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A Long-Term-Care Status Transition Model

by Jim Robinson

Introduction

Undeniable trends in the age composition of the population present significant challenges and opportunities to the actuarial profession. In an effort to manage the increasing demand for health care services, the public and private sectors are exploring new health care financing and delivery schemes such as managed care programs, capitated reimbursement, and private long-term care (LTC) insurance. Effective actuarial analysis of such programs requires an understanding of the migration of individuals between health statuses that affect health care utilization and costs. This paper presents a health status transition model developed as part of a larger LTC insurance pricing model. The model is fitted to longitudinal data on Medicare enrollee functional and cognitive impairments obtained from the 1982/84/89 National LTC Surveys. The paper discusses model features, estimation, and applications.

Status Transition Models: General

For this paper I limit the discussion to processes in which an individual is in one of a finite number of discrete statuses at any point in time. For most applications, the state space of the underlying process can be adequately approximated by a limited number of statuses.

A few of the status transition models available from the literature are described below.

Markov Chains

In this context, a Markov chain might be used to describe the evolution of an individual's health status over uniform increments in time, say, month-by-month. Such models assume that the probability of moving to status j in the next month depends only on the current status, ignoring the sequence of past statuses leading to the current status. Although this seems to be a very restrictive assumption, some imagination in the construction of status definitions combined with variation in the status transition probabilities over time allows such models to mimic the characteristics of many real processes. A wealth of analysis is available on the properties of Markov chains. See chapter 7 of Heyman and Sobel (1982) for an introduction or part 1 of Chung (1967) for a more exhaustive treatment.

Continuous-Time Markov Chains (CTMCs)

Such models allow for status jumps at any time, with the destination status determined by a Markov chain, and are described in terms of the instantaneous rates of transition at a given time from the current status to each of the possible destination statuses. The force of mortality from life contingencies is an example of such a transition rate in a model having only two statuses, living and dead. A common special case is the time-homogeneous CTMC in which the transition rates are assumed to be constant over time. Time-homogeneous CTMCs exhibit exponential interjump intervals. Chapter 8 of Heyman and Sobel (1982) and part 2 of Chung (1967) serve as useful references.

Semi-Markov Processes

These models are extensions of CTMCs that allow the instantaneous transition rates to vary with current status *and* duration in current status. The unconstrained CTMC and the semi-Markov process both allow for transition rates that vary over time. The CTMC indexes time relative to a fixed calendar date or a fixed initial age, whereas the semi-Markov process reinitializes time at each status change. Compared to the time-homogeneous CTMC, the semi-Markov process allows for interjump intervals with nonexponential distributions. Chapter 9 of Heyman and Sobel (1982) is an introduction to these processes.

Further extensions of these models are possible by allowing the transition rates to depend upon additional aspects of the individual's status history beyond the current status and the duration in the current status. However, such extensions present two challenges. First, the derivation and computation of process probabilities from transition rates become more complicated. Second, the number of model parameters required to represent the process greatly increases, making estimation more difficult.

LTC Status Transition Model

The LTC transition model in this paper is a CTMC that allows the transition rates to vary with the sex and attained age of the individual. This structure is selected to balance model complexity and flexibility with consideration for the nature of the available data. Extensions to the model would produce parameter identifiability issues. To simplify the analysis further, it is assumed that the model transition rates are constant over short age intervals of that they can be adequately approximated by constant rates over short age intervals.

I adopt the following notation to describe the model characteristics:

$r_{ij}(s, x)$ = annual rate of transition from status i to status j for an individual aged x of sex s. (Note: $s = 0$ for males and $s = 1$ for females.)

$p_{ij}(s, x, y)$ = probability that an individual of sex s is in status j at age y given status i at age x.

If we know the transition rates, $r(.)$, we can find the transition probabilities, $p(.)$, as solutions to the following system of differential equations:

$$\frac{dp_{ij}(s, x, y)}{dy} = \Sigma_k\, p_{ik}(s, x, y)\, r_{kj}(s, y)$$

for $y > x$, with initial conditions,

$$p_{ij}(s, x, y) = 0 \quad \text{for } i \neq j \text{ and } 1 \text{ for } i = j,$$

$$r_{jj}(s, y) = -\Sigma_{m \neq j}\, r_{jm}(s, y).$$

These equations are more conveniently expressed in matrix notation:

$$\frac{dP(s, x, y)}{dy} = P(s, x, y)\, R(s, y) \quad \text{for } y > x,$$

$$P(s, x, x) = 1.$$

The third condition sets the diagonal element of $R(.)$ so that each row sums to zero.

