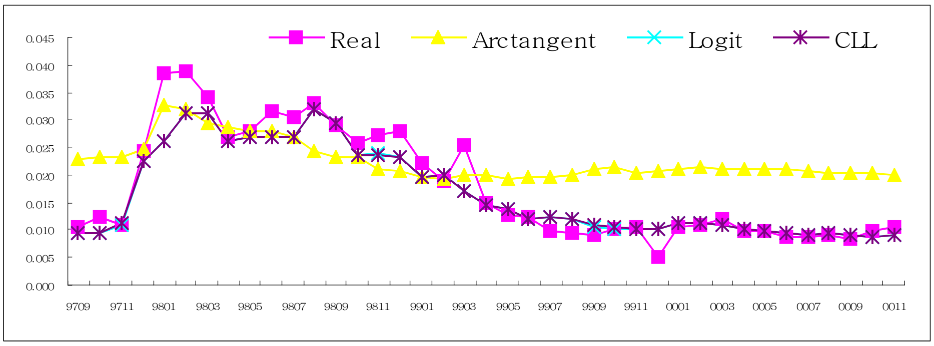


Figure A3. Continued







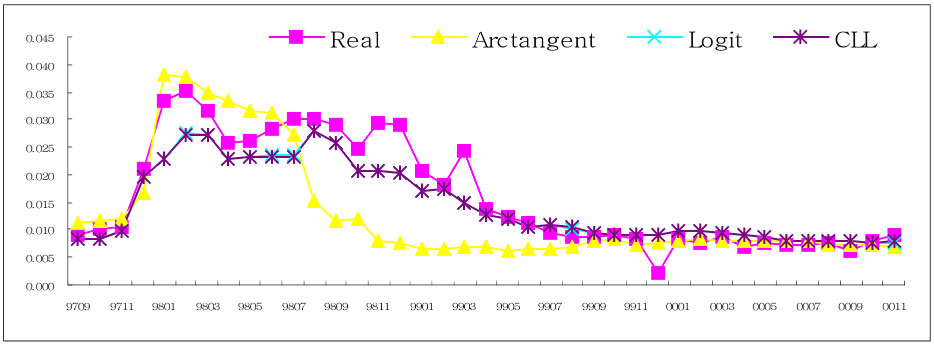
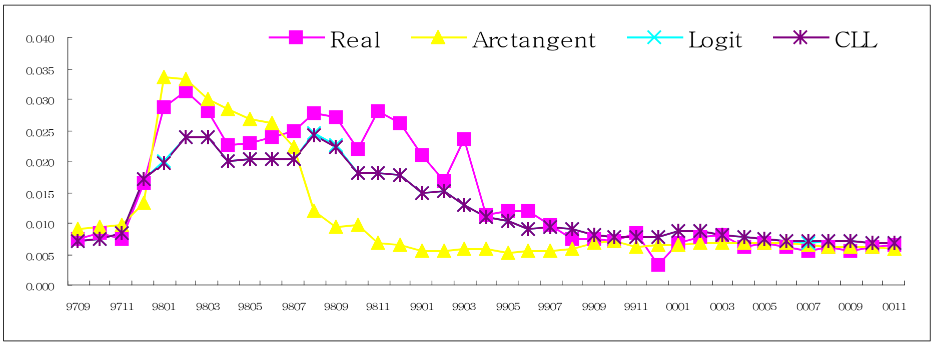
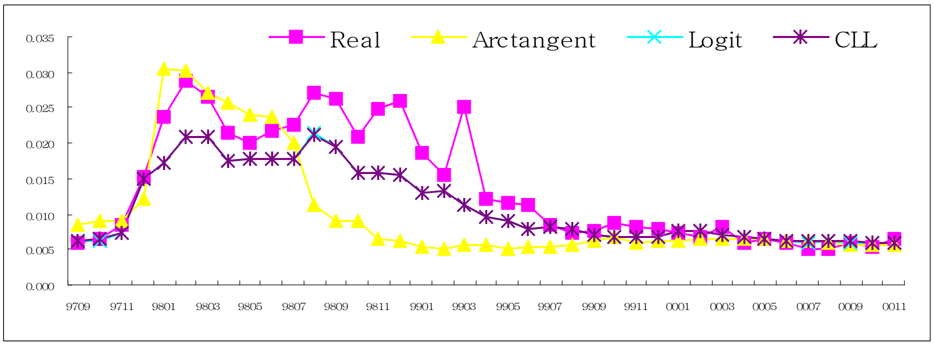


Figure A3. Continued





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