If we temporarily treat $P(.)$ and $R(.)$ as scalar functions of y, we might speculate that a solution to the system would be.

$$P(s, x, y) = \exp\left[\int_x^y R(s, z)\, dz\right]. \qquad (*)$$

This represents a valid solution under the following conditions:

1. The integral of a matrix is defined as the matrix of the integrals of each element.
2. Exp(A) for a square matrix A is defined to be the matrix summation, $\Sigma_k A^k/k!$, where the sum over k runs from zero to infinity
3. $R(s, z_0)\, R(s, z_1) = R(s, z_1)\, R(s, z_0)$ for all z_0 and z_1 in the interval $[x, y]$.

Condition (3) is satisfied when $R(s, z)$ is constant on $[x, y]$, that is, the process is time homogeneous on the interval. In this case, the transition probability in (*) simplifies to

$$P(s, x, y) = \exp\left[(y - x)\, R(s, x)\right] = \Sigma_k\, (y - x)^k \frac{R(s, x)^k}{k!}.$$

Probabilities extending over longer age intervals can be approximated by assuming constant transition rates within subintervals; that is,

$$P(s, x, y_n) = P(s, x, y_1)\, P(s, y_1, y_2) \ldots P(s, y_{n-1}, y_n),$$

where $R(s, z)$ is constant for z within each interval $[x, y_1]$, $(y_1, y_2), \ldots, (y_{n-1}, y_n)$.

Data: the National LTC Surveys

The 1982/84/89 National LTC Surveys (NLTCSs) consist of an ongoing longitudinal study of elderly

Medicare enrollee levels of disability and care utilization (Rudberg [1996] analyzes transition rates using another longitudinal survey, the Longitudinal Study of Aging). The initial sample in 1982 screened over 34,000 enrollees. Of these, approximately 26,000 were unimpaired, 2,000 were institutionalized, and 6,000 were chronically disabled community residents. The disabled community residents were interviewed in greater detail about their medical, functional, and cognitive health statuses as well as their family and economic situations. The 1984 interviews revisited all enrollees given detailed interviews in 1982, all enrollees institutionalized in 1982, a sample of 12,000 unimpaired enrollees in 1982, and a sample of 5,000 enrollees new to Medicare since 1982. Those found to be chronically disabled in 1984 were interviewed in greater detail, including those in institutions. A similar follow-up survey was conducted in 1989. (Data tapes containing questionnaire responses for the surveys are available from the Inter-University Consortium for Political and Social Research at the University of Michigan.)

The detailed questionnaires included several questions on the enrollee's functional and cognitive health status. Questions relating to difficulties with "activities of daily living" (ADLs), such as eating, bathing, dressing, toileting, transferring, and continence, were part of the functional assessment. Additional questions on "instrumental activities of daily living" (IADLs), such as shopping and food preparation, were also included. Enrollees were scored on cognitive skills using the "Short Portable Mental Status Questionnaire" (SPMSQ), a sequence of ten questions included in the detail survey form. For the purpose of fitting the LTC transition model, these following questions were summarized for each respondent into the following health statuses:

Status Code	Description
1	Well (no impairments)
2	IADL only; no ADL/cognitive impairments
3	1 ADL impaired; no cognitive impairment
4	2 ADLs impaired; no cognitive impairment
5	3+ ADLs impaired; no cognitive impairment
6	< 2 ADLs impaired; cognitive impairment
7	2+ ADLs impaired; cognitive impairment
8	Dead

Respondents were considered to be impaired in an ADL if they were unable to perform the activity without continuous human assistance. Respondents were classified as cognitively impaired if they had five or more incorrect answers on the SPMSQ or if they were unable to participate in the interview and were described as senile by the proxy.

A typical LTC insurance policy requires the insured to be in statuses 5–7 to be eligible for health care benefits. (Some policies may provide benefits to those in status 4 as well.) Consequently, LTC insurance pricing actuaries should be very interested in the movement of insureds between these health statuses.

Respondents have been grouped by sex (male and female), health status (status 1 to status 7), and age group (65–74, 75–84, and 85+) at the start of each of the two observation periods (1982–84 and 1984–89). Twelve observed transition matrices summarize the distribution of ending health statuses for each such cohort. Each matrix corresponds to a sex/age-group/observation-period combination. The rows and columns of each matrix relate to the starting and ending health status, respectively, of each individual in the cohort.

Each row of the observed transition matrices can be thought of as an independent trial from a multinomial process, $M(n, p)$, with n equal to the sum of the values in the row and p equal to a vector of two- or five-year health status transition probabilities, as appropriate.

Table 1 shows the contents of one of the 12 observed transition matrices, females aged 75 to 84 in 1982, observed from 1982 to 1984.

The diagonal shows the percentage of enrollees with unchanged health status. The last column displays the percent of each group dying during the two-year period. Cells above the diagonal generally relate to increased impairment; cells below the diagonal relate to recovery from impairment.

The NLTCS provides information on health status migration over two and five years. I adopt the LTC transition model in an effort to "fill in the gaps" between the beginning and end of these two observation periods. Just as I might adopt the constant-force-of-mortality assumption to interpolate to fractional ages in a life table, I assume constant-health-status transition rates over short age intervals to interpolate the data from the NLTCS to shorter transition periods.

TABLE 1
NLTCS OBSERVED HEALTH STATUS TRANSITION MATRIX
FEMALES AGED 75–84, 1982–84
(AVERAGE AGE: 78.7)

	1984 Health Status								
1982 Health Status	Count	1	2	3	4	5	6	7	8
Well	2,122	77.4%	11.0%	1.4%	0.4%	1.2%	2.2%	0.7%	5.7%
IADL Only	804	8.5	54.7	7.1	2.1	3.2	6.5	2.2	15.7
1 ADL	303	6.8	43.3	11.1	2.1	5.1	6.2	4.6	20.8
2 ADLs	72	0.0	22.4	13.3	8.2	17.4	3.1	3.9	31.7
3+ ADLs	53	0.0	11.0	4.0	11.1	24.4	2.0	15.0	32.5
< 2 ADLs, CI	168	2.7	19.1	3.9	4.0	11.0	28.2	12.8	18.3
2+ ADLs, CI	85	0.9	1.5	2.5	0.0	21.2	3.3	38.0	32.6

Model Parameter Estimation

In the analysis of mortality under the constant-force-of-mortality assumption, we can always extract a unique annual force of mortality that relates a starting population to the surviving population, so long as some survivors remain. One simply negates the log of the observed survival rate and divides by the length of the observation period in years. Unfortunately, the matrix counterpart of the log in the multistatus LTC analysis might not exit as a valid transition rate matrix (nonnegative off-diagonal elements and zero row sums), and, if it does exist, it may not be unique. Singer and Spilerman (1976) provide an excellent treatment of the problems encountered when we attempt to solve the equation $P = e^{(y-x)R}$, for R in terms of P. The following comments relate to this inversion problem:

1. The natural extension of the log function to matrix arguments is to use the Taylor expansion (about the identity) as we did to define the exponential function of a matrix. Just as with the scalar log series expansion, if the matrix log argument is too far removed from the expansion point, the series will not converge. And, just as with the scalar log function, this does not imply that the matrix log cannot be well-defined. Singer and Spilerman (1976, pp. 21–22) provide a detailed description of the construction of the matrix log function.
2. The log of a matrix is not single-valued. Some, none, or all of the "branches" of the log function might yield valid transition rate matrices. It is possible for more than one branch to produce a valid transition rate matrix.
3. Small changes in P can generate large changes in the corresponding value(s) of R. Thus, sampling error in estimating P can make estimation of R very unreliable.
4. As $y-x$ increases (that is, the length of the observation period) the number of alternate values of valid R matrices leading to the same P matrix increases. In some cases, the number of R matrices leading to the same P matrix may become infinite for observation periods beyond a certain threshold.

Therefore, estimation of annual transition rates using the formula $R=\log(P)/(y-x)$, where P is replaced with the observed transition probabilities, is more challenging than the simple form of the expression might suggest. Even when the true value of P leads to a valid R matrix, when the rows of P are estimated from a small number of observed individuals, the resulting sampling error might prevent extraction of reasonable estimates of R. Consequently, I do not attempt to estimate separately the annual transition rates for each of the 12 observed transition matrices. Rather, I employ maximum likelihood estimation to fit a parametric model of the annual transition rates as a function of the sex, age, starting status, and ending status of the individual. The parametric structure is guided by step-by-step inspection of residuals, that is, the observed minus the fitted transition probabilities across all 12 observation matrices. After some trial and error, the following parametric function captures most of the key characteristics of the data:

$$r_{ij}(s, x) = \exp\left[a_{ij} + b_{ij}(s - 0.5) + \frac{c_{ij}(x - 80)}{100}\right] \quad \text{for } i < 8 \text{ and } i \neq j.$$

Of course, r_{8j} is zero for all j since status 8 (death) is absorbing. No constraints are placed on the 49 values of a_{ij}. The sex-adjustment parameter b_{ij}, values are constrained to three values, one value for $i > j$ (recovery), one value for $j=8$ (mortality), and one value for other combinations of i and j (impairment). The age slope parameters, c_{ij}, are constrained to five values, the same structure used for sex adjustment with distinct values provided when $i=1$ (well). The resulting model contains 57 parameters.

The likelihood function for the observed data corresponds to 84 independent multinomial processes (12 matrices each with 7 starting health statuses). The probability vectors for each multinomial depend upon the model parameters. Numerical optimization techniques are employed to extract the maximum likelihood parameter estimates. For each set of candidate parameter values, R matrices are computed for each of the 12 observation cohorts using the sex and average age of the cohort over the observation period. The corresponding transition probability matrices are obtained by matrix exponentiation. Log-likelihood values are computed from the row probabilities and the observed distribution of ending health statuses for each of the 84 multinomial processes and are summed. Numerical estimates of the derivatives of the log-likelihood function with respect to variations in each of the 57 parameters are used to obtain the next set of candidate parameter values. The process is repeated to convergence.

Initial parameter estimates are obtained by combining the 12 observation matrices into a single matrix, ignoring the sex and age differences. The b and c parameters are set to zero. Initial values of the a parameters are set arbitrarily to 0.1. Maximum likelihood estimates of the a parameters for this constrained model are obtained using the procedure of the previous paragraph applied to the seven collapsed multinomial cohorts. The resulting a values, along with zero b and c values, are used as the initial parameter estimates for the full-model estimation routine.

Table 2 shows the final parameter estimates. The likelihood ratio test for goodness of fit (comparing this model with an unconstrained multinomial with $12 \times 49 = 588$ parameters) indicates that the model discards some statistically significant behavior exhibited by the data. Inspection of the model residuals does not suggest any obvious adjustments to the model structure, however, and I have elected to limit the model to the 57 parameters shown in Table 2. From a practical point of view, the model produces fitted transition probabilities reasonably close to the observed values. Table 3 shows the fitted values corresponding to the observed transition probabilities in Table 1. The last section of Table 3 also shows the annual forces of transition derived for this cohort from the parameter estimates.

Applications of the Model

In general terms, multistatus migration models have several possible actuarial applications. Such models might be used to explain life insurance policy lapsation, reinstatement, and mortality. Similarly, flexible premium annuity or universal life premium payment statuses ranging from "actively paying" to "paying minimum" to "paid-up" could be modeled. The evolution of insurance claim statuses from incurred-but-not-reported through closure and reopening offers another application. Of course, the most obvious applications are associated with private and public insurance programs in which benefit levels are tied to insured health status, such as disability income and LTC insurance policies.

A general multistatus insurance policy might pay a prescribed benefit while the insured is in a certain status, that is, a status-based annuity benefit or a benefit on entry to a new status, for example, death benefits and cash surrender values. If the benefit structure is simple, we can express actuarial present values for these benefits using the matrix notation of this paper.

Let $u_j(z)$ be the annual payment rate for an annuity payable if the insured is in status j at age z. Let $\boldsymbol{u}(z)$ denote the vector of status-based payment rates at age z.

Let $w_j(z)$ be the benefit paid if the insured jumps to status j at age z. Let $\boldsymbol{w}(z)$ denote the vector of benefit amounts at age z for each destination status.

Let $v(z)$ be the multiplicative scalar factor used to discount a payment at age z back to issue age x.

If the insured's status migration is described by a CTMC, we can compute the present value at issue of benefits from age x to age y as

TABLE 2
LTC TRANSITION RATE PARAMETER ESTIMATES

Starting Status, i	Ending Status, j: 1	2	3	4	5	6	7	8
				a_{ij}				
1	—	−2.53	−5.49	−9.17	−5.18	−4.45	−9.12	−3.56
2	−2.98	—	−1.88	−3.75	−4.99	−2.72	−3.99	−2.21
3	−3.87	−0.40	—	−2.11	−1.92	−2.33	−9.21	−1.66
4	−9.23	−3.28	−1.00	—	−1.12	−3.42	−1.97	−2.03
5	−9.32	−5.05	−4.23	−2.40	—	−9.22	−1.92	−1.19
6	−5.37	−2.24	−1.95	−3.11	−3.51	—	−1.34	−2.78
7	−7.07	−9.23	−4.67	−9.21	−1.11	−3.02	—	−1.10
				b_{ij} (sex adjustment factors)				
1	—	−0.062	−0.062	−0.062	−0.062	−0.062	−0.062	−0.570
2	−0.041	—	−0.062	−0.062	−0.062	−0.062	−0.062	−0.570
3	−0.041	−0.041	—	−0.062	−0.062	−0.062	−0.062	−0.570
4	−0.041	−0.041	−0.041	—	−0.062	−0.062	−0.062	−0.570
5	−0.041	−0.041	−0.041	−0.041	—	−0.062	−0.062	−0.570
6	−0.041	−0.041	−0.041	−0.041	−0.041	—	−0.062	−0.570
7	−0.041	−0.041	−0.041	−0.041	−0.041	−0.041	—	−0.570
				c_{ij} (age slopes)				
1	—	9.51	9.51	9.51	9.51	9.51	9.51	5.23
2	−4.73	—	3.11	3.11	3.11	3.11	3.11	3.35
3	−4.73	−4.73	—	3.11	3.11	3.11	3.11	3.35
4	−4.73	−4.73	−4.73	—	3.11	3.11	3.11	3.35
5	−4.73	−4.73	−4.73	−4.73	—	3.11	3.11	3.35
6	−4.73	−4.73	−4.73	−4.73	−4.73	—	3.11	3.35
7	−4.73	−4.73	−4.73	−4.73	−4.73	−4.73	—	3.35

Note: $T_{ij}(s, x) = \exp[a_{ij} + b_{ij}(s - 0.5) + c_{ij}(x - 80)/100]$

$$\boldsymbol{PV} = \int_x^y v(z)\,(P(s, x, z)\,\boldsymbol{u}(z) + P(s, x, z)\,R^*(s, z)\,\boldsymbol{w}(z))\,dz.$$

The matrix $R^*(s, z)$ is equal to the matrix $R(s, z)$ with the diagonal elements replaced by zeros. Note that $\boldsymbol{PV}$ is a vector with rows corresponding to the starting status of the insured at age x. If we set $v(z)=1$, $\boldsymbol{w}(z)=0$, and $\boldsymbol{u}(z)=1$ for status j and zero otherwise, then $\boldsymbol{PV}$ is the expected holding time in status j between ages x and y. If we set $v(z)=1$, $\boldsymbol{u}(z)=0$, and $\boldsymbol{w}(z)=1$ for status j and zero otherwise, then $\boldsymbol{PV}$ becomes the expected number of visits to status j between ages x and y.

If the CTMC and the vectors $\boldsymbol{u}(z)$ and $\boldsymbol{w}(z)$ are time homogeneous on the age interval $[x, y]$ and the force of interest is constant and greater than zero, then there is a closed form expression for $\boldsymbol{PV}$. Let the annual force of interest be denoted as δ. The conditions on $R(s, z)$ as a valid transition matrix (nonnegative off-diagonal elements and rows that sum to zero) ensure that the eigenvalues of $R(s, z)$ are all negative or zero. It can be demonstrated that

$$\boldsymbol{PV} = [R(s, x) - \delta I]^{-1}\,[v(y)\boldsymbol{P}(s, x, y) - I] \times [\boldsymbol{u}(x) + R^*(s, x)\boldsymbol{w}(x)].$$

The inverse on the right-hand side is well defined because $\delta > 0$ and the eigenvalues of $R(s, x)$ are not positive.

Although these expressions are interesting, they are not particularly useful in practice because of the strong constraints on $\boldsymbol{u}(z)$ and $\boldsymbol{w}(z)$. Most insurance policies contain benefit limitations that truncate benefits after an episodic or lifetime limit is reached. Other policy features such as elimination periods (episodic and lifetime) further complicate the benefit structure. So even when the status transition process can be adequately represented by CTMC, the complexity of the benefit structure will prevent us from obtaining a simple expression for the actuarial present value of benefits.

Table 3
NLTCS Observed and Fitted Health Status Transition Matrices
Females 75–84, 1982–84
(Average Age: 78.7)

	1984 Health Status								
1982 Health Status	Count	1	2	3	4	5	6	7	8
Observed Values									
Well	2,122	77.4%	11.0%	1.4%	0.4%	1.2%	2.2%	0.7%	5.7%
IADL Only	804	8.5	54.7	7.1	2.1	3.2	6.5	2.2	15.7
1 ADL	303	6.8	43.3	11.1	2.1	5.1	6.2	4.6	20.8
2 ADLs	72	0.0	22.4	13.3	8.2	17.4	3.1	3.9	31.7
3+ ADLs	53	0.0	11.0	4.0	11.1	24.4	2.0	15.0	32.5
< 2 ADLs, CI	168	2.7	19.1	3.9	4.0	11.0	28.2	12.8	18.3
2+ ADLs, CI	85	0.9	1.5	2.5	0.0	21.2	3.3	38.0	32.6
Fitted Values									
Well		79.8%	10.0%	1.3%	0.3%	1.0%	1.7%	0.6%	5.3%
IADL Only		6.7	53.5	8.3	2.4	3.6	6.1	3.2	16.2
1 ADL		4.0	34.8	14.7	4.5	9.5	6.9	4.1	21.5
2 ADLs		1.1	13.0	10.6	17.0	21.3	3.9	9.4	23.8
3+ ADLs		0.2	2.2	2.2	4.9	43.8	0.9	10.5	35.4
< 2 ADLs, CI		1.5	13.9	7.2	3.3	9.8	31.1	16.1	17.1
2+ ADLs, CI		0.2	1.3	1.3	1.8	23.2	3.1	31.9	37.3
Annual Transition Rates									
Well		—	7.5%	0.4%	0.0%	0.5%	1.1%	0.0%	2.1%
IADL Only		5.0	—	14.6	2.3	0.7	6.3	1.8	8.2
1 ADL		2.1	66.5	—	11.6	14.0	11.4	0.0	14.2
2 ADLs		0.0	3.8	36.4	—	31.4	3.1	13.4	9.8
3+ ADLs		0.0	0.6	1.4	9.1	—	0.0	14.0	22.8
< 2 ADLs, CI		4.6	10.6	14.2	4.4	3.0	—	25.0	4.6
2+ ADLs, CI		0.1	0.0	0.9	0.0	32.8	4.9	—	24.9

The LTC status transition model discussed in this paper was used to simulate monthly insured health status histories. A second-stage model was then applied that simulated LTC service utilization and policy benefit payments for each health status simulant. The approach allows for a very flexible benefit structure. Claim episodes are summarized to provide simulated claim costs, incidence rates, and claim termination rates. These values are graduated to provide input to pricing and valuation formulas.

Summary

Status transition models provide a means of extending traditional actuarial models to a much richer and more complicated class of processes, such as the evolution of an individual's health profile. The properties of such processes as the time-homogeneous continuous-time Markov chain examined in this paper have been studied extensively in the literature, but this wealth of information has been applied only sparingly to date by the actuarial profession. This paper demonstrates one application of these models that the author has found very helpful in analyzing private and public LTC benefit programs. Much remains to be studied and explored in this and other applications.

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